XXXII. *On the Construction of the New Imperial Standard Pound, and its Copies of Platinum; and on the Comparison of the Imperial Standard Pound with the Kilogramme des Archives.* *By* W. H. MILLER, *M.A.*, *F.R.S.*, *Professor of Mineralogy in the University of Cambridge.*

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History of the Standards of English Weight.

THE earliest legal standard of English weight, of which any very authentic account is preserved, is the weight called the pound of the Tower of London. According to FOLKES*, it was the old pound of the Saxon Moneyers before the Conquest. This pound was lighter than the troy pound by three-quarters of an ounce troy, and did not very sensibly differ from twelve ounces of the weight still used in the money affairs of Germany, and there known by the name of the Cologne weight. It is most probable that the pound of the Tower of standard silver was then cut up into 240 pennies; whence the weight of the penny will be 22·5 troy grains. The silver pennies of the first two kings after the Conquest agree, as near as can be judged, in weight and goodness, with the pennies of the Saxon kings their immediate predecessors. It is therefore reasonable to think that King William introduced no new weight into his Mints. CLARKE, in his Treatise on the connexion of Roman, Saxon and English Coins, p. 97, considers this evident from the words of William I.: 'Statuimus et præcipimus, quod habeant per universum regnum mensuras fidelissimas, et signatas; et pondera fidelissima, et signata, sicut boni prædecessores nostri statuerunt.' And also (p. 152) from one of the Conqueror's laws, where it is said, that the Saxon shilling was four pence (from the time of Athelstane), the preamble of which informs us, that these laws were in force during the Confessor's reign: 'Ice les meismes, que le Reis Edward sun Cosin tint devant lui.'

That the Tower pound was lighter than the troy pound by three-quarters of an ounce troy, appears by a verdict relating to the coinage dated 30th October, 1527, 18 Hen. VIII., in the Exchequer, in which are the following words: 'And whereas heretofore the merchaunte paid for coynage of every pounde Towre of fyne gold, weighing xi oz. quarter Troye, ii s. vi d. Nowe it is determyned by the King's highness, and his said councelle, that the foresaid pounde Towre shall be no more

* Table of English Coins, p. 1.

used and occupied, but al maner of golde and sylver shall be wayed by the pounde Troye, which maketh xii oz. Troye, which excedith the pounde Towre in weight iii quarters of the oz.' Hence it follows that the weight of the Tower pound was 5400 troy grains, and that of the ounce or the twelfth part thereof 450 like grains. He quotes a passage taken from the Register of Accounts in Paris, to prove that the Tower pound was also known in France, where it was called the Rochelle or English weight. The difference of the several pounds then made use of in France is there computed, and the proportion between the troy and English weights is thus estimated: 'Ou royaume souloit avoir iv marcs: c'est assavoir le marc de Troyes, qui poise xiv sols, ii den. Esterlins de poix.... le marc de la Rochelle, dit d'Angleterre, qui poise xiii s. iv den. Esterlins de poix.' It is supposed that this account was taken about the beginning of the reign of Edward III., not long after 1329*. Since the sol=12 esterlings, the ratio of the standards of Troyes and Rochelle is as 17 to 16; whence, supposing the weight of Troyes to be the same as the English troy weight, the Rochelle ounce =451·76 troy grains. He refers to a statute of the 51st of Henry III., called 'Assisa panis et cerevisiæ,' to show that the weights in use at that time, though commonly taken to have been troy weights, were not really so, but the money weights: 'By consent of the whole realm of England the king's measure was made, that is to say that an English pennie, which is called a sterling, round without clipping, shall weigh xxxii graines of wheat dry in the middest of the eare; and xx pence make an ounce, and xii ounces make a pound.' For otherwise the pennyweight here described, could never be, as the statute plainly implies, the true weight of the English coined penny.

Folkes determined the weights of a number of silver coins well preserved, or but little impaired, in troy grains (p. 159). Five pennies of Henry III. weighed 22·5 grains each, and one 22·25 grains. Of four pennies of Edward I., two weighed 22·5 grains each, and two others 22 grains each. Assuming the true weight of the penny at this time to be 22·5 grains, which was also that of the Saxon penny†, the weight of the pound will be 5400 grains. It is, however, just possible that the weights were adjusted in conformity with the words of the Act, but that the coin called the sterling fell 1·5 grain short of the full weight of 32 corns of wheat, or 24 grains troy, the weight of 4 corns of wheat being usually considered equivalent to 3 grains troy. On this supposition the pound defined in the statute of the 51st of Henry III., and in the 31st of Edward I., in precisely the same words, would be the pound of 5760 troy grains.

That another pound, the libra mercatoria, was in use at this time, is shown by the following extract from the Treatise on Arithmetic, by Dr. Peacock, in the Encyclopædia Metropolitana, Art. 166:—'Though this weight was the favourite of the legislature, there was another pound, one-fourth greater, which was in more general

* Clarke, p. 15.

† Ibid. p. 428.

use; it is mentioned in a Tractatus de Ponderibus of the same age (the time of Edward I.), where the two pounds are said to consist of 20 and 25 shillings respectively: in the statute of the 54th of Henry III., where the composition of the gallon and pound (troy ?) are given, there is mentioned also una libra, pondus vigintiquinque solidorum legalium sterlingorum. On many other occasions this libra mercatoria is referred to, and we may consider its use in mercantile transactions and ordinary sales as nearly universal.'

If the pound mentioned in the 51st and 54th of Henry III., and the 31st of Edward I., be supposed to contain 5400 grains troy, the libra mercatoria will contain 6750 grains, which does not differ very much from the old pounds of Villefranche (6741·9 grains), Zieriksee (6736 grains), Dresden stahlgewicht (6726·2 grains), Dantzig (6722 grains), Embrun (6714 grains), Murcia (6711 grains). This supposition is rendered probable by a passage from Fleta quoted by Clarke, p. 96, who says, 'Quindecim unciæ faciunt libram mercatoriam.' Fifteen Tower ounces of 450 troy grains, twelve of which make the Tower pound, are equal to 6750 troy grains. Sixteen of these ounces make 7200 troy grains, a weight which approaches very closely to the Ptolemaic mina of 7199·96 troy grains, the 100th part of the large Alexandrian talent, also divided into 16 ounces. This weight appears to have survived in the old pounds of Namur (7201·1 grains), Altenburg (7202 grains), Ciney (7202·5 grains), Valenciennes (7195·2 grains), Duerstadt (7206·1 grains), Wittenberg (7207 grains), Heidelberg (7207·22 grains), Aix la Chapelle (7208 grains), Liege (7209·1 grains), Bruchsal (7190 grains), Brunswick (7212·3 grains), Mons (7185·2 grains), Dresden (7215·4 grains), Binche (7185 grains), Gotha (7213·85 grains), Jemappe (7185·2 grains), in the well-known Cologne pound of 7216 grains, and in many others differing rather more largely from 7200 grains. If, on the other hand, the pound of Henry III. and Edward I. contained 5760 troy grains, the libra mercatoria would weigh 7200 troy grains.

In the Acts of the 2nd of Henry V. st. 2. c. 4, and of the 2nd of Henry VI. c. 13, relating to Goldsmiths, mention is made of the 'Pound Troy*.' Either the Tower pound was abolished, or the use of the troy pound as a legal standard confirmed in 1498, the 12th of Henry VII. A statute made in that year enacts 'That every gallon contain viii li of wheat of Troy weight....and every pound xii ounces of Troy weight, and every ounce contain xx sterlings.' The sterling here mentioned must have been a weight, and not the coin of that name; for, during the reign of Henry VII., the weight of the groat was 48 grains, and that of the shilling 144 grains, which gives only 12 grains for the weight of the coin called the penny or sterling†.

The troy pound appears to have been derived from the Roman weight of 5759·2 grains, the 125th part of the large Alexandrian talent, and which, like the troy

* Reynardson, Philosophical Transactions, vol. xlvi. p. 61. † Folkes, p. 16.

pound, was divided by the Romans into 12 ounces. So also the avoirdupois pound was probably derived from the large Attic mina of 6945·3 grains troy, the 60th part of the large Attic talent, divided by the Romans, as the pound avoirdupois is divided, into 16 ounces, of nearly the same weight as the modern Roman ounce*. According to GREAVES, it approximates much more closely to the ancient Roman ounce.

The word 'avoirdepois,' applied to commodities, occurs in statutes of the 9th and 27th of Edward III. By a statute of the 24th of Henry VIII., butchers were obliged to provide themselves with beams, scales and weights sealed, called 'haberdepois.' It is not known when the avoirdupois weight was first introduced. Two weights, one of which, in its present state, is about 650 grains less than seven pounds avoirdupois, in the possession of C. C. BABINGTON, Esq., of St. John's College, Cambridge, are marked with a crowned H, which is supposed to be of the time of Henry VII. Two weights, one of 2 pounds, the other of 4 pounds avoirdupois, ornamented with the Tudor rose, and marked with the letter H, and therefore probably of the reign of either Henry VII. or Henry VIII., are preserved in the University Library, Cambridge. In the reign of Elizabeth avoirdupois weights were deposited in the Exchequer, as appears from the name and inscription thereon. The Transactions of the Royal Society, for 1742 and 1743 (vol. xlii. p. 541), contain an account of the comparison of various standards of measure and weight, from which the following extracts are made:—'The weights in His Majesty's Exchequer, and which are esteemed the standards, are a pile, or box, of hollow brass weights, from 256 ounces (troy) downwards, to the 16th part of an ounce, all severally marked with a crowned E.' 'The weight mentioned in all our old Acts of Parliament, from the time of King Edward I., is universally allowed to be the troy weight, whose pound consisted of 12 ounces, each of which contained 20 pennyweights. And as the pound is the weight of the largest single denomination commonly mentioned in those Acts, 12 ounces taken from the pile of troy weights above mentioned, as there is no single troy pound weight, must now be reputed to be the true standard of the troy pound, used at this day in England.

'Besides which troy standards, there are also kept in the Exchequer the following standards for averdupois weights; that is to say, a fourteen pound bell-weight of brass, marked with a crowned E, and inscribed

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As also a seven pound, a four pound, a two pound, and a single pound, like averdupois bell-weights, and severally marked as follows, excepting the variations for the number of pounds in each respective weight.'

(The marks are: VII. A., AN° DO, a crowned E. L., 1588, A° REG. XXX.)

* DOURSTHER, Dictionnaire des Poids et Mesures, p. 281.

'With which are also kept a pile of flat averdupois weights, from 14 pounds to the 64th part of a pound.'

The comparisons were made with considerable care, the weights being interchanged so as to eliminate the error produced by the inequality of the arms of the balance.

The results, in ounces troy, and grains of which the ounce contains 480, are—

14 lb.+1 lb. (bell-weights)=218 oz. 335·25 grains=104975·25 grains.
7 lb. (bell-weight)=102 oz. 45 grains=49005 grains.
1 lb. (bell-weight)= 14 oz. 282 grains= 7002 grains.
1 lb. (flat weight) = 1 lb. (bell-weight)−2·5 grains=6999·5 grains.

In the year 1758, a Committee of the House of Commons, appointed to inquire into the standards of weight and measure, recommended that the troy pound should be made the unit or standard by which the avoirdupois and other weights should be regulated. By order of the Committee, three several troy pounds were adjusted with great care, under the direction of Mr. Harris, the then Assay Master of the Mint, by a mean of a great number of comparisons with the old weights in the Exchequer. The details of the comparisons are given at length in the Report of the Committee, presented by Lord Carysfort on the 26th of May, 1758. One of these weights (the imperial standard troy pound which was destroyed by the burning of the Houses of Parliament in 1834) was placed in the custody of the Clerk of the House of Commons. The bill, however, brought in by Lord Carysfort in 1760 to give effect to the recommendations of the Committee, in the pressure of business attending the death of George II. and the accession of George III., and the dissolution of Parliament soon following, was not carried through all its stages.

In the year 1818, Sir Joseph Banks, Sir George Clerk, Mr. Davies Gilbert, Dr. W. H. Wollaston, Dr. Thomas Young, and Captain Kater, were appointed Commissioners for considering how far it might be practicable to establish a more uniform system of weights and measures. In that part of their Report which relates to the subject of Weights, they recommend that the Parliamentary standard of 1758 should remain unaltered: they state that the avoirdupois pound, which has long been in general use, though not established by any act of the Legislature, is so nearly 7000 troy grains, that they recommend that 7000 such troy grains be declared to constitute a pound avoirdupois: also, that from Sir George Shuckburgh's weighings of a cube, cylinder and sphere in air and in water in 1797, and Captain Kater's measurements of the linear dimensions of the same in 1821, they determined the weight of a cubic inch of distilled water, weighed in air by brass weights, at the temperature of 62° Fahr., the barometer being at 30 inches, to be equal to 252·458 grains, of which the Imperial standard troy pound contains 5760.

The chief recommendations of this Committee passed into law by an Act of Par-

liament on the 17th of June, 1824. In the fourth clause of this Act it is enacted, that the old troy pound of 1758, now in the custody of the Clerk of the House of Commons, shall continue to be the original unit or only standard of weight from which all other weights shall be derived; and that it is to be denominated 'The Imperial Standard Troy Pound;' and that the avoirdupois pound, now in use, shall contain 7000 grains, of which the troy pound contains 5760. In the sixth clause it is enacted, that if the standard troy pound should be lost or destroyed, it is to be restored by a reference to the weight of a cubic inch of distilled water, which has been found and is declared to be 252·458 troy grains, weighed in air with brass weights, at the temperature of 62° FAHR., the barometer being at 30 inches.

The Imperial standard troy pound was compared with five troy pounds of gunmetal, destined for the use of the Exchequer, the Royal Mint, and the cities of London, Edinburgh and Dublin, by Captain KATER in 1824 or 1825*. Denoting the standard troy pound by U, and the troy pounds of the Exchequer, the cities of London, Edinburgh, Dublin, and the Royal Mint, by Ex, L, Ed, D, RM respectively, it was found that

No. of Comps.	grain.
16	Ex $=U+0{\cdot}0010$
12	L $=U+0{\cdot}0005$
15	Ed $=U-0{\cdot}0015$
18	D $=U+0{\cdot}0022$
20	RM $=U+0{\cdot}0021$

In the year 1829 the standard troy pound was compared with extraordinary care by Captain v. NEHUS with two brass troy pounds and a platinum troy pound, all in the custody of Professor SCHUMACHER, and with a platinum troy pound, the property of the Royal Society.

Let Sb, K denote the two brass troy pounds, Sp the platinum troy pound in the custody of Professor SCHUMACHER, RS the platinum troy pound the property of the Royal Society, t the temperature of the air in degrees of FAHRENHEIT's scale, b the height of the mercury in the barometer in English inches, and reduced to the temperature of melting snow. Let Δ prefixed to the symbol of any weight denote the ratio of the density of the weight at the temperature of melting snow to the maximum density of water. The symbol $\triangleq$ placed between the symbols of two weights will be used to denote that they appear to be equal when weighed in air. The two weights in this case will not be equal unless their volumes are equal. When the weighings have been made in air of constant density, or have been reduced to what they would have been in air of given density; or when, the volumes of the weights and the temperature and pressure of the air being unknown, we are compelled to assume the equality of their volumes, the symbol $=$ may be substituted for $\triangleq$.

* Philosophical Transactions for 1826, Part II. p. 18.

According to the observations of Captain v. NEHUS,—

No. of Comps.	gr.	b.	t.
300	Sp $\triangleq$ U−0·00857	29·722	65·62
140	RS $\triangleq$ U−0·00205	29·806	65·73
60	Sb $\triangleq$ U−0·01034	29·965	64·50
92	K $\triangleq$ U+0·03389	29·646	65·09
16	RM $\triangleq$ U+0·00887	29·679	65·91

10−log ΔSp=8·67392, 10−log ΔRS=8·67392, 10−log ΔSb=9·08471, 10−log ΔK=9·09724 *.

In the burning of the Houses of Parliament in 1834, all the standards of measure and weight were either totally destroyed, or injured to such an extent as to render them quite useless as standards. The Imperial standard troy pound was never recovered from the ruins.

In the year 1838, the Astronomer Royal, Mr. F. BAILY, Mr. J. E. D. BETHUNE, Mr. DAVIES GILBERT, Sir J. F. W. HERSCHEL, Mr. J. S. LEFEVRE, Mr. J. W. LUBBOCK, the Rev. GEORGE PEACOCK, and the Rev. R. SHEEPSHANKS, were appointed Commissioners to consider the steps to be taken for the restoration of the standards of weight and measure, to replace those which were destroyed by the burning of the Houses of Parliament. They found provisions for the restoration of the lost standards prescribed to them by Sections 3 and 5 of the Act 5th George IV., whereby it is directed that, in case of the loss of the standards, the yard shall be restored by taking the length which shall bear a certain proportion to the length of the pendulum, vibrating seconds of mean time in the latitude of London, in a vacuum, at the level of the sea; and that the pound shall be restored by taking the weight which bears a certain proportion to the weight of a cubic inch of water weighed in a certain manner. The Commissioners, however, in their Report, dated December 21, 1841, decline to recommend the adoption of these provisions for the following reasons: ‘Since the passing of the said Act, it has been ascertained that several elements of reduction of the pendulum experiments therein referred to are doubtful or erroneous.... It is evident, therefore, that the course prescribed by the Act would not necessarily reproduce the length of the original yard. It appears also that the determination of the weight of a cubic inch of water is yet doubtful (the greatest difference between the best English, French, Austrian, Swedish and Russian determinations being about $\frac{1}{1200}$ of the whole weight, whereas the mere operation of weighing may be performed to the accuracy of $\frac{1}{1,000,000}$ of the whole weight). Several measures, however, exist, which were most carefully compared with the former standard yard; and several metallic weights exist, which were most accurately compared with the former standard pound; and by the use of these, the values of the original standards can be respectively restored without sensible error. And we are fully persuaded that, with reasonable precautions, it will always be possible to provide for the accurate restoration of standards by means

* SCHUMACHER, Philosophical Transactions for 1836, p. 437.

of material copies which have been carefully compared with them, more securely than by reference to experiments referring to natural constants.'

The weight of a given volume of water at 62° FAHR., by means of which the Act of Geo. IV. directs the pound to be restored, was deduced from the weighings in air and in water of a brass cube of 5 inches, of a cylinder of 4 inches diameter and 6 inches long, and of a sphere of 6 inches diameter, by Sir GEORGE SHUCKBURGH in 1797, and the measurements of their linear dimensions by Captain KATER in 1821. The resulting values of the weight of a cubic inch of water at 62° FAHR. in vacuum, in grains of which the lost standard pound contained 5760, were,—by the cube 252·741, by the cylinder 252·685, by the sphere 252·741. Mean 252·722*. Similar observations had been made in France by MM. LEFÈVRE-GINEAU and FABBRONI for the purpose of establishing the value of the kilogramme, which was intended to be the weight of a cubic décimètre of water at its maximum density, in a vacuum: the solid used on this occasion was a cylinder the diameter and axis of which were nearly 243·5 millimètres each†. In Sweden, MM. BERZELIUS, SVANBERG and AKERMANN, who employed a cylinder 4 inches in diameter and 6 inches long, found the weight of a cubic décimètre of water, at 62° FAHR. in a vacuum, to be 2·350595 Swedish pounds‡. In Austria, Professor STAMPFER, who used a cylinder of about 3·11 inches diameter and 3·11 inches long, found the weight of a Vienna cubic inch of water, at 62° FAHR. in a vacuum, equal to 18·2492 grammes§. Lastly, in Russia, Professor KUPFFER has determined the weight of an English cubic inch of water in vacuum at 62° FAHR., in doli of which a kilogramme contains 22504·86, to be 368·380 by a cylinder the axis and diameter of which were nearly 80 millimètres each, and 368·341 by a cylinder the axis and diameter of which were 4 English inches each. Mean 368·361 doli. At the end of the work entitled 'Travaux de la Commission pour fixer les Mesures et les Poids de l'Empire de Russie,' Professor KUPFFER has collected the different results expressed in doli. The English observations are affected by a small error arising from the uncertainty of the value of Professor SCHUMACHER's troy pound K, which was used by Professor KUPFFER in finding the relation between the English and French standards of weight. This error, however, is quite insignificant compared with the differences between the results obtained by the several observers.

French observations	368·365
English observations	368·542
Swedish observations	368·474
Austrian observations	368·237
Russian observations	368·361

* Philosophical Transactions for 1821, Part I. p. 326.

† Base du Système Métrique, t. iii. p. 558.

‡ Memoirs of the Royal Academy of Stockholm, 1825.

§ Jahrbücher des k. k. Polytech. Institutes zu Wien, B. 16, S. 53.

Assuming the Russian observations to be the most accurate, as they probably are, it will be seen that even if we leave entirely out of the question the injurious effect of the error likely to arise in establishing the standard of length, a troy pound deduced according to the method prescribed by the Act would be 2·829 grains too heavy; while, if the Austrian observations had been accepted as the best, the troy pound would have been 4·707 grains too heavy. On the other hand, it was possible to recover the weight of the lost standard in air to within a fraction of 0·001 grain, by means of the troy pounds which had been compared with it, and could be easily brought together for recomparison. Seeing, then, that the error of one of these two methods of restoring the lost standard is at least 2829 times as large as the error of the other method, the Committee could not hesitate to recommend the adoption of the latter. The Commissioners recommend also that the avoirdupois pound, being universally used through the kingdom, while the troy pound is wholly unknown to the great mass of the population, be adopted as the standard of weight; that the troy pound be no longer recognized; and that the use of the troy ounce be confined to gold, silver, and precious stones.

In the year 1843 a Committee was appointed to superintend the construction of the new Parliamentary standards of length and weight, to replace those which were destroyed by the burning of the Houses of Parliament. The members of this Committee were, the Astronomer Royal, the Marquis of Northampton, the Earl of Rosse, the Lord Wrottesley, Sir J. W. Lubbock, Bart., Sir J. F. W. Herschel, Bart., the Rev. G. Peacock, Dean of Ely, the Rev. R. Sheepshanks, F. Baily, Esq., J. E. D. Bethune, Esq., J. G. S. Lefevre, Esq., and Professor W. H. Miller. To the last of these was intrusted the construction of the new standards of weight.

The evidence for ascertaining the weight of the lost Standard Troy Pound, placed at the service of this Committee, consisted of the following weights:—The brass troy pound of the Exchequer Office. The brass troy pounds from the cities of London, Edinburgh and Dublin. The platinum troy pound and the two brass troy pounds then in the custody of Professor Schumacher. The platinum troy pound of the Royal Society. The troy pound used by the late Mr. Robinson of Devonshire Street, Portland Place, purchased by the Committee. Two troy pounds, formerly in the possession of Mr. Bingley (one of which, lately in the possession of Stansby Alchorne, Esq., of the Royal Mint, has been purchased by the Committee). The troy pound formerly the property of Mr. Freeman, now the property of Messrs. Vandome and Titford. To these has very recently been added a troy pound the property of the Bank of England.

The results of the comparisons of the troy pounds of the Exchequer Office, of the cities of London, Edinburgh and Dublin, and of the three troy pounds in the custody of Professor Schumacher, and the troy pound of the Royal Society, with the lost standard, have already been given. Mr. Robinson's troy pound is also said to have been compared by Captain Kater, but no record has been discovered of the

comparison. Mr. VANDOME's pound, Mr. BINGLEY's troy pounds, and the Bank of England troy pound, were all constructed, along with the lost standard, in 1758 by Mr. HARRIS, Assay Master of the Mint. These were referred to, at the suggestion of Professor SCHUMACHER, in the hope of arriving at a knowledge of the volume of the lost standard, which unfortunately had never been determined by weighing it in water. For, as long as the volume of the lost standard remains unknown, the weight of the air displaced by it, and, consequently, its absolute weight, is uncertain within limits far exceeding the errors of weighing.

The first step in the process of arriving at the weight of the lost standard, was obviously to compare among themselves the different troy pounds with which the lost standard had been compared by Captain KATER and Captain v. NEHUS. These comparisons were made with a balance of extreme delicacy procured from Mr. BARROW. In its construction it nearly resembles the balances of the late Mr. T. C. ROBINSON. The beam is made sufficiently strong to carry a kilogramme in each pan. The middle knife-edge is about 1·93 inch long, and rests, when the balance is in action, throughout its whole length on a single plane surface of quartz. The surfaces of quartz which rest upon the extreme knife-edges, and from which the pans are suspended, are also plane. The distance between the extreme knife-edges is about 15·06 inches, the length of each about 1·05 inch.

Instead of having an index pointing downwards, as is usual in balances of this description, the beam has a pointer at each end, and a graduated scale is carried by an arm attached to the pillar of the balance at a little distance behind the left-hand pointer. Affixed to the right-hand end of the beam is a thin slip of ivory, a little more than half an inch long, divided into spaces of about 0·01 inch each, or subtending an angle of about 5′ each at the middle knife-edge. This scale is viewed through a compound microscope, having a single horizontal wire in the focus of the eye-piece. The distance between two divisions of the scale, as seen through the microscope, subtends an angle of about 37′. This contrivance for determining the position of the beam at the extremity of an oscillation, was found so superior to a scale and pointer viewed with the naked eye, that after a trial of a few days, the scale at the left hand was found to be a useless incumbrance and was accordingly removed. A screen was interposed between the observer and the front of the balance case, having a small opening opposite to the eye-piece of the microscope, through which the scale could be seen.

In order to admit of the employment of a large vessel of water in observations for finding specific gravities, the base of the balance has an opening immediately under the right-hand pan, capable of being closed when not in use by a sliding plate of brass. A corresponding opening exists in the table on which the balance stands. The vessel of water is placed under the table, and the wire by which the object to be weighed in water is suspended from a hook under the right-hand pan, passes through the openings in the base of the balance under the table.

In the comparisons of weights, I at first employed the method of weighing invented by the Père AMIOT, more commonly known as BORDA's*. The counterpoise was invariably placed in the left-hand pan, and the weights to be compared alternately in the right-hand pan. The reading of the divided scale was noted at the end of each of three consecutive oscillations. One-fourth of (first reading+third reading+2 second reading) was taken as the reading of the scale in the position of equilibrium of the balance.

In all the more important weighings the reading of the scale was noted at the end of each of four consecutive oscillations, of which the last three only were used in finding the reading corresponding to the position of equilibrium of the beam. The first reading is apt to exhibit small irregularities, especially when it follows very soon after the interchange of the weights. Hence the employment of it in finding the position of equilibrium would not be likely to increase the accuracy of the result. The observation of an additional reading is not, however, without its use; for by comparing the first with the third, as well as the second with the fourth, the error of an integer in either of the readings, if it occurred, would be instantly detected.

Let P, Q be the apparent weights in air of two bodies P, Q, either of which in the right-hand pan is nearly in equilibrium with the counterpoise C in the left-hand pan; (C, P), (C, Q) the scale readings in the position of equilibrium of the balance when P, Q respectively are in the right-hand pan; and let m be the weight equivalent to one part of the scale, the readings increasing with an increase of the weight in the right-hand pan. Then $Q=P+m[(C,Q)-(C,P)]$.

Subsequently I used the method attributed to GAUSS†. Let P, Q be the apparent weights in air of two bodies P, Q, and (P, Q) the reading of the scale in the position of equilibrium of the balance, when P is in the left-hand pan, and Q is in the right-hand pan. Now let P be placed in the right-hand pan, and Q in the left-hand pan, and let P, Q become R, S respectively, by the addition of small weights, in order to bring the balance nearly into its former position of equilibrium. Let (S, R) be the reading of the scale in the position of equilibrium of the balance, when R is in the right-hand pan, and S is in the left-hand pan. Then, m being the weight equivalent to one division of the scale, the reading increasing with an increase of weight in the right-hand pan, $Q+S=P+R+m[(P,Q)-(S,R)]$.

When the weights P, Q are very nearly equal, the balance may be so adjusted by placing a small constant weight in one of the pans or hanging it on the beam, that, on interchanging the weights P, Q, the position of equilibrium may still be near the middle of the scale. Supposing the balance to be so adjusted, let (P, Q) be the reading of the scale in the position of equilibrium of the balance, when P is in the left-hand pan and Q is in the right-hand pan; and let (Q, P) be the reading of the scale

* PÉCLET, Cours de Physique, p. 48.

* STEINHEIL, Denkschriften der K. Akademie der Wissenschaften zu München für die Jahre 1844–46 B. iv. S. 222.

when the balance is in its position of equilibrium, with Q in the left-hand pan and P in the right-hand pan. Then $2Q=2P+m[(P,Q)-(Q,P)]$.

In making a large number of comparisons, the weights are exposed to the risk of being injured by wear. In order to obviate this danger, two light pans were procured of very nearly equal weight, each of which has a loop of wire forming an arch the ends of which are attached to the pan at opposite extremities of a diameter of the pan, by which the pan could be lifted with the hook at the end of a long handle, into or out of either of the pans of the balance. Calling the pans X and Y, and the weights to be compared P and Q, P was placed in X and Q in Y, and $P+X$ compared with $Q+Y$ n times; then P was placed in Y and Q in X, and $P+Y$ compared with $Q+X$ n times. The weights were thus exposed to the wear of two ordinary comparisons only in the course of $2n$ comparisons. The mean of the $2n$ comparisons gives the difference between P and Q, unaffected by the very small but unknown difference between the weights of the pans X and Y. This contrivance was found to be especially useful when either of the weights to be compared consisted of several parts.

An improvement upon this was made by using the pans of one of the balances employed by the French Pharmaciens, which resemble those above described, with the addition of an iron hook at the highest point of the wire loop. Either pan is suspended by a wire of suitable length bent into a hook at each end, from the ring attached to the agate-plane. In using the method of double weighing, the original left-hand pan of the balance was suffered to remain with the counterpoise in it, and the pan X containing the weight P, and the pan Y containing the weight Q, alternately suspended from the right-hand end of the beam, and the positions of equilibrium observed (usually about twenty times). The weights were then interchanged, and pan Y containing the weight P, and pan X containing the weight Q, suspended from the right-hand end of the beam, and the positions of equilibrium observed the same number of times. The weights of X and Y were frequently reduced in order to make them as nearly equal as possible, and sometimes in order to remove rust from the iron hooks.

In using GAUSS's method, it was desirable to be able to transfer the pans, and the weights contained in them, from one end of the beam to the other without opening the doors of the balance-case, and thus avoid sudden changes of temperature of air within the balance-case, and consequent production of currents of air. In order to effect this, various contrivances were tried. Of these the following proved the most successful. A slender brass tube, 38 inches long, passes freely through two holes in the ends of the balance-case, which is 22·75 inches long, near the top of the case, and half-way between the balance and the front of the case. To the middle of the tube is attached a descending loop of wire. Suppose that by sliding the rod, the loop is brought near to the right-hand end of the beam, and a pan with a weight in it transferred from the end of the beam to the wire loop by a brass rod having a deep groove filed round

it near the end, which is inserted through a hole in the middle of the right-hand end of the balance-case. By sliding the rod in the opposite direction, the loop with the pan and weight suspended from it, is brought near to the left-hand end of the beam, to which the pan is transferred by a brass rod passing through a hole in the left-hand end of the balance-case. Pins inserted in holes at each end of the tube at right angles to it, prevent it from being pushed too far. A similar tube half-way between the balance and the back of the case, serves to transfer the other pan and weight from one end of the beam to the other. In this manner any number of comparisons may be made without opening the balance-case, except in the middle of the series, for the purpose of changing the pans. The transfer of the pans and weights from one end of the beam to the other, might be effected still more conveniently by means of two coarse screws of the same length as the balance-case, turned by small winches at each end, and provided with loosely-fitting nuts with wire loops from which to suspend the pans and weights.

In the course of making the preliminary observations some peculiarities of the instrument were discovered, which, though they probably exist in other balances, do not appear to have been hitherto noticed. One of these is, that the expansion of one arm by heat, the left in the present case, is a little greater than that of the other arm. Hence, when the weights in the two pans are nearly equal and of equal volume, the reading of the scale in the position of equilibrium diminishes as the temperature of the beam increases. Another is, that the sensibility of the balance, as measured by the number of parts of the scale equivalent to a given weight, was found to diminish with an increase of temperature. The cause of this is obvious. The beam being of bronze and the knife-edges of steel, the balance-beam becomes an over-compensated pendulum, and an increase of temperature increases the distance between the middle knife-edge and the centre of gravity of the beam and weights, supposing the latter concentrated in the extreme knife-edges. Possibly also, the flexure of the beam may increase with the temperature, or the mean expansion of the upper bar of the beam may be greater than that of the under bar. The variation of the sensibility of the balance is so large, that it is necessary to determine the weight equivalent to a given number of parts of the scale for each set of observations, except in cases where the temperature is very nearly the same.

For the comparison of the smaller weights two excellent balances by ROBINSON were used, one having a beam 10·5, the other a beam 5·5 inches long. The reading of the scale of these balances increases on the addition of a small weight to the weight in the left-hand pan.

Comparisons of Sp, RS, Sb, K, Ex, L, Ed, R.

T is a platinum troy pound left a little in excess; P the sum of five weights of platinum making together a troy pound; *b* is the height of the mercury in the

barometer in inches, reduced to the freezing-point; t the temperature of the air in the balance-case in degrees of FAHRENHEIT's scale.

1844, March 29.		100 parts = 0·1810 grain.
gr.	gr.	gr. pt.
T=Sp+0·0147	T=RS+0·0092	RS=Sp+0·010−3·70
+0·0122	+0·0124	+0·010−2·60
+0·0181	+0·0128	+0·010−3·00
+0·0189	+0·0090	+0·010−3·15
+0·0217	+0·0085	+0·006+1·48
+0·0155	+0·0078	+0·006+0·60
+0·0147	+0·0072	+0·006+1·40
+0·0136	+0·0103	+0·006+0·25
+0·0161	+0·0108	+0·006+1·95
+0·0156	+0·0110	Mean RS=Sp+0·00640 grain.
+0·0124	+0·0081	
+0·0174	+0·0087	
+0·0209	+0·0138	
+0·0194	+0·0107	
+0·0175	+0·0111	
+0·0185	+0·0090	
Mean T=Sp+0·01670 grain.	Mean T=RS+0·01002 grain.	

The numbers in the columns headed Sp+X, P+Y, Sp+Y, P+X are the readings of the scale in the positions of equilibrium of the beam with the counterpoise in the left-hand pan, and the weights Sp+X, P+Y, Sp+Y, P+X respectively in the right-hand pan. The scale readings of the alternate weighings are arranged in separate columns for the sake of greater convenience in adding up the results.

June 18. 100 parts=0·22082 grain.

Sp+X.	P+Y.	Sp+Y.	P+X.
25·30	27·80	22·50	22·90
26·10	27·00	21·90	22·90
24·05	25·20	22·05	21·40
24·60	25·30	26·50	26·45
24·40	24·60	27·15	29·50
23·50	24·40	28·50	28·65
22·75	23·55	28·50	27·30
23·00	23·00	28·90	29·30
22·20	22·55	29·65	30·10
21·90	22·40	29·75	30·40
237·80	245·80	265·40	268·90

10(Sp+X)=10(P+Y)−8 parts. 10(Sp+Y)=10(P+X)−3·5 parts.

Mean Sp=P−0·00127 grain.

June 12. 100 parts =0·23641 grain.

RS+X.	P+Y.	RS+Y.	P+X.
19·60	18·60	20·70	21·20
20·20	18·10	21·10	19·00
20·00	18·70	21·60	19·30
20·00	18·50	20·80	19·50
19·30	19·10	21·30	19·40
19·85	18·00	20·80	18·70
118·95	111·00	126·30	117·10

6(RS+X)=6(P+Y)+7·95 parts. 6(RS+Y)=6(P+X)+9·2 parts.

Mean RS=P+0·00318 grain.

April 22, 23. 100 parts =0·14798 grain.

	gr.	pt.
Sp≏Sb	+0·01	−0·45
	+0·01	−0·45
	+0·01	−0·60
	+0·01	−1·90
	+0·01	−0·70
	+0·01	−0·95
	+0·01	−1·05
	+0·01	−3·10
	+0·01	−3·00
	+0·01	−2·90

Mean Sp≏Sb+0·00777 gr. in air (t=62·2, b=30·145).

April 24. 100 parts =0·14009 grain.

	gr.	pt.
K≏Sp	+0·027	−0·10
	+0·027	−1·15
	+0·027	−0·90
	+0·024	+0·20
	+0·024	+0·50
	+0·024	+0·25
	+0·024	−0·10
	+0·024	+0·30
	+0·024	−0·30

Mean K≏Sp+0·02479 gr. in air (t=62·2, b=30·186).

April 23. 100 parts =0·14798 grain.

	gr.	pt.
RS≏Sb	+0·016	−0·35
	+0·016	−1·45
	+0·016	−0·90
	+0·016	−1·45
	+0·016	−1·70
	+0·016	−1·10
	+0·016	−1·05
	+0·016	−1·10
	+0·016	−0·65
	+0·016	−1·70
	+0·016	−0·95

Mean RS≏Sb+0·01433 gr. in air (t=62·33, b=30·235).

	gr.	pt.
K≏RS	+0·016	+1·00
	+0·016	+1·85
	+0·016	+1·80
	+0·016	+1·70
	+0·016	+1·45
	+0·016	+2·00
	+0·016	+1·60
	+0·016	+1·45

Mean K≏RS+0·01837 gr. in air (t=62·33, b=30·196).

April 23. 100 parts=0·144 grain.

	gr.	pt.
K=Sb	+0·040	−4·95
	+0·030	+0·12
	+0·034	−2·37
	+0·030	+6·40
	+0·030	+3·80
	+0·030	+4·05
	+0·030	+3·45
	+0·030	+2·40
	+0·020	+8·20
	+0·030	+2·40
	+0·030	+0·95
	+0·030	+1·80
	+0·030	+0·70
	+0·030	+1·00

Mean K=Sb+0·03316 grain.

April 8. 100 parts=0·2244 grain.

	gr.	pt.
K=Sb	+0·031	+1·00
	+0·031	+1·65
	+0·031	+1·95
	+0·031	+1·80
	+0·031	+1·20
	+0·031	+1·40
	+0·031	+1·90

Mean K=Sb+0·03449 grain.

April 22. 100 parts=0·148 grain.

	gr.	pt.
Ex=Sb	+0·016	−0·25
	+0·020	−2·72
	+0·013	−2·00
	+0·010	+0·05
	+0·010	−0·63
	+0·010	+0·60
	+0·010	+1·45
	+0·010	+0·40
	+0·010	+0·95
	+0·010	+1·00
	+0·010	+1·20
	+0·010	+1·10

Mean Ex=Sb+0·01172 grain.

	gr.	pt.
K=Ex	+0·010	+3·13
	+0·014	+0·37
	+0·014	+0·35
	+0·015	−1·95
	+0·015	−1·70
	+0·015	−0·65
	+0·015	+0·05
	+0·015	−0·50
	+0·015	−0·05
	+0·015	−0·25
	+0·015	−0·20
	+0·015	−0·10

Mean K=Ex+0·01406 grain.

	gr.
L=Sb	+0·0233
	+0·0206
	+0·0203
	+0·0158
	+0·0171
	+0·0169
	+0·0181
	+0·0178
	+0·0196

Mean L=Sb+0·01883 grain.

100 parts=0·116 grain.

	gr.	pt.
K=L	+0·007	−0·35
	+0·007	+0·25
	+0·007	+1·40
	+0·007	+2·80
	+0·010	+1·75
	+0·010	+2·70
	+0·013	+0·50
	+0·013	−0·35
	+0·013	−0·01

K=L+0·01079 grain.

	gr.
Ed=Sb	+0·0285
	+0·0290
	+0·0287
	+0·0227
	+0·0235
	+0·0248
	+0·0276
	+0·0250
	+0·0204
	+0·0232

Mean Ed=Sb+0·02534 grain.

	gr.
K=Ed	+0·0065
	+0·0082
	+0·0062
	+0·0059
	+0·0054
	+0·0064
	+0·0037
	+0·0078
	+0·0060
	+0·0059

Mean K=Ed+0·00621 grain.

100 parts=0·121 grain.

	gr.	pt.
D=Sb	+0·031	−1·40
	+0·031	−1·90
	+0·031	−1·60
	+0·030	+0·65
	+0·030	−1·30
	+0·030	−1·80
	+0·030	−1·80
	+0·030	−1·40
	+0·030	−1·40

Mean D=Sb+0·02873 grain.

March 27, April 13.

	gr.	pt.
K=D	+0·006	−2·65
	+0·003	−0·60
	+0·003	−0·20
		−1·60
		−2·30
		−1·20
		−0·70
		+1·20
		+3·50
		+1·90
		+0·60
		+1·70
		+2·20
		+3·10

Mean K=D+0·00129 grain.

April 16. 100 parts=0·1310 grain.

	gr.	pt.
Sb=R	+0·006	+3·75
	+0·010	+0·70
	+0·010	−1·15
	+0·010	−0·97
	+0·008	+0·68
	+0·008	−1·63
	+0·008	−2·27
	+0·008	−1·97
	+0·006	−1·00
	+0·006	−0·75

Mean Sb=R+0·00740 grain.

	gr.	pt.
K=R	+0·035	−0·10
	+0·035	+2·60
	+0·037	+2·20
	+0·038	+0·60
	+0·038	−0·73
	+0·038	+2·50
	+0·038	+2·23
	+0·038	+2·20
	+0·040	+1·25
	+0·040	+0·27

Mean K=R+0·03941 grain.

April 25. 100 parts=0·1456 grain.

	gr.	pt.
L=Ex	+0·008	+0·20
	+0·008	−0·65
	+0·008	−1·35
	+0·008	−1·25
	+0·006	−0·90
	+0·006	−0·80
	+0·006	−0·65
	+0·006	−0·60
	+0·006	−1·75
	+0·006	−0·65
	+0·006	−0·55
	+0·006	−1·00

Mean L=Ex+0·00546 grain.

April 26. 100 parts=0·1456 grain.

	gr.	pt.
Ed=Ex	+0·014	−0·90
	+0·014	−2·10
	+0·014	−3·10
	+0·014	−1·50
	+0·010	+1·40
	+0·010	+0·85
	+0·010	+0·50
	+0·010	+0·55
	+0·010	+1·75
	+0·010	+0·10
	+0·010	+0·55
	+0·010	+0·90

Mean Ed=Ex+0·01121 grain.

April 25. 100 parts=0·1456 grain.

	gr.	pt.
D=Ex	+0·014	+0·30
	+0·014	+0·70
	+0·014	+0·50
	+0·014	+0·85
	+0·014	−0·80
	+0·014	+0·70
	+0·014	+0·40
	+0·014	−0·50
	+0·014	−0·40
	+0·014	−0·50
	+0·014	−0·55
	+0·014	−0·25

Mean D=Ex+0·01405 grain.

April 25.

	gr.	pt.
Ed=L	+0·000	+4·90
	+0·010	−2·70
	+0·010	−2·20
	+0·000	+5·90
	+0·007	+0·50
	+0·007	+0·15
	+0·007	+1·00
	+0·007	+0·75
	+0·007	+1·20
	+0·007	+0·20
	+0·007	+1·30
	+0·007	+1·40

Mean Ed=L+0·00784 grain.

April 27.

	gr.	pt.
D=L	+0·007	+2·30
	+0·010	+0·05
	+0·010	+0·35
	+0·010	+1·05
	+0·010	+0·35
	+0·010	+0·20
	+0·010	+0·70
	+0·010	+0·50
	+0·010	+0·60
	+0·010	+0·05
	+0·010	+0·05

Mean D=L+0·01055 grain.

April 24.

	pt.
Ed=D	+1·85
	+0·80
	+0·60
	+0·30
	+0·30
	−0·80
	−0·35
	−0·50
	−0·45
	−1·20

Mean Ed=D+0·00008 grain.

Before any use can be made of these weighings, the results must be reduced to what they would have been either in a vacuum or in air of given density. The latter is to be preferred, for then the result will be less affected by any uncertainty in the densities of the weights, or the value of the density of air, and also because we are not in possession of the data requisite for the reduction of the weighings in 1824.

Let P appear to weigh as much as Q in air (thermometer $=t$, barometer $=b$), p, q being the weights of the air displaced by P and Q respectively. Let P appear to weigh as much as Q+R in air (thermometer $=t'$, barometer $=b'$), p', q' being the weights of the air displaced by P and Q respectively,—

$$P-p=Q-q \text{ and } P-p'=Q+R-q'.$$

Whence $P+p'-p$ appears to weigh as much as $Q+q'-q$ in air (thermometer $=t'$, barometer $=b'$).

In calculating the densities of Sp, K, Sb, Professor SCHUMACHER adopted the formulæ and constants given in BESSEL's paper on the reduction of weighings, in the Astronomische Nachrichten, B. vii. S. 373. It will therefore be proper to use his tables in reducing the weighings of 1844, even though the values of some of the constants, according to recent and more accurate observations, differ slightly from those employed by BESSEL.

Let vP be the volume of the weight P at the temperature of melting snow, the unit of volume being the volume of one grain of water at its maximum density; p the weight in grains of the air displaced by P; t the temperature of the air in degrees of FAHRENHEIT's scale; and b the height of the mercury in the barometer in English inches reduced to the temperature of melting snow. Then

$\log p = \log b + \log vP + \log$ from Table A. $+ \log$ from Table B. or P., according as the weight is of brass or platinum, t being the argument in each of the tables.

Of the following tables, copied as far as they are wanted from the Philosophical Transactions for 1836, p. 486, A contains the logarithm of the ratio of the density of air temperature t, and under the pressure of one inch of mercury at the temperature of melting snow, to the maximum density of water. B and P contain the logarithms of the ratio of the density of brass and platinum, respectively, at the temperature of melting snow, to the density at the temperature t of FAHRENHEIT's scale.

t.	A.	t.	B.	t.	P.
61	5·61177	61	0·000394	61	0·000189
62	5·61092	62	0·000408	62	0·000195
63	5·61007	63	0·000421	63	0·000202
64	5·60922	64	0·000435	64	0·000209
65	5·60837	65	0·000449	65	0·000215
66	5·60752	66	0·000462	66	0·000222
67	5·60668	67	0·000476	67	0·000228

Let Δ prefixed to the symbol by which any weight is designated denote the ratio of the density of the weight at the temperature of melting ice to the maximum density of water. Then (Philosophical Transactions for 1836, pp. 490–493), $\Delta Sp=21·1874$, $\Delta RS=21·1874$, $\Delta K=7·994$, $\Delta Sb=8·228$. The density of U is unknown. Let it be assumed equal to a mean between the densities of K and Sb, or $\Delta U=8·111$. The platinum weights contain about 5759·5 grains, the brass weights about 5760 grains each. Hence $\log vSp=2·43430$, $\log vRS=2·43430$, $\log vK=2·85766$, $\log vSb=2·84513$, $\log vU=2·85135$.

The temperature and pressure to which it will be most convenient to reduce the weighings, is the mean of the temperatures and pressures observed during the comparisons of Sp and RS with U. This, taking into account the number of observations in the two cases, is,—thermometer $=65·66$ FAHRENHEIT, barometer $=29·75$ English inches. When both the weights are of brass, or both of platinum, the reduction is so small as to be insensible.

Calculation of the weights of air displaced by Sp, RS, K, &c. when (t=65·66, b=29·75).

b	29·75	1·47349		b	29·75	1·47349
A	65·66	5·60781		A	65·66	5·60781
P	65·66	0·00022		B	65·66	0·00046
		7·08152				7·08176

	Log v.	Log air displaced.	Air displaced.
Sp	2·43430	9·51582	0·32796
RS	2·43430	9·51582	0·32796
K	2·85766	9·93942	0·86980
Sb	2·84513	9·92689	0·84506
U	2·85135	9·93311	0·85726

Reduction of the weighings of Sp, RS, K, &c. to air (t=65·66, b=29·75).

Sp≏U−0·00857 grain in air (t=65·62, b=29·722).

29·722	1·47308	29·722	1·47308
65·62	5·60784	65·62	5·60784
65·62	0·00022	65·62	0·00046
	2·43430		2·85135
0·32767	9·51544	0·85650	9·93273

Sp displaces 0·32767 gr. of air.
U displaces 0·85650 gr. of air.

Sp≏U−0·00810 grain in air (t=65·66, b=29·75).

RS≏U−0·00205 grain in air (t=65·73, b=29·806).

29·806	1·47430	29·806	1·47430
65·73	5·60775	65·73	5·60775
65·73	0·00022	65·73	0·00046
	2·43430		2·85135
0·32853	9·51657	0·85874	9·93386

RS displaces 0·32853 gr. of air.
U displaces 0·85874 gr. of air.

RS≏U−0·00296 grain in air (t=65·66, b=29·75).

Sp≏Sb+0·00777 grain in air (t=62·2, b=30·145).

30·145	1·47921	30·145	1·47921
62·2	5·61075	62·2	5·61075
62·2	0·00020	62·2	0·00041
	2·43430		2·84513
0·33455	9·52446	0·86198	9·93550

Sp displaces 0·33455 gr. of air.
Sb displaces 0·86198 gr. of air.

Sp≏Sb−0·00256 grain in air (t=65·66, b=29·75).

K≏Sp+0·02479 grain in air (t=62·2, b=30·186).

30·186	1·47980	30·186	1·47980
62·2	5·61075	62·2	5·61075
62·2	0·00041	62·2	0·00020
	2·85766		2·43430
0·88842	9·94862	0·33501	9·52505

K displaces 0·88842 gr. of air.
Sp displaces 0·33501 gr. of air.

K≏Sp+0·03636 grain in air (t=65·66, b=29·75).

RS≏Sb+0·01433 grain in air (t=62·4, b=30·235).

30·235	1·48051	30·235	1·48051
62·4	5·61058	62·4	5·61058
62·4	0·00020	62·4	0·00041
	2·43430		2·84513
0·33542	9·52559	0·86424	9·93663

RS displaces 0·33542 gr. of air.
Sb displaces 0·86424 gr. of air.

RS≏Sb+0·00261 grain in air (t=65·66, b=29·75).

K≏RS+0·01837 grain in air (t=62·4, b=30·196).

30·196	1·47995	30·196	1·47995
62·4	5·61058	62·4	5·61058
62·4	0·00041	62·4	0·00020
	2·85766		2·43430
0·88838	9·94860	0·33499	9·52503

K displaces 0·88838 gr. of air.
RS displaces 0·33499 gr. of air.

K≏RS+0·02992 grain in air (t=65·66, b=29·75).

Let U, Sp, RS, &c. denote the apparent weights in air ($t=65{\cdot}66$, $b=29{\cdot}75$) of the troy pounds U, Sp, RS, &c. Then,—

	In 1824. gr.	No. of obs.		gr.	No. of obs.
a	Ex =U +0·0010	16	7	K =Sp +0·0364	9
b	Ed =U −0·0015	15	8	RS=Sb +0·0026	11
c	L =U +0·0005	12	9	K =RS+0·0299	8
d	D =U +0·0022	18	10	K =Sb +0·0336	21
e	RM=U +0·0021	20	11	Ex=Sb +0·0117	12
	In 1829.		12	K =Ex +0·0141	12
f	Sp =U −0·0081	300	13	L =Sb +0·0188	9
g	RS =U −0·0030	140	14	K =L +0·0108	9
h	Sb =U −0·0103	60	15	Ed=Sb +0·0253	10
k	K =U +0·0339	92	16	K =Ed +0·0062	10
l	RM=U +0·0089	16	17	D =Sb +0·0287	9
	In 1844.		18	K =D +0·0013	14
1	T =Sp +0·0167	16	19	Sb=R +0·0074	10
2	T =RS+0·0100	16	20	K =R +0·0394	10
3	RS =Sp +0·0064	9	21	L =Ex +0·0055	12
4	Sp =P −0·0013	20	22	Ed=Ex +0·0112	12
5	RS =P +0·0032	12	23	D =Ex +0·0140	12
6	Sp =Sb −0·0026	12	24	Ed=L +0·0078	12
			25	D =L +0·0106	11
			26	Ed=D +0·0001	10

The results 11 and 12 appear to be considerably in error.

The numbers in the first column in the following Table indicate the equations from which the results are deduced; those in the last column express roughly the weights of the results, supposing all the observations to be equally good.

	gr.	
1, 2	RS=Sp+0·0067	1
3	RS=Sp+0·0064	1
4, 5	RS=Sp+0·0045	1
6, 8	RS=Sp+0·0052	1
	Mean ...RS=Sp+0·0057	

	gr.	
6	Sp=Sb−0·0026	2
3, 8	Sp=Sb−0·0038	1
7, 10	Sp=Sb−0·0028	1
	Mean ...Sp=Sb−0·0030	

	gr.	
8	RS=Sb+0·0026	2
3, 6	RS=Sb+0·0038	1
9, 10	RS=Sb+0·0037	1
	Mean ...RS=Sb+0·0032	

	gr.	
7	K=Sp+0·0364	2
3, 9	K=Sp+0·0363	1
6, 10	K=Sp+0·0362	2
	Mean ...K=Sp+0·0363	

	gr.	
9	K=RS+0·0299	2
3, 7	K=RS+0·0300	1
8, 10	K=RS+0·0310	2
	Mean ...K=RS+0·0304	

	gr.	
6, 7	K=Sb+0·0338	1
8, 9	K=Sb+0·0325	1
10	K=Sb+0·0336	4
11, 12	K=Sb+0·0258	1
13, 14	K=Sb+0·0296	1
19, 20	K=Sb+0·0315	1
17, 18	K=Sb+0·0300	1
19, 20	K=Sb+0·0320	1

Mean, omitting the 4th result on account of the large difference between it and the others,

K=Sb+0·0324

	In 1829. gr.	In 1844. gr.
f, g	RS=Sp +0·0051	RS=Sp +0·0057
f, h	Sp =Sb +0·0022	Sp =Sb −0·0030
g, h	RS=Sb +0·0073	RS=Sb +0·0032
f, k	K =Sp +0·0420	K =Sp +0·0363
g, k	K =RS+0·0369	K =RS+0·0304
h, k	K =Sb +0·0442	K =Sb +0·0324

In the interval between 1829 and 1844, the difference between the two platinum troy pounds Sp and RS had undergone no very sensible relative change. If, as appears highly probable, Sp and RS have undergone no sensible absolute change, Sb has gained 0·0046 grain, and K has lost 0·0061 grain.

In 1844, $Sb+K=Sp+RS+0{\cdot}0333$ grain. Assuming Sp and RS to have experienced no change since 1829, $Sp+RS=2U-0{\cdot}0111$ grain; whence $Sb+K=2U+0{\cdot}0222$ grain.

The equations 11...18 give—

$$\begin{aligned}
2Ex &= Sb+K-\overset{gr.}{0{\cdot}0024}=Sp+RS+\overset{gr.}{0{\cdot}0309}\\
2L &= Sb+K+0{\cdot}0080=Sp+RS+0{\cdot}0413\\
2Ed &= Sb+K+0{\cdot}0191=Sp+RS+0{\cdot}0525\\
2D &= Sb+K+0{\cdot}0274=Sp+RS+0{\cdot}0607
\end{aligned}$$

The first column of the following Table exhibits the errors of Ex, L, Ed, D, as deduced from the above equations; the second column exhibits the errors of the same weights, as determined by Captain KATER, in 1824, by direct comparison with U; the third shows the increase of weight of the several troy pounds in the course of twenty years.

In 1844.	In 1824.	
gr.	gr.	gr.
Ex−U=+0·0099	Ex−U=+0·0010	0·0089
L −U=+0·0151	L −U=+0·0005	0·0146
Ed−U=+0·0206	Ed−U=−0·0015	0·0221
D −U=+0·0248	D −U=+0·0022	0·0226

In 1824, $RM=U+0{\cdot}0021$ grain; in 1829, $RM=U+0{\cdot}0089$; consequently RM gained 0·0068 grain in five years. With the single exception of K, all the new brass weights have become heavier since they were first compared with U, in consequence probably of the oxidation of their surfaces, while U, which was made in 1758, was preserved from further change by the coat of oxide already formed. One of these, Sb, appeared to have been protected by gilding, though imperfectly, as parts of its surface were slightly tarnished. Ex and L were brighter than Ed and D. K, though it had become lighter, was much tarnished, yet exhibited no traces of abrasion. The discordances presented by the different weighings of K previous to 1844 were highly perplexing, and were probably the cause of the very numerous and accurate comparisons of the various troy pounds placed at the disposal of the Committee, with the lost standard, on which alone the possibility of restoring it with sufficient accuracy depends.

Professor SCHUMACHER received K in March 1827, accompanied by a statement that it had been found by Captain KATER to exceed the standard very little, not more than 0·006 grain*. In June 1828, Captain KATER compared a second weight Kn with each of two troy pounds in his possession, the errors of which were well determined. One of these was 0·0122 grain too heavy, the other 0·0267 grain too heavy. Let 2W denote the sum of these two troy pounds. Then $W=U+0{\cdot}0194$ grain. By a

* Philosophical Transactions, 1836, p. 457.

mean of eight comparisons Kn=W−0·0170 grain. In September 1828, by a mean of twenty comparisons, Professor SCHUMACHER found K=Kn+0·0198 grain; whence K=U+0·0223 grain. In February and March 1829, by a mean of twenty comparisons, Captain KATER found K=W+0·0065; whence K=U+0·0259 grain. This differs from the first result, 0·0199 grain (not 0·0299 grain as it is erroneously printed). By ninety-two direct comparisons of K with the standard by Captain v. NEHUS in June and July 1829, K=U+0·0339 grain. In the autumn of 1829 Professor SCHUMACHER compared K again with the sum of three brass weights of 5000 grains, 400 grains, 300 grains and 60 grains of platinum, with which it had been compared on its arrival at Altona in 1827, and there was no sensible difference from the first comparison. By thirty comparisons in October 1829 and February 1830, Professor SCHUMACHER found K=Kn+0·0200 grain. This differs but 0·0002 grain from the result obtained in 1828. In April 1844, 2K=Sp+RS+0·0667 grain, Sp+RS=2U−0·0111 grain. Therefore K=U+0·0278 grain.

On taking K out of its case after I had received it from Professor SCHUMACHER in March 1844, I observed a small fragment of wood, like a grain of coarse sawdust, adhering to the under surface of the weight so firmly that I was unable to brush it off with a feather, and had some difficulty in removing it with a pointed bit of quill. The adhesion of the bit of wood to the weight is due apparently to the pressure produced by screwing down very tightly the lid of the box in which it was contained. Two or three similar grains were imbedded in the velvet lining of the case. In all probability this bit of wood had been attached to K immediately after its first comparison by Captain KATER, when it appeared to be 0·006 grain too heavy, and previous to its comparison by Professor SCHUMACHER with the brass weights of 5000 grains, 400 grains and 300 grains, and the platinum weights of 60 grains.

By the observations of February and March 1829, K=W+0·0065 grain, and by those of June and July 1829, K=U+0·0339 grain; whence W=U+0·0274 grain. But W=U+0·0194 grain when first compared. Therefore in 1829 W had gained 0·0080 grain. At the same time RM, which has been very carefully preserved, had gained 0·0068 grain. In 1844 the well-preserved troy pounds Ex and L had gained 0·0089 grain and 0·0146 grain respectively, and Ed and D, which were in a less perfect state of preservation, had gained 0·0221 grain and 0·0226 grain respectively. The whole gain of K up to 1844 appears to be 0·0218 grain, about the same as that of Ed or D. If, as seems probable, K was compared with W about the end of 1826 or the beginning of 1827, this error of +0·0218 must include the gain of W up to that period. The discordances in the weighings of K may be explained by supposing the gain of K, including that of W up to 1829, to be 0·014 grain, and the gain from 1829 to 1844 to be 0·008 grain, since it may be assumed that brass having a recently polished surface gains weight faster than when its surface is protected by a film of oxide; also, that in the same interval, Sb, which was in some measure protected by gilding, had gained rather less than K, and that the bit of wood weighed 0·014 grain.

Then, original error of K$=+0{\cdot}006$ grain. Error of K in 1829=original error $+0{\cdot}014$ grain (wood) $+0{\cdot}014$ grain (oxygen)$=+0{\cdot}034$ grain. Error of K in 1844 = error of K in 1829 $-0{\cdot}014$ grain (wood) $+0{\cdot}008$ grain (oxygen)$=+0{\cdot}028$ grain.

The comparisons of the troy pounds Ex, L, Ed, D with each other in 1814 give

$$\mathrm{Ex}+0{\cdot}0102 \text{ grain} = \mathrm{L}+0{\cdot}0061 \text{ grain} = \mathrm{Ed} = \mathrm{D}.$$

This result agrees with the conclusion already derived from the comparison of Ex, L, Ed, D with Sb and K, in showing that the differences between Ex, L, Ed, and D have very sensibly changed in the course of twenty years.

The troy pound R, which is much tarnished, is about 0·012 grain lighter than U, and therefore cannot be either of the weights used by Captain KATER in finding the errors of K and Kn.

The discrepancies presented by the weighings of the brass troy pounds at different times, due to the effect of oxidation or other causes, are so large, that I resolved, with the consent of the Astronomer Royal, to rest for the evidence of the weight of the lost standard entirely on the comparisons of the two platinum troy pounds Sp and RS. In a note appended to Professor SCHUMACHER's paper in the Transactions of the Royal Society for 1836, p. 471, Mr. BAILY observes, that, for some unexplained reason, Mr. CARY, who was commissioned to construct the troy pound RS, used for this purpose some platinum of his own instead of that which was supplied to him by the Royal Society. The exchange, whatever may have been the cause of it, does not appear to have been detrimental, for the surface of RS, though certainly inferior to that of the newly made platinum kilogrammes and mètre bars which I saw in Paris in 1844, is superior to that of Sp, in which plugs have been inserted to fill up holes left by drilling out defective places, and is much better than that of the other pound weights made since of platinum prepared in England.

If we consider the discordances presented by the weighings of the brass troy pounds simply as errors of observation, without paying any regard to their probable causes, the resulting value of U will not be very different from that given by the platinum troy pounds alone.

By the observations of 1824 and 1829,—

	gr.	weight.
U=	Sp $+0{\cdot}0081$	30
U=	RS$+0{\cdot}0030$	14
U=	Sb $+0{\cdot}0103$	6
U=	K $-0{\cdot}0339$	9
U=$\frac{1}{4}$(Ex+L+Ed+D)$-0{\cdot}0022$		6

By the observations of 1844,—

		gr.
RS	=Sp	$+0{\cdot}0057$
Sb	=Sp	$+0{\cdot}0030$
K	=Sp	$+0{\cdot}0363$
Ex+L+Ed+D=2(Sb+K)		$+0{\cdot}0260$

Whence, supposing the errors of the weighings to be insensible compared with the discordances of the brass troy pounds,—

		weight.
	gr.	
1	U=Sp+0·0081	30
2	U=Sp+0·0087	14
3	U=Sp+0·0133	6
4	U=Sp+0·0024	9
5	U=Sp+0·0261	6

The mean of all the equations gives

U=Sp+0·0096 grain.

Excluding the last, which depends upon the weighings in 1824,

U=Sp+0·0079 grain.

Excluding all except the result of the comparisons of U with the two platinum troy pounds,

U=Sp+0·0083 grain.

Comparison of Thermometers.

The thermometer K was supplied by the Committee of the Kew Observatory. It bears the inscription "No. 43, Kew Observatory, July 1853." It is graduated by lines etched upon the tube at every fifth of a centesimal degree. The distance between the freezing- and boiling-points is about 18·1 inch. Mr. Welsh, under whose superintendence it was constructed, examined it by the method employed by Mr. Sheepshanks, and concluded that the graduation was correct throughout the scale to one-tenth of a small division, or 0°·02 C. He obtained the following data for determining its boiling-point at the Kew Observatory, the stem being vertical:—

1853.	Barom.	Att. therm.	Barom. in millims. of mercury at 0° C.	Reading of K.	Temp. by Regnault's Tables.	Error.
July 27.	30·039	65·5	760·47	99·98	100·02	−0·04
August 16.	29·726	64·3	752·61	99·70	99·73	−0·03
August 17.	29·640	63·2	750·51	99·58	99·65	−0·07
					Mean......	−0·047

Hence the boiling-point with the stem vertical under the pressure of 760 millimètres of mercury at 0° C., is 99°·953. The freezing-point, with the stem vertical, was −0°·04, before boiling, and −0°·12, after boiling.

Assuming 100° C. to be the temperature of steam under Laplace's standard atmospheric pressure, or the pressure of a column of mercury at 0° C., the height of which in millimètres is 760+1·946 cos 2 latitude +0·0001492 height in mètres above the sea, the temperature of steam at Kew under the pressure of 760 millimètres of mercury at 0° C., will be 100°·016. But the reading of K was 99°·953. Hence, denoting by K the reading of No. 43 at the temperature t by a thermometer the freezing- and boiling-points of which are accurately determined, when $t=0°$, $t-\mathrm{K}=+0°{\cdot}12$, and when $t=100°$, $t-\mathrm{K}=+0°{\cdot}063$. In August 1853 it was heated to rather above 100° C. On December 15 the freezing-point had ascended to −0°·04. The reading was 0°·00 when the thermometer was surrounded with broken ice, May 26, 1855, and also when immersed in pounded ice, July 10, 1855. The determination of the zero of a

thermometer by ice is said to be less accurate than when snow is used. In the present instance it appears from the following comparisons to have been sufficiently exact. In February 1844 the freezing-point of a thermometer G, having an arbitrary scale, was 37·4. In March 1845 it was 37·5, the thermometer having been immersed in snow in both cases. In May 1855, the bulbs of G and K being almost in contact with each other and surrounded with broken ice, the reading of G was 37·55, while that of K was 0°·00. In July 1855, the reading of G, when immersed in pounded ice, was again found to be 37·55. In March 1855, by a mean of ten comparisons, the reading of G was 48·541, when that of K was 2°·365. By a mean of ten other comparisons, the corresponding readings of G and K were 90°·797 and 11°·432 respectively. Hence, one part of G=0°·2145. If we suppose the zero of G unchanged since 1845, the freezing-point of K would be −0°·004. But if we suppose the zero of G to have been correctly determined by immersion in ice in May and July 1855, or that the zero had ascended 0·05 part =0°·011 since 1845, the freezing-point of K would be +0°·007.

Assuming the freezing-point of K in 1855 to be 0°·00, as given by observation, the corrections of K will be,—

t.	*t*−K.	*t.*	*t*−K.	*t.*	*t*−K.
0°	0·000	35°	−0·020	70°	−0·040
5	−0·003	40	−0·023	75	−0·043
10	−0·006	45	−0·026	80	−0·046
15	−0·009	50	−0·029	85	−0·048
20	−0·011	55	−0·031	90	−0·051
25	−0·014	60	−0·034	95	−0·054
30	−0·017	65	−0·037	100	−0·057

The thermometers B, C, D were made by the late M. BUNTEN of Paris. They are all divided into centesimal degrees by lines etched upon the tubes. The graduation of B extends from −23° up to +107°. It bears the inscription '25 Mai 1843. Divisé le 18 Mai 1844.' The graduation of C extends from −24° up to +41°. It is dated 1841. The graduation of D, which is also dated 1841, extends from −25° up to +53°.

The graduation of B was examined at certain points of the scale by the method described by Professor FORBES in the Transactions of the Royal Society for 1836, p. 578, and the boiling- and freezing-points determined, in order that the other thermometers might be referred to it as a standard. It was not, however, used as a standard, in consequence of the acquisition of the Kew thermometer, which has a scale of much larger dimensions, and is more accurately and closely divided, and also on account of the inconvenience in using it as a standard, arising from the large amount of the displacement of its zero. Immediately after boiling, February 1, 1845, the freezing-point of B was −0°·20; on February 4 it was −0°·15; on March 3 it was −0°·11; in December 1846 it was 0°·00; and in July 1855 it was +0°·11. The depression of the zero, which in the present case amounts to 0°·31 C., depends upon

the composition of the glass, and perhaps also upon the manner in which it is worked by the glass-blower. LEGRAND observed depressions of from 0°·3 to 0°·5 in thermometers by M. BUNTEN *. DESPRETZ found changes amounting to 0°·47, 0°·45, 0°·23, 0°·30, 0°·57, 0°·61, 0°·60, 0°·60; and that the freezing-point became stationary at the end of about four years †. Mr. WELSH has observed a depression in the thermometers constructed at Kew varying from 0°·09 to 0°·11. In the thermometers employed by Mr. SHEEPSHANKS it amounts to about 0°·17. In ten different thermometers examined by Dr. LAMONT, the depressions were 0°·31, 0°·28, 0°·45, 0°·25, 0°·31, 0°·37, 0°·62, 0°·25, 0°·31, 0°·27 respectively. He also found that it takes about five years for the zero to regain its permanent position after boiling ‡.

The depression of the freezing-point of B below +0°·11, from March 1845 up to January 1851, is represented with sufficient accuracy by $0·0044m^2$, where m is the time in months up to the middle of January 1851. The computed depressions are,—

1845. March, April	°0·22	1846. May, June	°0·14	1847. December—February	°0·06
May	0·21	July, August	0·13	1848. March—June	0·05
June, July	0·20	September, October	0·12	July—September	0·04
August	0·19	November, December	0·11	October—February	0·03
September, October	0·18	1847. January—March	0·10	1849. March—July	0·02
November, December	0·17	April—June	0·09	August—February	0·01
1846. January, February	0·16	July, August	0·08	1850. March—December	0·00
March, April	0·15	September—November	0·07		

Means of Comparisons of B and K in February and March 1855.

No. of comparisons.	B.	K.	B−K.	K−t.	B−t.
10	4·895	4·712	0·183	0·003	0·186
10	5·344	5·177	0·167	0·003	0·170
10	6·319	6·143	0·176	0·003	0·179
10	9·219	9·016	0·203	0·006	0·209
10	10·740	10·493	0·247	0·006	0·253
10	12·893	12·652	0·241	0·008	0·249
10	14·532	14·279	0·253	0·009	0·262
10	15·336	15·095	0·241	0·009	0·250
10	16·379	16·126	0·253	0·009	0·262
10	17·581	17·329	0·252	0·010	0·262
10	18·000	17·749	0·251	0·010	0·261
10	18·000	17·750	0·250	0·010	0·260
10	18·180	17·917	0·263	0·010	0·273
10	21·382	21·116	0·266	0·011	0·277
10	23·202	22·919	0·283	0·013	0·296
10	25·331	25·045	0·286	0·014	0·300

t.	B−t.	t.	B−t.	t.	B−t.	t.	B−t.
0	0·11	7	0·19	14	0·25	21	0·28
1	0·12	8	0·20	15	0·25	22	0·29
2	0·13	9	0·21	16	0·26	23	0·30
3	0·14	10	0·23	17	0·26	24	0·30
4	0·16	11	0·25	18	0·26	25	0·30
5	0·17	12	0·25	19	0·27		
6	0·18	13	0·25	20	0·28		

* Annales de Chimie, 1836, tome lxiii. p. 368.

† Ibid. 1837, tome lxiv. p. 312.

‡ Jahresbericht der Münchener Sternwarte für 1852, S. 64, 93, 101; and Annalen für Meteorologie, 1842, Heft iv. S. x. xv.

The observations for determining the freezing-points of C and D gave,—

	C.	D.	
1845, March	0·00	0·05	in snow.
1846, December	0·01	0·08	in snow.
1855, May	0·02	0·07	in ice.
1855, July	0·00	0·08	in ice.
Means in 1855	0·01	0·07	

Comparisons of C and D with K in March and May 1855.

No. of comparisons.	C.	K.	D.
10	6·047	6·004	6·064
6	7·780	7·772	7·783
5	10·314	10·316	10·310
10	13·097	13·137	13·098
24	16·000	16·042	16·037
33	17·000	17·056	17·030
24	18·000	18·062	18·032
11	20·161	20·268	20·228
17	24·955	25·079	25·086

t.	t−K.	K−C.	t−C.	K−D.	t−D.
0	0·000		−0·010		−0·070
6	−0·004	−0·043	−0·047	−0·060	−0·064
8	−0·005	−0·008	−0·013	−0·011	−0·016
10	−0·006	+0·002	−0·004	+0·006	0·000
13	−0·008	+0·040	+0·032	+0·039	+0·031
16	−0·009	+0·042	+0·033	+0·005	−0·004
17	−0·010	+0·056	+0·046	+0·026	+0·016
18	−0·010	+0·062	+0·052	+0·030	+0·020
20	−0·011	+0·107	+0·096	+0·040	+0·029
25	−0·014	+0·124	+0·110	−0·007	−0·021

t.	t−C.	t−D.
0	−0·01	−0·07
1	−0·02	−0·07
2	−0·03	−0·07
3	−0·03	−0·07
4	−0·04	−0·07
5	−0·04	−0·06
6	−0·05	−0·06
7	−0·03	−0·04
8	−0·01	−0·02
9	−0·01	−0·01
10	0·00	0·00
11	+0·01	+0·01
12	+0·02	+0·02
13	+0·03	+0·03
14	+0·03	+0·02
15	+0·03	+0·01
16	+0·03	0·00
17	+0·05	+0·02
18	+0·05	+0·02
19	+0·07	+0·02
20	+0·10	+0·03
21	+0·10	+0·02
22	+0·10	+0·01
23	+0·10	0·00
24	+0·11	−0·01
25	+0·11	−0·02

The zero-points of C and D do not appear to have undergone any very appreciable change. The comparisons of B, C, D with each other at distant intervals may, therefore, be used to check the values of the depression of the zero of B deduced from the observations of the freezing-point in 1845, 1846 and 1855.

Comparisons of C, B, D, July 31, 1846.

No. of comparisons.	C.	B.	D.
10	20·492	20·769	20·620
10	19·697	19·969	19·805
10	20·707	20·977	20·841
10	18·347	18·589	18·425
Mean of 40 comparisons ...	19·810	20·076	19·923

Hence, at 20°, B−C=+0·266, B−D=+0·153.

Comparisons of C, B, D in July 1855.

No. of comparisons.	C.	B.	D.
10	19·884	20·258	19·967
10	20·535	20·885	20·616
Mean of 20 comparisons ...	20·209	20·571	20·291

Hence, at 20°, B−C=+0·362, B−D=+0·280.

Also, by a mean of 11 comparisons at 20°, K−C=+0·107, K−D=+0·040. By 30 comparisons at 18°, and 10 at 21°, B−K=+0·261. Therefore B−C=+0·368, B−D=+0·301. The resulting mean value of the depression of the zero of B is 0°·12. The value deduced from the observations of the freezing-point is 0°·13.

Comparisons of C, B, D in September 1846.

No. of comparisons.	C.	B.	D.
10	26·551	26·884	26·750
10	25·385	25·695	25·516
10	24·401	24·670	24·531
10	23·448	23·725	23·577
10	26·634	26·949	26·796
10	23·646	23·940	23·774
Mean of 60 comparisons ...	25·011	25·311	25·157

Hence, at 25°, B−C=+0·30, B−D=+0·153.

By a mean of 16 comparisons of C, B, D between 23°·9 and 26°·2, in July 1855, their corresponding readings were

C.	B.	D.
25·081	25·487	25·204

Hence, at 25°, B−C=+0·406, B−D=+0·283.

Also, 17 comparisons of C, K, D, gave K−C=+0·124, K−D=−0·007, and 10 comparisons of B, K, gave B−K=+0·286. Therefore B−C=+0·410, B−D=+0·279. The resulting mean value of the depression of the zero of B is 0°·12, the same as its calculated value.

The thermometers G, L have arbitrary scales, the divisions of which are traced on the tubes with a diamond point at every tenth of an inch. The parts of the scales of G, L, as well as those of B, C, D, K, are subdivided to hundredths of an inch by sliding scales of glass. The division of the sliding scale which is brought into apparent coincidence with the extremity of the thread of mercury, is viewed through a hole in a plate of brass attached to a very light brass frame which carries the scale, so that the direction of vision may be perpendicular to the axis of the tube. The hundredths of an inch are subdivided by estimation.

Means of Comparisons of G and L with K in February 1855.

No. of comparisons.	G.	K.	No. of comparisons.	L.	K.
10	48·541	2·365	10	51·465	15·753
10	90·797	11·432	10	54·545	16·089
10	111·350	15·753	10	73·010	18·389
10	124·050	18·389	10	92·384	20·828
			10	94·698	21·055
			10	94·869	21·079
			10	97·790	21·448
			10	101·515	21·911

By a mean of 30 comparisons of G and L between 17° and 20° in July 1846, their corresponding readings were

G.	L.
120·897	67·591

Means of Comparisons of G, K, L in March and May 1855.

No. of comparisons.	G.	K.	L.
11	117·79	17·08	62·38
10	124·24	18·43	73·36
	6·45	1·35	10·98

1 part of G=0°·2093, 1 part of L=0°·12295, 1 part of G=1·702 parts of L.

	G.	K.	L.
Mean of all	120·860	17·723	67·609

In July 1846 the corresponding readings of G, L were 120·897 and 67·591.

By the observations of 1855, K=17·731 when G=120·897, and K=17·721 when L=67·591.

Hence L stood 0°·01 higher, compared with G, in 1855 than it did in July 1846. Between March 1845 and May 1855 the zero of G appears to have ascended 0°·01. It is probable that the whole or, at any rate, the greater part of this ascent occurred before July 1846. On this supposition, in July 1846, the depression of the zero of G below its permanent place would be 0°·00, and that of L would be 0°·01.

The thermometers H, P were used only in the earlier observations. The divisions of H were on the tube. The scale of P was traced on paper and enclosed between the tube of the thermometer and an exterior tube of glass joined by fusion to the thermometer tube at the lower end, and sealed at the upper end. The freezing-point of H was +0°·50. The freezing-point of P was +0°·25.

By a mean of 13 comparisons of H, P, C, D, their corresponding readings were

H.	P.	C.	D.
3·31	3·13	2·93	2·99

Let 2M=C+D. Then the corresponding readings of H, P, M, t will be

H.	P.	M.	t.	t−H.	t−P.
3·31	3·13	2·96	2·91	−0·40	−0·22

Means of Comparisons of H, B, P, March 3, 1845.

No. of comparisons.	H.	B.	P.
8	6·09	5·66	6·02
10	11·45	11·37	11·84
9	17·06	17·19	17·94
10	19·67	19·82	20·73
10	23·40	23·71	24·82

Hence, observing that the zero of B was 0·22 lower in March 1845 than in 1855,—

t.	$t-$H.	$t-$P.
5·70	−0·39	−0·32
11·34	−0·11	−0·50
17·15	+0·09	−0·79
19·76	+0·09	−0·97
23·63	+0·23	−1·19

Means of Comparisons of H, P with G and L in July and August 1844.

No. of comparisons.	H.	P.	G.	L.
25	16·43	17·13	114·96	——
27	17·46	18·24	——	65·90
14	18·37	19·32	124·75	——
13	20·73	21·82	——	93·03

The corresponding readings of G, K, t; L, K, t in 1855 were,—

G.	K.	t.	L.	K.	t.
114·96	16·504	16·49	65·90	17·503	17·49
124·75	18·535	18·52	93·03	20·909	20·90

At a given temperature the readings of G and L were higher in 1855 than in July 1844 by 0°·02 C. and 0°·04 C. respectively. Hence the corresponding readings of G, L, t, in July 1844, would have been,—

G.	t.	L.	t.
114·96	16·51	65·90	17·53
124·75	18·54	93·03	20·94

The corresponding readings of H, P, t, in July 1844, would have been,—

H.	P.	t.	$t-$H.	$t-$P.
16·43	17·13	16·51	+0·08	−0·62
17·46	18·24	17·53	+0·07	−0·71
18·37	19·32	18·54	+0·17	−0·78
20·73	21·82	20·94	+0·21	−0·88

By a graphic interpolation the following values of $t-$H, $t-$P were obtained:—

t.	$t-$H.	t.	$t-$H.	t.	$t-$P.	t.	$t-$P.
0	−0·50	13	−0·07	0	−0·25	13	−0·53
1	−0·48	14	−0·02	1	−0·25	14	−0·57
2	−0·46	15	+0·01	2	−0·26	15	−0·62
3	−0·43	16	+0·04	3	−0·27	16	−0·67
4	−0·41	17	+0·07	4	−0·28	17	−0·72
5	−0·38	18	+0·10	5	−0·29	18	−0·78
6	−0·35	19	+0·14	6	−0·31	19	−0·84
7	−0·31	20	+0·17	7	−0·33	20	−0·90
8	−0·27	21	+0·19	8	−0·36	21	−0·97
9	−0·24	22	+0·21	9	−0·39	22	−1·04
10	−0·20	23	+0·22	10	−0·42	23	−1·12
11	−0·16	24	+0·23	11	−0·45	24	−1·20
12	−0·11			12	−0·49		

The thermometer R was used in some of the earlier observations. It has an ivory scale the dimensions of which vary with the quantity of moisture present in the atmosphere, and consequently the error is very sensibly different at different times.

By a mean of twenty comparisons of R, B in October 1846, their corresponding readings were:—

R.	B.	B—R.
19·31	18·90	—0·41

But in October 1846, at 19°, $t-B=-0{\cdot}15$. Therefore $t-R=-0^{\circ}{\cdot}56$.

Comparison of Barometers.

Up to the end of August 1844 a siphon barometer by the late Mr. ROBINSON was employed. It resembles BUNTEN's improved GAY-LUSSAC's barometer in all respects except that it is graduated in English inches, and the attached thermometer in degrees of FAHRENHEIT's scale. The observations were reduced, for the mercury to 32° FAHR., and for the scale to 62° FAHR., by the tables in SCHUMACHER's Jahrbuch für 1837.

From the beginning of September 1844 a cistern barometer by ERNST of Paris was used. It is graduated in millimètres, and the attached thermometer in centesimal degrees. In the following comparisons with the barometer of the Paris Observatory, made by one of the Assistants of the Observatory, O, T denote the readings of the Observatory barometer, and of its attached thermometer; F, E those of ERNST, and of its attached thermometer.

O.	T.	F.	E.	O—F.
mm.		mm.		mm.
753·16	21·4	752·90	21·0	0·26
753·64	21·8	753·30	21·3	0·34
754·00	22·3	753·60	21·7	0·40
754·14	22·3	753·70	21·7	0·44
754·32	22·5	753·90	21·9	0·42
757·40	21·1	757·00	20·9	0·40
756·90	22·4	756·50	21·7	0·40
758·64	19·5	758·25	19·3	0·39
758·20	21·7	757·70	21·1	0·50
			Mean	0·393

ERNST stands 0·393 millimètre lower than the Observatory barometer. The latter requires no correction. ERNST was suspended close to the Observatory barometer all night previous to the day on which the comparisons were made, their temperatures must, therefore, have been very nearly equal. Yet E is less than T, while, by a subsequent comparison with B, E was found to be 0°·45 too great. There is reason to believe that this discrepance is due to the lodgment of a small quantity of mercury in the upper end of the tube of the thermometer, which occurs sometimes after the instrument has been conveyed in a carriage over a rough pavement, as was done previous to its comparison with the Observatory barometer.

In March 1845 the thermometer B was suspended so that its bulb was in contact

with the brass covering of the bulb of E, and the corresponding readings of B and E observed. At this time the freezing-point of B was 0°·22 lower than in 1855. The readings of a thermometer free from error are denoted by *t*.

No. of comparisons.	B.	E.	B−E.	*t*−B.	*t*−E.
10	5·25	3·75	−0·50	+0·05	−0·45
10	7·46	7·94	−0·48	+0·03	−0·45
17	9·73	10·12	−0·39	−0·01	−0·40
9	11·65	12·09	−0·44	−0·03	−0·47
9	15·10	15·52	−0·42	−0·03	−0·45
7	21·58	22·03	−0·45	−0·06	−0·51
				Mean	−0·45

Since the reduction of F to 0° C., with a temperature E 0°·45 too great, would make the reduced value of F 0·055 mm. too small, the error of E will be corrected by adding 0·45 mm. instead of 0·39 mm. to the reduced value of F.

Subsequently, by the kindness of the Rev. W. T. Kingsley, an opportunity was afforded of comparing F with a standard barometer having a tube of very large bore, also constructed by Ernst, belonging to the Taylor Library in Sidney-Sussex College, Cambridge. According to the maker's statement, the reading of this barometer is 0·04 mm. too great.

	Taylor.	Att. therm.	F.	E.
April 11, 1851.	761·28	9·50	760·80	10·00
	761·28	9·60	760·85	10·10
	761·32	9·75	760·90	10·20
	761·22	9·75	760·87	10·15
	761·18	9·75	760·80	10·30
	761·26	9·69	760·84	10·15
April 12.	760·78	9·25	760·38	9·6
	760·80		760·42	
	760·83	9·5	760·40	9·8
	760·74	9·6	760·30	10·1
	760·74		760·36	
	760·74	9·75	760·30	10·2
	760·76	9·52	760·36	9·92
	T.	Att. therm.	F.	E.
Means ...	761·01	9·60	760·60	10·03

Taylor—F=0·41 mm. But Taylor stands 0·04 mm. too high. Therefore F is 0·37 mm. too low. E is 0°·43 higher than the attached thermometer of Taylor. The reduction due to 0°·43 is 0·053 mm. Therefore the error of E will be corrected by adding 0·423 mm. instead of 0·37 mm. to the reduced height of F.

The mean of the two corrections gives 0·44 millimètre to be added to the height of F reduced to 0° C. The reductions were made by the tables in Schumacher's Jahrbuch für 1838.

Weight of Air.

According to RITTER* the observations of REGNAULT† show that in Paris, lat. 48° 50′ 14″, 60 mètres above the level of the sea, a litre of dry atmospheric air at 0° C., under the pressure of 760 millimètres of mercury, weighs 1·2932227 gramme. Assuming that atmospheric air contains on an average 0·0004 of its volume of carbonic acid the density of which is 1·529 of that of atmospheric air, the weight of a litre of dry atmospheric air containing its average amount of carbonic acid, under the circumstances already stated, will be 1·2934963 gramme. It appears from the discussion of pendulum experiments by Mr. BAILY‡, that if we take G to denote the force of gravity at the mean level of the sea in lat. 45°, the force of gravity in lat. λ, at the mean level of the sea,

$$=G(1-0{\cdot}0025659\cos 2\lambda).$$

POISSON§ has proved that the force of gravity in a given latitude at a place on the surface of the earth at the height z above the mean level of the sea

$$=\left\{1-\left(2-\frac{3}{2}\frac{\varrho'}{\varrho}\right)\frac{z}{r}\right\}\times \text{ (force of gravity at the level of the sea in the same latitude)},$$

where r is the radius of the earth, ϱ its mean density, and ϱ' the density of that part of the earth which is above the mean level of the sea. If, as is probable,

$$\varrho':\varrho=5:11,\quad 2-\frac{3}{2}\frac{\varrho'}{\rho}=1{\cdot}32 \text{ nearly},\quad r=6366198 \text{ mètres}.$$

Hence the weight in grammes of a litre of dry atmospheric air containing the average amount of carbonic acid, at 0°, and under the pressure of 760 millimètres of mercury at 0°, at the height z above the mean level of the sea in lat. λ, is

$$1{\cdot}2930693\left(1-1{\cdot}32\frac{z}{r}\right)(1-0{\cdot}0025659\cos 2\lambda).$$

REGNAULT found the expansion of air from 0° to 100°, under constant pressure, equal 0·36706 of its volume at 0°; also that, at 50°, the mercurial thermometer was a little in advance of the air thermometer ‖. The difference between the mercurial and air thermometers, at 50°, amounts to about 0°·2¶. Hence, the expansion of air between 0° and 50°·2 is 0·18353 of its volume at 0°; or, between 0° and 50°, the ratio of the density of air at 0° to its density at t° is $1+0{\cdot}003656t$. The density of the vapour of water is 0·622 of that of air. Hence, if t be the temperature of the air, b the barometric pressure, v the pressure of the vapour present in the air, b and v being expressed in millimètres of mercury at 0°, at a place on the surface of the earth at a height z above the mean level of the sea, in lat. λ, the weight in grammes of a litre

* Mémoires de la Société de Physique et d'Histoire Naturelle de Genève, t. xiii. p. 361.

† Mémoires de l'Institut, t. xxi. p. 157.

‡ Memoirs of the Astronomical Society, vol. vii. p. 94.

§ Traité de Mécanique, t. ii. p. 629.

‖ Mémoires de l'Institut, tome xxi. pp. 91, 238.

¶ Annales de Chimie, 3me série, tome v. p. 99.

of air will be

$$\frac{1\cdot2930693}{1+0\cdot003656t}\,\frac{b-0\cdot378v}{760}\left(1-1\cdot32\frac{z}{r}\right)(1-0\cdot0025659\cos 2\lambda).$$

In the cellar under the Mineralogical Museum in Cambridge, where the weights were compared, in lat. 52° 12′ 18″, about 8 mètres above the mean level of the sea, this becomes

$$\frac{1\cdot293893}{1+0\cdot003656t}\,\frac{b-0\cdot378v}{760}.$$

Since a litre is the volume of 1000 grammes of water at its maximum density, the above expression, divided by 1000, gives the ratio of the density of air to the maximum density of water. The logarithm of the ratio of the density of air at $t°$, to the maximum density of water, is obtained by adding the logarithm of $b-0\cdot378v$ in millimètres to $\log A_t$.

Table I.

$$10+\log 1\cdot293893-3-\log 760-\log(1+0\cdot003656\,t).$$

t.	$10+\log A_t$.	Diff.	t.	$10+\log A_t$.	Diff.
0	4·231085		15	4·207898	
1	4·229500	1585	16	4·206396	1502
2	4·227921	1579	17	4·204898	1498
3	4·226347	1574	18	4·203406	1492
4	4·224780	1567	19	4·201919	1487
5	4·223217	1563	20	4·200436	1483
6	4·221661	1556	21	4·198959	1477
7	4·220110	1551	22	4·197488	1471
8	4·218565	1545	23	4·196020	1468
9	4·217025	1540	24	4·194558	1462
10	4·215490	1535	25	4·193101	1457
11	4·213961	1529	26	4·191648	1452
12	4·212437	1524	27	4·190201	1447
13	4·210919	1518	28	4·188758	1443
14	4·209406	1513	29	4·187320	1438
15	4·207898	1508	30	4·185887	1433

The logarithms in the preceding Table, when diminished by 0·000028, serve for the reduction of the weighings in Somerset House, lat. 51° 30′ 40″, 29·56 mètres above the mean level of the sea; and when diminished by 0·000132, they may be used for the reduction of weighings in Paris.

According to a document to which the names of Biot, Regnault, and Bianchi are appended*, the pressure of vapour in rooms that are not heated artificially, in Paris, is two-thirds of the maximum pressure due to the temperature. It is probable that in Cambridge the hygrometric condition of the air is very nearly the same.

* Memorie di Matematica e di Fisica della Società Italiana della Scienze residente in Modena, tomo xxv. p. 1.

Table II.

Values of $0{\cdot}378 \times \frac{2}{3} v_t$, where v_t is the maximum pressure of vapour at the temperature t, in millimètres of mercury at 0°, according to Regnault's observations*.

t.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	1·16	1·17	1·18	1·18	1·19	1·20	1·21	1·22	1·23	1·24
1	1·25	1·25	1·26	1·27	1·28	1·29	1·30	1·31	1·32	1·33
2	1·34	1·35	1·36	1·37	1·37	1·38	1·39	1·40	1·41	1·42
3	1·43	1·44	1·45	1·46	1·47	1·48	1·49	1·50	1·51	1·53
4	1·54	1·55	1·56	1·57	1·58	1·59	1·60	1·61	1·63	1·64
5	1·65	1·66	1·67	1·68	1·69	1·70	1·72	1·73	1·74	1·75
6	1·76	1·78	1·79	1·80	1·81	1·82	1·84	1·85	1·86	1·87
7	1·89	1·90	1·91	1·93	1·94	1·95	1·96	1·98	1·99	2·01
8	2·02	2·03	2·05	2·06	2·08	2·09	2·10	2·12	2·13	2·15
9	2·16	2·17	2·19	2·21	2·22	2·24	2·25	2·27	2·28	2·29
10	2·31	2·32	2·34	2·35	2·37	2·39	2·40	2·42	2·44	2·45
11	2·47	2·48	2·50	2·52	2·53	2·55	2·57	2·58	2·60	2·62
12	2·64	2·65	2·67	2·69	2·71	2·72	2·74	2·76	2·78	2·80
13	2·81	2·83	2·85	2·87	2·89	2·91	2·93	2·94	2·96	2·98
14	3·00	3·02	3·04	3·06	3·08	3·10	3·12	3·14	3·16	3·18
15	3·20	3·22	3·24	3·26	3·28	3·31	3·33	3·35	3·37	3·39
16	3·41	3·43	3·46	3·48	3·50	3·52	3·54	3·57	3·59	3·61
17	3·63	3·66	3·68	3·71	3·73	3·75	3·78	3·80	3·82	3·85
18	3·87	3·90	3·92	3·95	3·97	4·00	4·02	4·04	4·07	4·09
19	4·12	4·15	4·17	4·20	4·22	4·25	4·28	4·30	4·33	4·36
20	4·38	4·41	4·44	4·47	4·49	4·52	4·55	4·58	4·61	4·63
21	4·66	4·69	4·72	4·75	4·78	4·81	4·84	4·87	4·90	4·92
22	4·95	4·99	5·02	5·05	5·08	5·11	5·14	5·17	5·20	5·23
23	5·26	5·30	5·33	5·36	5·39	5·43	5·46	5·49	5·52	5·56
24	5·59	5·62	5·66	5·69	5·73	5·76	5·80	5·83	5·87	5·90
25	5·93	5·97	6·01	6·04	6·08	6·12	6·15	6·19	6·22	6·26
26	6·30	6·34	6·37	6·41	6·45	6·49	6·53	6·56	6·60	6·64
27	6·68	6·72	6·75	6·79	6·83	6·87	6·91	6·95	6·99	7·03
28	7·08	7·12	7·17	7·21	7·25	7·29	7·34	7·38	7·42	7·46
29	7·51	7·55	7·59	7·64	7·68	7·73	7·77	7·82	7·86	7·91
30	7·95	8·00	8·04	8·09	8·14	8·18	8·23	8·28	8·32	8·37

Expansion of Brass.

The mean rate of expansion of cast brass from 0° to 100°, usually assumed 0·0000187 of its length at 0° for 1°, is considerably larger than the rate at ordinary atmospheric temperatures, according to the observations of Mr. Sheepshanks, who found that at about 17° the coefficient of the linear expansion of cast brass =0·00001722.

* Annales de Chimie, 1845, 3me série, tome xv. p. 138.

TABLE III.

Logarithms of the ratio of the density of brass at 0° to its density at $t°$, and their arithmetical complements, assuming the coefficient of linear expansion $=0·00001722$.

t.	log Br.	10 – log Br.	t.	log Br.	10 – log Br.		
0	0·0000000	10·0000000	16	0·0003588	9·9996412		
1	0·0000224	9·9999776	17	0·0003812	9·9996188		
2	0·0000449	9·9999551	18	0·0004037	9·9995963		
3	0·0000673	9·9999327	19	0·0004261	9·9995739		
4	0·0000897	9·9999103	20	0·0004485	9·9995515	0·1	22
5	0·0001122	9·9998878	21	0·0004709	9·9995291	0·2	45
6	0·0001346	9·9998654	22	0·0004933	9·9995067	0·3	67
7	0·0001570	9·9998430	23	0·0005158	9·9994842	0·4	90
8	0·0001795	9·9998205	24	0·0005382	9·9994618	0·5	112
9	0·0002019	9·9997981	25	0·0005606	9·9994394	0·6	135
10	0·0002243	9·9997757	26	0·0005830	9·9994170	0·7	157
11	0·0002467	9·9997533	27	0·0006054	9·9993946	0·8	179
12	0·0002691	9·9997309	28	0·0006277	9·9993723	0·9	202
13	0·0002916	9·9997084	29	0·0006502	9·9993498		
14	0·0003140	9·9996860	30	0·0006726	9·9993274		
15	0·0003364	9·9996636					

Expansion of Bronze.

TABLE IV.

Logarithms of the ratio of the density of the alloy adopted by Mr. BAILY for the standard bars at 0° to its density at $t°$, the coefficient of its linear expansion at 17° being 0·00001705, according to the observations of Mr. SHEEPSHANKS.

t.	log Ba.	10 – log Ba.	t.	log Ba.	10 – log Ba.		
0	0·0000000	10·0000000	16	0·0003553	9·9996447		
1	0·0000222	9·9999778	17	0·0003775	9·9996225		
2	0·0000444	9·9999556	18	0·0003997	9·9996003	0·1	22
3	0·0000666	9·9999337	19	0·0004219	9·9995781	0·2	44
4	0·0000888	9·9999112	20	0·0004441	9·9995559	0·3	67
5	0·0001111	9·9998889	21	0·0004663	9·9995337	0·4	89
6	0·0001333	9·9998667	22	0·0004884	9·9995116	0·5	111
7	0·0001555	9·9998445	23	0·0005106	9·9994894	0·6	133
8	0·0001777	9·9998223	24	0·0005328	9·9994672	0·7	155
9	0·0002009	9·9997991	25	0·0005550	9·9994450	0·8	178
10	0·0002221	9·9997779	26	0·0005772	9·9994228	0·9	200
11	0·0002443	9·9997557	27	0·0005994	9·9994006		
12	0·0002665	9·9997335	28	0·0006216	9·9993784		
13	0·0002887	9·9997113	29	0·0006437	9·9993563		
14	0·0003109	9·9996891	30	0·0006659	9·9993341		
15	0·0003331	9·9996669					

Expansion of Platinum.

SCHUMACHER has given two tables of the expansion of platinum, one calculated on the supposition that the coefficient of the linear expansion is 0·00000900, the other with the coefficient 0·00000909, as if in doubt which deserved the preference. STEINHEIL adopts 0·0000085655 in the reduction of the weighings of the platinum kilogramme des Archives. In comparing the standard platinum mètre with a glass mètre the expansion of which was known, at temperatures, however, not differing much from each other, he obtained 0·00000905 for the coefficient of expansion.

Table V.

Logarithms of the ratio of the density of platinum at 0° to its density at $t°$, and their arithmetical complements, assuming the coefficient of the linear expansion of platinum to be 0·00000900.

t.	log P.	10−log P.	t.	log P.	10−log P.		
0	0·0000000	10·0000000	16	0·0001876	9·9998124		
1	0·0000117	9·9999883	17	0·0001993	9·9998007		
2	0·0000235	9·9999765	18	0·0002110	9·9997880	0·1	12
3	0·0000352	9·9999648	19	0·0002227	9·9997773	0·2	23
4	0·0000469	9·9999531	20	0·0002345	9·9997655	0·3	35
5	0·0000586	9·9999414	21	0·0002462	9·9997538	0·4	47
6	0·0000704	9·9999296	22	0·0002579	9·9997421	0·5	59
7	0·0000821	9·9999179	23	0·0002696	9·9997304	0·6	70
8	0·0000938	9·9999062	24	0·0002813	9·9997187	0·7	82
9	0·0001055	9·9998945	25	0·0002931	9·9997069	0·8	94
10	0·0001173	9·9998828	26	0·0003048	9·9996952	0·9	105
11	0·0001290	9·9998710	27	0·0003165	9·9996835		
12	0·0001407	9·9998593	28	0·0003282	9·9996718		
13	0·0001524	9·9998476	29	0·0003399	9·9996601		
14	0·0001641	9·9998359	30	0·0003516	9·9996484		
15	0·0001759	9·9998241					

Expansion of Water.

The latest and best observations for determining the expansion of water by heat, are those of STAMPFER*, DESPRETZ†, PIERRE‡, tabulated by FRANKENHEIM§, KOPP‖, and PLÜCKER and GEISSLER¶. The observations of HALLSTRÖM and MUNCKE are not included in this list, on account of certain sources of error pointed out by DESPRETZ.

The following Table exhibits the ratios of the maximum density of water to its density at $t°$ C, according to the different observers above mentioned.

t.	Stampfer.	Despretz.	Pierre.	Kopp.	Plücker and Geissler.
Min.	1·000000	1·0000000	1·0000000	1·000000	1·000000
5	1·000012	1·0000082	1·0000091	1·000006	1·000013
6	1·000038	1·0000309	1·0000336	1·000026	1·000038
7	1·000079	1·0000708	1·0000716	1·000061	1·000079
8	1·000135	1·0001216	1·0001232	1·000109	1·000134
9	1·000205	1·0001879	1·0001882	1·000171	1·000204
10	1·000289	1·0002684	1·0002670	1·000247	1·000287
11	1·000387	1·0003598	1·0003580	1·000336	1·000381
12	1·000497	1·0004723	1·0004608	1·000437	1·000495
13	1·000620	1·0005862	1·0005745	1·000552	
14	1·000757	1·0007146	1·0007065	1·000679	
15	1·000906	1·0008751	1·0008463	1·000818	
16	1·001066	1·0010215	1·0009972	1·000969	
17	1·001239	1·0012067	1·0011592	1·001133	
18	1·001422	1·00139	1·0013320	1·001307	
19	1·001617	1·00158	1·0015153	1·001493	
20	1·001822	1·00179	1·0017128	1·001690	
21	1·002039	1·00200	1·0019185	1·001899	
22	1·002265	1·00222	1·0021296	1·002118	
23	1·002502	1·00244	1·0023498	1·002348	
24	1·002749	1·00271	1·0025836	1·002588	
25	1·003005	1·00293	1·0028263	1·002838	

* Jahrbücher des k. k. polytechnischen Institutes in Wien, B. xvi. S. 1.

† Annales de Chimie et de Physique, 1839, 2me série, tome lxx. p. 47.

‡ Ibid. 1845, 3me série, tome xv. p. 350.

§ POGGENDORFF's Annalen, B. lxxxvi. S. 451.

‖ Ibid. B. lxxii. S. 1.

¶ Ibid. B. lxxxvi. S. 238.

The discordances which these tables exhibit are partly due to errors in the assumed expansions of brass, glass and mercury, on which, by the nature of the experiments, the value of the expansion of water is made to depend.

STAMPFER deduced the expansion of water from the apparent weight of a hollow cylinder of brass suspended in water. The coefficient of the linear expansion of the brass was found to be 0·0000192, by experiments in which the variations of temperature amounted to from 38° to 62° (the absolute temperatures are not given). At about 17° Mr. SHEEPSHANKS found the coefficient of expansion of cast brass equal to 0·00001722. This is 0·00000153 less than the mean coefficient of expansion from 0° to 100°, assuming the latter to be 0·00001875. The error of the assumed expansion of the cylinder at ordinary atmospheric temperatures will probably be not quite so large. If taken equal to 0·00000133, the correction will be $-0·000004(t-4)$. DESPRETZ experimented with thermometers filled with water. The expansion of the glass was inferred from the apparent expansion of mercury in the thermometer from 0° to 28°, using for the coefficient of the expansion of mercury 0·00018018, the value obtained by DULONG and PETIT. But the expansion of mercury from 0° to 28° is 0·005032 according to REGNAULT*. The resulting mean coefficient of expansion is 0·00017971. Hence the expansions obtained by DESPRETZ must be diminished by $0·00000047(t-4)$. PIERRE and KOPP, who employed the same method, deduced the expansion of the glass from the apparent expansion of the mercury from 0° to 100°, assuming its absolute expansion between those points to be 0·018018. But the absolute expansion of mercury from 0° to 100° is 0·018153. The glass used by PIERRE contained oxide of lead, and probably had very nearly the same rate of expansion at both high and low temperatures. It is not known how far the glass used by KOPP possessed this property. Hence these expansions require the addition of $0·00000135(t-4)$. The observations of PLÜCKER and GEISSLER extend only to 12°. They were made with a thermometric apparatus the capacity of which is compensated by mercury so as to be invariable, or very nearly so. Assuming the expansion of mercury from 0° to 100° to be 0·018018, the cubic expansion of the glass from 0° to 100°, deduced from the apparent expansion of mercury, is 0·002818. But according to REGNAULT† the coefficient of the cubic expansion of a glass free from lead was 0·00002761 from 0° to 100°, and 0·00002628 from 0° to 10°. It is therefore probable that the coefficient of the cubic expansion of the glass has been taken 0·00000133 too great. Also the quantity of mercury used for compensating the expansion of the glass will be too small in the ratio of 0·000179714, the rate of expansion of mercury from 0° to 10°, to 0·00018153, the rate of expansion of mercury from 0° to 100°. Hence, upon the whole, the expansion must be diminished by $0·0000013(t-4)$.

The ratios of the maximum density of water to its density at $t°$, according to the

* Mémoires de l'Institut, tome xxi. p. 328. † Ibid. p. 237.

observations of STAMPFER, DESPRETZ, PIERRE, KOPP, and PLÜCKER and GEISSLER, respectively, after applying the corrections above indicated, become,—

t.	S.	D.	P.	K.	P. G.
Min.	1·000000	1·000000	1·000000	1·000000	1·000000
5	1·000008	1·000008	1·000010	1·000007	1·000013
6	1·000030	1·000030	1·000036	1·000029	1·000036
7	1·000067	1·000069	1·000076	1·000065	1·000076
8	1·000119	1·000120	1·000129	1·000114	1·000129
9	1·000185	1·000186	1·000195	1·000178	1·000198
10	1·000265	1·000266	1·000275	1·000255	1·000279
11	1·000359	1·000357	1·000368	1·000345	1·000372
12	1·000465	1·000469	1·000472	1·000448	1·000485
13	1·000584	1·000582	1·000587	1·000564	
14	1·000717	1·000710	1·000720	1·000693	
15	1·000862	1·000870	1·000861	1·000833	
16	1·001018	1·001016	1·001013	1·000985	
17	1·001187	1·001201	1·001177	1·001151	
18	1·001366	1·001383	1·001351	1·001326	
19	1·001557	1·001573	1·001536	1·001513	
20	1·001758	1·001783	1·001734	1·001712	
21	1·001971	1·001992	1·001942	1·001922	
22	1·002193	1·002212	1·002154	1·002142	
23	1·002426	1·002431	1·002376	1·002374	
24	1·002669	1·002701	1·002611	1·002615	
25	1·002921	1·002920	1·002855	1·002866	

On account of the uncertainty of the correction to be applied to STAMPFER'S observations, and the small range of those of PLÜCKER and GEISSLER, it appears best to rely exclusively on the corrected observations of DESPRETZ, PIERRE and KOPP. In the details of the observations, I am unable to detect any particular that warrants the acceptance or rejection of any one in preference to the other two. The mean of all three will probably approximate more nearly to the truth than any one of them taken by itself. Assuming 3°·945 C. to be the temperature at which the density of water is a maximum, in accordance with the result obtained by Messrs. PLAYFAIR and JOULE*, the logarithms of the means, to seven places of decimals considered as integers, are represented with sufficient accuracy, between 4° and 25°, by

$$32{\cdot}72(t-3{\cdot}945)^2-0{\cdot}215(t-3{\cdot}945)^3.$$

Comparison of the means of D, P, K with the formula.

t.	Means of D, P, K.	Formula.	t.	Means of D, P, K.	Formula.
3·945	1·000000	1·000000	15	1·000855	1·000854
5	1·000008	1·000008	16	1·001005	1·001009
6	1·000032	1·000031	17	1·001176	1·001175
7	1·000070	1·000069	18	1·001353	1·001352
8	1·000121	1·000121	19	1·001541	1·001540
9	1·000186	1·000186	20	1·001743	1·001739
10	1·000265	1·000265	21	1·001952	1·001948
11	1·000356	1·000357	22	1·002169	1·002167
12	1·000463	1·000460	23	1·002394	1·002396
13	1·000578	1·000581	24	1·002642	1·002634
14	1·000708	1·000712	25	1·002880	1·002882

* Memoirs of the Chemical Society, vol. iii. p. 199.

Table VI.

Logarithms of the ratios of the maximum density of water to its density at t, for every fifth of a centesimal degree, according to the formula.

t.	$\log W_t$.	Diff.
3·945	0·0000000	
4·2	0·0000002	
4·4	0·0000007	5
4·6	0·0000014	7
4·8	0·0000024	10
5·0	0·0000036	12
5·2	0·0000051	15
5·4	0·0000069	18
5·6	0·0000089	20
5·8	0·0000111	22
6·0	0·0000136	25
6·2	0·0000164	28
6·4	0·0000194	30
6·6	0·0000227	33
6·8	0·0000262	35
7·0	0·0000299	37
7·2	0·0000339	40
7·4	0·0000381	42
7·6	0·0000426	45
7·8	0·0000474	48
8·0	0·0000524	50
8·2	0·0000576	52
8·4	0·0000630	54
8·6	0·0000687	57
8·8	0·0000747	60
9·0	0·0000808	61
9·2	0·0000872	64
9·4	0·0000939	67
9·6	0·0001007	68
9·8	0·0001079	72
10·0	0·0001152	73
10·2	0·0001228	76
10·4	0·0001306	78
10·6	0·0001386	80
10·8	0·0001468	82
11·0	0·0001553	85
11·2	0·0001640	87
11·4	0·0001729	89
11·6	0·0001821	92
11·8	0·0001915	94
12·0	0·0002011	96
12·2	0·0002109	98
12·4	0·0002209	100
12·6	0·0002312	103
12·8	0·0002416	104
13·0	0·0002523	107
13·2	0·0002632	109
13·4	0·0002743	111
13·6	0·0002857	114
13·8	0·0002972	115
14·0	0·0003089	117
14·2	0·0003209	120
14·4	0·0003331	122
14·4	0·0003331	
14·6	0·0003455	124
14·8	0·0003580	125
15·0	0·0003708	128
15·2	0·0003838	130
15·4	0·0003970	132
15·6	0·0004104	134
15·8	0·0004240	136
16·0	0·0004378	138
16·2	0·0004518	140
16·4	0·0004660	142
16·6	0·0004804	144
16·8	0·0004950	146
17·0	0·0005098	148
17·2	0·0005248	150
17·4	0·0005400	152
17·6	0·0005554	154
17·8	0·0005709	155
18·0	0·0005867	158
18·2	0·0006026	159
18·4	0·0006187	161
18·6	0·0006351	164
18·8	0·0006516	165
19·0	0·0006682	166
19·2	0·0006851	169
19·4	0·0007022	171
19·6	0·0007194	172
19·8	0·0007368	174
20·0	0·0007544	176
20·2	0·0007722	178
20·4	0·0007902	180
20·6	0·0008083	181
20·8	0·0008266	183
21·0	0·0008451	185
21·2	0·0008637	186
21·4	0·0008826	189
21·6	0·0009016	190
21·8	0·0009207	191
22·0	0·0009400	193
22·2	0·0009596	196
22·4	0·0009793	197
22·6	0·0009991	198
22·8	0·0010191	200
23·0	0·0010393	202
23·2	0·0010596	203
23·4	0·0010801	205
23·6	0·0011008	207
23·8	0·0011216	208
24·0	0·0011426	210
24·2	0·0011637	211
24·4	0·0011850	213
24·6	0·0012065	215
24·8	0·0012281	216
25·0	0·0012498	217

Reduction of Weighings.

If the weights P and Q appear to be equal when compared in air,—
weight of P— weight of air displaced by P=weight of Q— weight of air displaced by Q.

Let t be the temperature of the air in centesimal degrees, b its pressure in millimètres of mercury at 0°, v the pressure of the vapour contained in it, also in millimètres of mercury, $h=b-0{\cdot}378v$, ΔP, ΔQ ratios of the densities of P and Q at 0° to the maximum density of water; ePt, eQt the expansions in volume of P and Q. Then log weight in grains of the air displaced by P

$$=\log h+\log A_t+\log(1+ePt)+\log \text{ weight of P in grains} - \log \Delta P.$$

If vP be taken to denote the volume of P at 0°, the unit of volume being the volume of a grain of water at its maximum density,

$$\log vP = \log \text{ weight of P in grains} - \log \Delta P.$$

The expression for the weight of air displaced by Q differs from the above only in the substitution of Q for P.

The value of h is deduced from b by means of Table II., assuming that the amount of vapour in the air is two-thirds of the quantity in saturated air. Table I. gives the second term for the expression for the weight of the air displaced, and Tables III., IV., V. give the third term according as the weight is of brass, bronze or platinum.

Calculation of Densities.

Let P in water at $t°$ appear to weigh as much as Q in air. Then weight of water at $t°$ displaced by P= weight of P− weight of Q+ weight of air displaced by Q, $\log vP = \log$ weight in grains of the water displaced by $P+\log W_t-\log(1+ePt)$, where W_t is the ratio of the maximum density of water to its density at $t°$ obtained from Table VI. $\log \Delta P = \log$ weight of P in grains $-\log vP$.

An approximate value of vP having been found by assuming the weight of P equal to its apparent weight in air, this value of vP may be used in reducing the weight of P, and thus a more accurate value of vP obtained, by means of which a closer approximation to the values of the absolute weight of P and of ΔP may be found. This process is to be repeated when greater exactness is required.

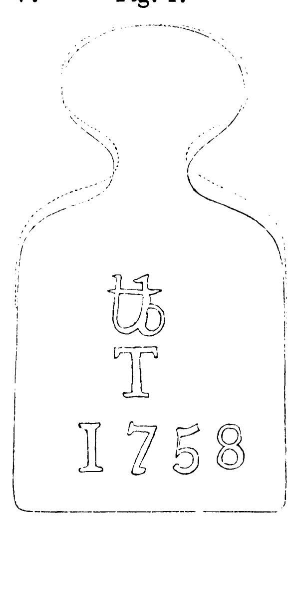
Densities of the Troy Pounds constructed in 1758.

Though it appears that only two of the five weights with which U was compared are in a state of unexceptionable preservation, and the number of trustworthy comparisons is reduced from 608 to 440, these are amply sufficient for the purpose of ascertaining the apparent weight of U in air ($t=65{\cdot}66$ F, $b=29{\cdot}75$ inches). But, in order to find the absolute weight of U, or indeed its apparent weight in air of a density different from that which it has when $t=65{\cdot}66$, $b=29{\cdot}75$, a knowledge of the volume of the lost standard is requisite. It is not probable that U was ever weighed in water, and certainly no record of any such weighing is known to exist. There is therefore no direct method of finding its volume. An indirect way of arriving at it was suggested to Professor SCHUMACHER by an examination of three Parliamentary Reports, the first presented May 26, 1758, the second April 11, 1759, and a third ordered to be printed March 2, 1824.

It appears from the first of these Reports that Mr. Harris, then Assay-Master of the Mint, presented to the first Committee three troy pounds made under his direction, one of which was the lost Imperial Standard Troy Pound. The third Report contains the evidence of Dr. Kelly, who in reply to the query, "What was effected with regard to weights and measures by the Committee of 1758?" answers, "They ordered three several troy pounds to be adjusted, under the direction of Mr. Harris, the then Assay-Master of the Mint. One of these was placed in the custody of the Clerk of the House of Commons; another was left with Mr. Harris, and is now in the possession of Mr. Bingley; and the third was, I understand, delivered to Mr. Freeman, weight-maker to the Mint, the Exchequer and the Bank of England, who used it as his standard, and it is still so employed by his successor Mr. Vandome." The same page contains the following note:—This weight [Mr. Bingley's pound] was produced to the Committee, by Mr. Bingley, who said it had formerly belonged to Mr. Harris when he held the situation of Assay-Master. There was a memorandum on the lid of the box in which it was kept, stating that Mr. Harris had made from it the pound weight which was placed in the custody of the Clerk of the House of Commons by direction of the Committee of 1758, and which is commonly called the Parliamentary Pound.

Professor Schumacher then observes that "if Dr. Kelly's statements be exact, as there is no doubt they are, and Messrs. Bingley's and Vandome's pound be *really* the two remaining weights of the often mentioned three which Mr. Harris presented to the Committee of 1758, we can still either determine, with the highest degree of probability, the specific gravity of the lost Imperial standard troy pound, or know with certainty that all hope to arrive at this knowledge is lost. It will be only requisite to ascertain with the greatest care the specific gravities of both pounds, the one in the possession of Mr. Bingley, the other in the possession of Mr. Vandome. If the specific gravity of both is found the *same*, we might from that circumstance draw the highly probable conclusion, that the three single pounds of Mr. Harris, according to my hypothesis, were really made of the same identical metal; and the specific gravities of the two remaining pounds might with safety be considered as that of the lost standard. If, on the contrary, the two remaining pounds prove to be of *different* specific gravities, the hypothesis that all three were made of the same metal is evidently erroneous, and nothing can be inferred from the specific gravity of either of the two remaining."

V. Fig. 1.



These two weights were found to be still in existence. Mr. Vandome readily consented to allow the troy pound in his possession to be experimented upon by the Committee. In form and size this weight very closely resembles the figure of the lost standard given by Captain v. Nehus. The

upright stroke of the 5 in the type appears to have been broken off, and the defect supplied in the inscription by a cut with a chisel. The letters SF are impressed diametrically opposite to the T. This weight, as well as the others of the same date, is of one piece of metal, without any cavity for adjustment by the addition of bits of wire. Mr. SIMMS, to whom it was shown, pronounced it to be of soft gun-metal, as hard as cast brass, but not so hard as hammered brass, and, for such an alloy, a very bad casting.

As the balance ordered of Mr. BARROW was not yet ready, Mr. VANDOME's troy pound (V) was weighed in water with a balance of $10\frac{1}{2}$ inches beam by ROBINSON. The case of this was too small to admit a large cylinder of water, the use of which is considered essential to the accuracy of observations of this kind, and some unaccountable discordances in the weighings of V in air impair the probable accuracy of the result. For these reasons the result alone is given, omitting the details of the observations. By a mean of six weighings in water in July and August, 1843, the density of V at 0° C. appeared to be 8·15105 times the maximum density of water. This value, notwithstanding its uncertainty, was sufficiently exact for a preliminary comparison of the densities of the weights made in 1758. The accurate determination of the density of V and of the other weights of the same date presents considerable difficulty; for the pores in the metal are so deep, that the complete expulsion of the air contained in them is very questionable, even after prolonged immersion in boiling water.

The following observations were made with BARROW's balance under circumstances more favourable to accuracy. The glass jar containing the distilled water in which V was weighed, was 6·7 inches in diameter and 6·5 inches deep. V was suspended, from the pan of the balance, by a hook attached to a fine copper wire, 7·5 inches of which weighed about one grain. In order to expel the air adhering in bubbles to the weight, or contained in the cavities in the metal, it was placed, with the fine wire attached to it, in water in a bell-shaped jar of thin glass, just large enough to contain the weight. The jar was suspended over the flame of a spirit-lamp by a stout wire bent at its lower end into a ring into which the jar descended to its rim, and the water allowed to boil till it was supposed that the air was entirely expelled. The small jar containing the weight was then immersed in the water which very nearly filled the large jar, the suspending wire hooked on to the under side of the scale pan, and the small jar lowered till the weight hung clear of it, and then removed. The transfer of the weight from the small jar to the large one was thus effected without taking it out of the water. The counterpoise was placed in the left-hand pan of the balance; V was suspended in water from the right-hand pan. Small weights were placed in the right-hand pan till equilibrium was produced, and the readings of the scale observed. V was then removed, leaving the hook suspended in water, and a volume of water equal to that of V added to the water in the jar; the weights A, B, C, D, &c. were placed in the right-hand pan till equilibrium was again produced, and the

readings of the scale observed. The thermometer B was suspended with its bulb in a horizontal plane through the centre of gravity of V. The thermometer C was in the balance case. F denotes the reading of Ernst's barometer, E that of the attached thermometer. 100 parts of the micrometer scale $=0{\cdot}2$ grain nearly, when V is suspended in water, or when the hook alone is suspended in water, and the right-hand pan contains 5053·3 grains.

Observations for finding the density of V.

Weighing of V in air.

1845, July 7.

100 parts$=0{\cdot}3302$ grain.

$S=A+B+C+D+F.$

$V+\overset{\text{gr.}}{0{\cdot}31}+X \simeq S+Y+\overset{\text{pt.}}{0{\cdot}50}$	$V+\overset{\text{gr.}}{0{\cdot}31}+Y \simeq S+X+\overset{\text{pt.}}{0{\cdot}20}$
+0·25	+0·30
−0·50	+0·40
0·00	+0·65
+0·20	+1·00
−0·20	+0·80
−0·10	+1·15
0·00	+0·75
+0·20	+0·80
+0·20	+0·70
+0·055	+0·675

$$V+\overset{\text{gr.}}{0{\cdot}31} \simeq A+B+C+D+F+\overset{\text{gr.}}{0{\cdot}0012}.$$

$V \simeq 5759{\cdot}160$ grains of platinum in air ($t=16{\cdot}13$, $b=761{\cdot}27$).

Weighing of V in water.

1845, April 2.

	V and hook in water.			In right-hand pan.	
B.	C.	F.	E.	gr.	Scale.
				0·14	18·0
8·5	8	765·8	8·5	0·14	20·0
				0·14	20·0
				0·14	19·6
				0·18	26·5
				0·10	12·3
8·45				0·14	20·1
				0·14	20·0
				0·14	20·3
				0·14	20·2
8·47	8	765·8	8·5	0·140	19·7

Hook in water.	In right-hand pan. gr.	Scale.
$A+B+C+D+L+(8)+(4)+(1)+$	0·80	20·0
	0·80	18·5
	0·80	22·5
	0·80	19·2
	0·80	19·4
	0·80	19·6
	0·80	20·2
	0·800	19·9

V in water ($B=8{\cdot}47$) $\simeq A+B+C+D+L+(8)+(4)+(1)+\overset{\text{gr.}}{0{\cdot}7596}$ in air ($C=8$, $F=765{\cdot}8$, $E=8{\cdot}5$).

May 2. V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
				0·24	22·8
				0·20	20·4
10·5	11·1	758·3	11·5	0·20	18·0
				0·24	19·1
				0·24	25·0
				0·20	21·0
				0·20	21·5
				0·16	12·0
				0·20	23·7
				0·16	17·0
				0·20	22·5
10·55				0·16	14·5
10·52	11·1	758·3	11·5	0·200	19·81

Hook in water. In right-hand pan.

	gr.	Scale.
A+B+C+D+N+(8)+(4)+(1)	+0·92	19·2
	+0·92	20·0
	+0·92	20·0
	0·920	19·7

V in water (B=10·52)≏A+B+C+D+N+(8)+(4)+(1)+0·7202 gr. in air (C=11·1, F=758·3, E=11·5).

June 28. V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
14·45				1·08	20·6
				1·08	20·7
14·5				1·08	22·0
14·5	15·7			1·07	19·0
		744·5	16	1·08	22·3
14·6	15·85			1·07	20·0
				1·07	20·3
				1·07	20·4
14·51	15·77	744·5	16	1·075	20·66

Hook in water. In right-hand pan.

	gr.	Scale.
A+B+C+D+N+(8)+(4)+(2)	+0·88	22·8
	0·87	19·7
	0·87	19·5
	0·88	22·5
	0·87	19·8
	0·87	19·2
	0·88	24·2
	0·87	20·0
	0·8737	20·96

June 28. V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
14·75	15·7			1·08	27·6
				1·06	21·8
				1·04	14·5
		747·9	15·7	1·06	22·0
				1·05	18·0
				1·06	21·5
14·75				1·05	17·5
				1·06	21·1
				1·05	19·2
14·75	15·7	747·9	15·7	1·0567	20·35

V in water (B=14·63)≏A+B+C+D+N+(8)+(4)+(2)−(1)+0·807 gr. in air (C=15·73, F=746·2, E=15·85).

July 1. V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
	14·7	750·8	15·2	1·08	19·5
				1·08	19·4
14·35				1·08	20·2
				1·08	20·5
				1·08	20·3
				1·08	20·8
14·4				1·08	21·1
14·37	14·7	750·8	15·2	1·080	20·3

Hook in water. In right-hand pan.

	gr.	Scale.
$A+B+C+D+N+(8)+(4)+(2)+$	0·90	22·0
	0·87	19·5
	0·88	20·9
	0·87	19·7
	0·87	19·8
	0·87	18·7
	0·88	20·5
	0·87	18·1
	0·88	20·5
	0·88	20·6
	0·877	20·03

V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
14·7				1·08	18·8
	16	750·8	16·3	1·09	19·6
				1·09	20·1
14·75				1·09	21·6
				1·08	20·3
14·85				1·08	20·3
				1·08	20·8
				1·07	19·9
				1·07	20·5
14·95				1·07	21·5
14·83	16	750·8	16·3	1·080	20·34

V in water (B=14·6) $\triangleq$ A+B+C+D+N+(8)+(4)+(2)−(1)+0·798 gr. in air (C=15·35, F=750·8, E=15·75).

July 2. V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
14·3	14·6	759·2	15·1	1·07	18·2
				1·08	19·2
14·3				1·08	20·4
				1·07	19·1
				1·08	19·8
				1·08	20·0
14·4				1·08	20·3
				1·08	20·8
14·4	15·3	759·4	15·7	1·08	21·1
14·35	14·95	759·3	15·4	1·0778	19·9

Hook in water.	In right-hand pan.	
	gr.	Scale.
A+B+C+D+N+(8)+(4)+(2)+	0·88	23·1
	0·88	22·2
	0·87	20·1
	0·87	20·0
	0·87	19·6
	0·87	20·2
	0·87	19·7
	0·87	18·4
	0·87	18·0
	0·88	22·0
	0·87	18·8
	0·8727	20·19

V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
14·6	15·6	759·5	15·8	1·05	19·9
				1·05	19·9
				1·05	20·6
				1·05	20·0
14·65				1·05	20·0
				1·05	19·8
				1·05	20·0
14·65	15·6	759·6	15·9	1·05	20·7
14·63	15·6	759·55	15·85	1·050	20·1

V in water (B=14·49)≏A+B+C+D+N+(8)+(4)+(2)−(1)+0·808 gr. in air (C=15·27, F=759·42, E=15·62).

	t.	grs. of platinum.	t.	b.
V in water...	8·49	≏5053·194 in air	7·99	765·19
V in water...	10·49	≏5053·257 in air	11·09	757·34
V in water...	14·58	≏5053·344 in air	15·76	744·73
V in water...	14·55	≏5053·335 in air	15·38	749·33
V in water...	14·44	≏5053·345 in air	15·30	757·95

Where t is the temperature in centesimal degrees, b the height of the mercury in the barometer in millimètres corrected and reduced to 0° C.

For the platinum of which the weights, A, B, C, &c. are made $\log \Delta = 1{\cdot}32564$.

The resulting values of vV, ΔV and log ΔV are,—

vV.	ΔV.	log ΔV.
706·580	8·1515	0·911238
706·555	8·1518	0·911253
706·661	8·1505	0·911188
706·669	8·1505	0·911183
706·656	8·1506	0·911191

Means, giving to the third, fourth and fifth results twice the weight of the first and second, because each of these was deduced from two separate series of weighings in water, while the first and second were deduced each from one series of weighings in water,—

vV.	ΔV.	log ΔV.
706·638	8·15084	0·911202

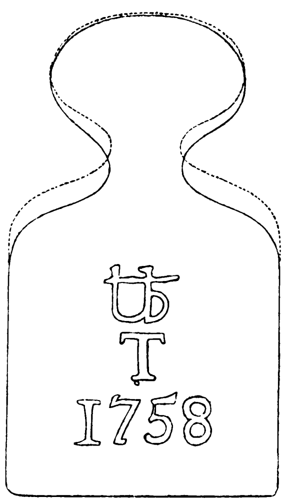
In air (t=16·13, b=761·27). V≏A+B+C+D+F−0·3088 gr. But A+B+C+D+F≏T−0·0017 gr. Therefore V≏T−0·3105 gr.

V displaces $\overset{\text{gr.}}{0{\cdot}8616}$, and T$-0{\cdot}310$ displaces $\overset{\text{gr.}}{0{\cdot}3317}$ of air ($t=16{\cdot}13$, $b=761{\cdot}27$). Hence $V=T+0{\cdot}2194$ grain.

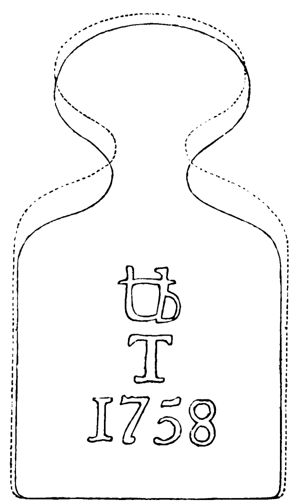
V displaces $\overset{\text{gr.}}{0{\cdot}8468}$, and T$-\overset{\text{gr.}}{0{\cdot}310}$ displaces $\overset{\text{gr.}}{0{\cdot}3261}$ of air ($t=18{\cdot}7$, $b=755{\cdot}64$). Hence, in air ($t=18{\cdot}7$, $b=755{\cdot}64$), $V \bumpeq T-0{\cdot}3013$ gr. But $T \bumpeq U-0{\cdot}0074$ gr. Therefore $V \bumpeq U-0{\cdot}3087$ grain in air ($t=18{\cdot}7$, $b=755{\cdot}64$).

Mr. Bingley had in his possession two troy pounds of the same date. One of these (O) is said to be the original weight from which the standard was made for the House of Commons in 1758. It is distinguished by a small dot under the T, and the imperfection in the type of the 5 is remedied by a cut with a chisel as in V. This weight has since (in 1851) been purchased by the Committee. The other (M), in which the 5 is left imperfect, and which has the mark ◎ ◎ impressed on its under surface, has since been presented to the Mint by its former possessor.

O. Fig. 2.



M. Fig. 3.



Mr. Bingley was unwilling to permit either of these troy pounds to be weighed in water; Messrs. Troughton and Simms were therefore commissioned to construct an instrument on the principle of the Stereometer invented by M. Say for the purpose of determining the specific gravity of gunpowder*, but with some improvements which I had described in the Philosophical Magazine for July to December, 1834, vol. v. p. 203. It consists of two glass tubes, PQ, DB (fig. 4), of equal diameter, cemented into cylindrical cavities communicating with each other at their lower ends, in an oblong piece of iron G. In the axes of the two cavities are holes concentric with the tubes. The hole under PQ is closed by a screw K, into the other is screwed an iron stopcock L. The upper end of the tube PQ is cemented into an iron cylinder N carrying a ring which surrounds the upper end of the tube DB. The inside of the cylinder is tapped to receive the screw of the stopcock, and the outside tapped so as to screw into the under end of a cup F, having its rim ground plane,

* Annales de Chimie, 1797, tome xxiii. p. 1.

and capable of being closed so as to be air-tight by a plate of glass E, smeared with lard. The tube PQ is graduated by lines traced upon the glass. The original tube, graduated in inches, having been broken, was replaced by a tube graduated in centimètres by M. BUNTEN. The subdivision is effected by an ivory scale S, of ten millimètres divided on the side next to the glass tube, to every fifth of a millimètre, attached to a rectangular rod of deal carrying frames on which filaments of silk TU, VW are stretched, and slips of brass having eye-holes so adjusted that the planes through the threads and the corresponding eye-holes may be perpendicular to the rod, the tubes being between the eye-holes and the threads, as shown in the section fig. 5. A weak brass spring attached to the rod keeps it in contact with the tubes, with the silk threads and ivory scale close to that part of PQ which is graduated, so that it can be easily moved up and down, and is retained in the position in which it is left by the pressure of the spring. The support of the stereometer is adjusted by three foot screws till a thread of unspun silk by which a small weight is suspended, hangs coinciding with the axis of the tube DB. Within E is a cup in which is placed the solid the volume of which is sought. Mercury having been poured into D till its surface rises to P, the first division of the graduation, the mouth of the cup is closed so as to be air-tight by the plate of glass. The stopcock is then opened and the mercury allowed to escape till the difference of the altitude of the mercury in the two tubes is nearly equal to half the height of the mercury in the barometer at the time of the observation. Let the point M of the graduation mark the height of the mercury in PQ, and C the height of the mercury in DB. Let u be the volume of the air in the cup F before the solid was placed in it; v the volume of the solid; b the altitude of the mercury in the barometer reduced to the temperature of the mercury in PQ and BD. Then

Fig. 5.

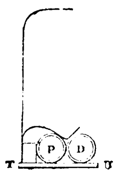
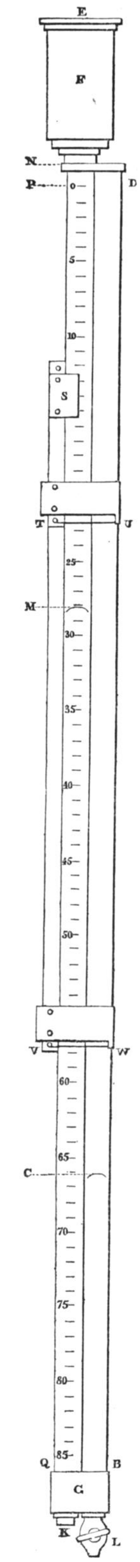


Fig. 4.



$$u-v=\frac{b-\text{MC}}{\text{MC}}\text{ vol. PM.}$$

In order to find the capacity of the portion of the tube included between P and any point M in the graduation, the cup F is taken off, and the stopcock L screwed into the iron collar N. The screw K is taken out, and the tubes placed vertical in an inverted position. The tube PQ is then filled up to about 50 cm with mercury poured through a slender glass tube inserted into the opening at K. This precaution is necessary in order to prevent the formation of air-bubbles on the inner surface of the tube, which would interfere with the correct estimation of the capacity of the tube. The stopcock is then opened, and the mercury contained in a

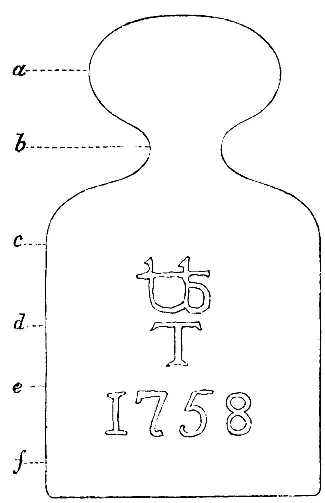
known number of divisions of the tube suffered to run into a light glass jar in which it is weighed. This process is to be repeated till the upper end of the column of mercury descends to the point P.

The stereometer was mounted in a room in Mr. BINGLEY's house at the Mint, September 12, 1843, and a few comparisons made of the volumes of V, O, M. The results, however, in consequence of the unequal heating of different parts of the stereometer in putting it together, did not prove satisfactory. On the 16th, the volumes of O, M and C, a hollow cylinder of brass, were compared with better success. The unit of volume being the volume of a grain of water at its maximum density, these observations gave $v\mathrm{O}+5{\cdot}3=v\mathrm{M}=v\mathrm{C}-0{\cdot}6$.

By observations made August 19, 1843, C $\bumpeq$ 600·001 grains of brass +78·832 grains of platinum in air ($t=24{\cdot}23$, $b=756{\cdot}78$). C in water ($t=18{\cdot}1$) $\bumpeq -9{\cdot}707$ grains of platinum. Hence C in air ($t=24{\cdot}23$, $b=756{\cdot}78$) $\bumpeq$ C in water ($t=18{\cdot}1$) +600·001 grains of brass +88·539 grains of platinum in air ($t=24{\cdot}23$, $b=756{\cdot}78$). The weights displace 0·092 grain of air; C displaces 0·810 grain of air. Hence C displaces 689·258 grains of water at 18·1, and the volume of C at 0° is equal to the volume of 689·562 grains of water at its maximum density. Hence $v\mathrm{O}=683{\cdot}66$, $v\mathrm{M}=688{\cdot}96$. By weighing in air and in water it was found that $v\mathrm{V}=706{\cdot}34$. The large differences between these numbers show that the volume of the lost standard cannot be inferred with any high degree of probability from a comparison of the volumes of the three remaining pounds.

The only resource now remaining was indicated by Professor SCHUMACHER's remarks on the figure of the lost troy pound:—"As soon as the Imperial standard troy pound was brought to Somerset House, Captain NEHUS's first care was to make an accurate drawing of its shape and marks, measuring all its dimensions with the greatest care. The annexed drawing represents this pound in its actual dimensions, and is now, since the original has been destroyed by the calamitous fire that consumed the two Houses of Parliament in 1834, the only thing remaining which can preserve an idea of it." An application was made to Professor SCHUMACHER for the original drawing, if still in existence, or for any information that would show how far the accuracy of the wood engraving might be depended upon. In his reply, dated October 3, 1843, he wrote as follows:—"The dimensions of the lost standard were only taken with a bow-legged compass in order to give an accurate *drawing* of the standard pound, and in this respect I called them in my papers accurate, but they certainly are not sufficient to give a near approximation of its volume. I find that he (Captain v. NEHUS) has immediately transferred the taken dimensions to paper. This paper, with the original drawing, has served to give the woodcut in the Philosophical Transactions, but to my best recollection Mr. BAILY

U. Fig. 6.



has returned it, though I cannot find it amongst my papers. Even if you had NEHUS's original drawing you would not be able to find the volume, the only height he has *measured* being that of the whole pound. The heights of the points *a*, *b*, *c* are only found by holding a scale in a vertical position near the pound. The diameters on the contrary are measured."

By a comparison of the figure of U with a profile of V traced mechanically, and with careful measurements of its axis and diameters, the axis and the extreme diameters of the knob and cylindrical portion of U appear to be a very little greater than the corresponding dimensions of V, the differences in other parts being exactly where we might expect the drawing to be inaccurate from the manner in which it was made. (In the figures of the weights V, O, M, B, the dotted line is the profile of U.) The diameters and axis of U being measured with a bow-legged compass, were more likely to err in excess than in defect. Making every allowance for this, it did not seem possible, on looking at the profiles of U and V, to suppose that the volume of U was less than that of V. But the volume of O, as well as that of M, being less than that of V, it appeared that of the three weights V, O, M, V approximated most nearly to U in volume. As the existing data were utterly insufficient to determine how much, if at all, U exceeded V in volume, it appeared safest to assume the volumes of U and V to have been equal. This course was recommended also by Professor SCHUMACHER in his letter of October 3, 1843.

Long after this resolution had been taken and acted upon, and the new standard constructed in accordance therewith, the troy pound O came by purchase into the hands of the Committee. The surface of O was studded with numerous small pores, showing it to be an extremely bad casting. It was only after repeatedly boiling the water in which it was suspended that the air-bubbles which attached themselves to the pores ceased to appear. It was weighed in water April 2, and then left to dry till April 28, when on being weighed in air it appeared to be about 16 grains too heavy. By heating it to above the boiling-point, the water that had been retained in the cavities was expelled, and the weight reduced to 5759·83 grains. Afterwards, by placing it in a jar containing water, under the receiver of an air-pump, and alternately exhausting the receiver and boiling the water, the cavities communicating with the surface were found capable of containing 21·37 grains of water. This explains the seeming paradox, that although the linear dimensions of O are hardly less than those of V, and sensibly greater than those of M and B, its specific gravity is considerably greater than that of V, and slightly exceeds that of either M or B.

Of the weights used in the following weighings, those marked (100), (200), (400) ..., of nearly 100, 200, 400 ... grains respectively, are of bronze, for which $\log \Delta = 0{\cdot}92260$. The smaller weights are of platinum. By a mean of four observations, April 30, 1853,

$O \triangleq (3200)+(1600)+(800)+(100)+(32)+(16)+(8)+(4)-0{\cdot}1360$ grain in air $(D=12,\ C=12{\cdot}2,\ F=756{\cdot}4,\ E=11{\cdot}8)$.

$O \triangleq 5699{\cdot}9704$ grains of bronze $+59{\cdot}8607$ grains of platinum in air $(t=12{\cdot}1,\ b=755{\cdot}4)$.

By one observation, April 2, 1853, O in water ($B=10{\cdot}4$) $\stackrel{\frown}{=}$ (3200)+(1600)+(200) (bronze)+(64)+(16)+(2)+(1)−(8)−0·075 grain (platinum) in air ($D=10{\cdot}9$, $C=11$, $F=754{\cdot}7$, $E=10{\cdot}8$).

O in water ($t=10{\cdot}16$) $\stackrel{\frown}{=}$ 4999·9640 grains of bronze + 74·7759 grains of platinum in air ($t=10{\cdot}96$, $b=753{\cdot}82$).

Hence $\quad v\mathrm{O}=685{\cdot}665, \quad \Delta\mathrm{O}=8{\cdot}40036, \quad \mathrm{O}=5759{\cdot}8333$ grains.

The weight of O is given more accurately by the following observations :—

July 15, 1853. $D=17{\cdot}55$, $C=17{\cdot}6$, $F=747{\cdot}55$, $E=17{\cdot}6$. 100 parts=0·3916 grain.

Left-hand. gr.	Right-hand.	Scale.	Left-hand.	Right-hand. gr.	Scale.
O+0·1113+X	T+Y	27·15	T+Y	O+0·1113+X	16·66
O+0·13 +X	T+Y	21·06	T+Y	O+0·13 +X	20·50
O+0·13 +X	T+Y	21·15	T+Y	O+0·13 +X	20·97
O+0·13 +Y	T+X	21·70	T+X	O+0·13 +Y	19·79
O+0·13 +Y	T+X	22·04	T+X	O+0·13 +Y	20·45
O+0·13 +Y	T+X	21·41	T+X	O+0·13 +Y	20·59

O $\stackrel{\frown}{=}$ T−0·13196 grain in air ($t=17{\cdot}61$, $b=745{\cdot}87$).

O displaces 0·8146 grain, T−0·132 grain displaces 0·3231 grain of air ($t=17{\cdot}61$, $b=745{\cdot}87$). Hence O=T+0·3595 grain.

In air ($t=18{\cdot}7$, $b=755{\cdot}64$), O $\stackrel{\frown}{=}$ T−0·1363 gr., and T $\stackrel{\frown}{=}$ U−0·0074 gr. Therefore O $\stackrel{\frown}{=}$ U−0·1437 gr.

By the good offices of Sir J. F. W. HERSCHEL, at that time Master of the Mint, permission was obtained from the Treasury to weigh M in air, and to repeat the observations with the stereometer for finding its volume with more care and more leisurely than on the former occasion, when they were made in Mr. BINGLEY's house.

By three series of comparisons made with the stereometer,

$$\text{May 11, 12, 1855} \quad . \; . \; . \quad v\mathrm{M}=v\mathrm{O}+4{\cdot}33=690{\cdot}00$$

$$\text{September 16, 1843} \quad . \; . \quad \begin{cases} v\mathrm{M}=v\mathrm{C}-0{\cdot}6 \;=688{\cdot}96 \\ v\mathrm{M}=v\mathrm{O}+5{\cdot}3 \;=690{\cdot}97 \end{cases}$$

According to a mean of all three results, the volume of M at 0° is equal to the volume of 689·98 grains of water at its maximum density.

Comparison of M with T, July 14, 1853.

$D=18{\cdot}2$, $C=18{\cdot}3$, $F=743{\cdot}2$, $E=17{\cdot}7$. 100 parts=0·3916 grain.

Left-hand. gr.	Right-hand.	Scale.	Left-hand.	Right-hand. gr.	Scale.
M+0·06+Y	T+X	15·30	T+X	M+0·06+Y	24·55
M+0·04+Y	T+X	20·02	T+X	M+0·04+Y	18·80
M+0·04+Y	T+X	20·31	T+X	M+0·04+Y	18·97
M+0·04+X	T+Y	19·05	T+Y	M+0·04+X	17·59
M+0·04+X	T+Y	18·12	T+Y	M+0·04+X	18·44
M+0·04+X	T+Y	18·94	T+Y	M+0·04+X	19·01

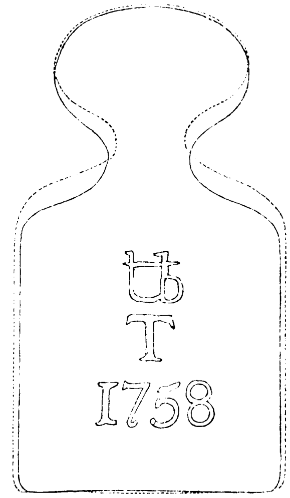
M $\stackrel{\frown}{=}$ T−0·0415 grain in air ($t=18{\cdot}28$, $b=741{\cdot}52$).

M displaces 0·81284 grain, T − 0·0415 grain displaces 0·32045 grain of air ($t=18·28$, $b=741·52$). Hence $M=T+0·4509$ grain. $\Delta M=8·3491$.

In air ($t=18·7$, $b=755·64$), $M \bumpeq T-0·04012$ gr. But $T \bumpeq U-0·0074$ gr. Therefore $M \bumpeq U-0·0475$ gr.

In a letter from William Miller, Esq., of the Bank of England, dated August 22, 1855, I was apprised of the existence at the Bank of a fourth troy pound of 1758, and soon afterwards received it from him with permission to weigh it in air and also in water. This weight (B) is a bad casting, though much better than O. The upright stroke of the 5 is left incomplete. The under surface is slightly concave, the depth of the concavity being about 0·01 inch.

B. Fig. 7.



By one comparison, October 6, 1855, $B \bumpeq (3200)+(1600)+(800)+(100)$ (bronze) $+(32)+(16)+(8)+(4)+0·271$ grain (platinum) in air ($C=16·0$, $F=746·3$, $E=16·4$).

$B \bumpeq 5699·9704$ grains of bronze $+$ 60·2678 grains of platinum in air ($t=16·03$, $b=744·77$).

By observations made October 2, 1855, B in water ($K=17·67$) $\bumpeq (3200)+(1600)+(200)$ (bronze)$+(64)+(4)+0·2505$ grain (platinum) in air ($C=17·0$, $F=754·55$, $E=17·4$).

B in water ($t=17·66$) $\bumpeq$ 4999·9640 grains of bronze $+$68·3459 grains of platinum in air ($t=17·05$, $b=752·88$).

Hence $vB=692·815$, $\Delta B=8·3175$, $B=U+0·2487$ grain. $B \bumpeq U+0·2653$ grain in air ($t=18·7$, $b=755·64$).

The magnitude of the differences between these weights is very remarkable, considering that O, M, B are in a state of very excellent preservation, V alone exhibiting traces of rough usage.

Linear dimensions of the different weights in inches.

	U.	V.	O.	M.	B.
Axis	2·576	2·57	2·568	2·538	2·564
Diameter of knob	1·012	0·98	0·973	1·020	1·024
Diameter of neck	0·359	——	0·383	0·391	——
Diameter at *d*	——	1·43	1·416	1·391	1·387
Diameter at *e*	1·445	1·43	1·424	1·389	——
Diameter at *f*	——	——	1·434	1·385	1·377

The above comparison shows that the linear dimensions of U were very sensibly larger than those of M or B, and so far justifies the assumption that the volume of U was larger than that of M or B, probably not less than that of V; for it cannot be supposed that the weight selected as the standard was a bad and porous casting like O, the linear dimensions of which are nearly equal to those of V, though its volume is considerably less in consequence of the numerous cavities that exist in it communicating with its surface.

If U, the lost standard, be supposed to have the same density as V, the volume of U at 0° C. will be equal to the volume of 706·676 grains of water of maximum density.

Sp ≏ U−0·00857 grain in air (t=18·74, b=754·93).

RS ≏ U−0·00205 grain in air (t=18·74, b=757·06).

Sp displaces 0·32544 grain, U displaces 0·84646 grain of air. Therefore

Sp=U−0·52956 grain.

RS displaces 0·32629 grain, U displaces 0·84865 grain of air. Therefore

RS=U−0·52441 grain.

The density of V is nearly the average density of brass or bronze weights, as appears by the following list, which includes all the determinations of densities of such weights I have been able to find. The specific gravities of the Russian weights and of some others marked (K), taken from the second volume of KUPFFER's work, entitled 'Travaux de la Commission pour fixer les Mesures et les Poids de l'Empire de Russie,' are reduced from 13°·33 R. to the specific gravity as defined by BESSEL, by the addition of 0·002. Those marked (M) are from my own observations.

Russian lb. No. 8 (K)	7·872
Russian lb. No. 9 (K)	7·872
Russian lb. No. 7 (K)	7·882
Russian lb. No. 6 (K)	7·932
Prussian divided pound (K)	7·952
Russian lb. (K)	7·972
G lb. No. 21 (M)	7·974
Russian lb. No. 5 (K)	7·992
SCHUMACHER's troy pound K	7·994
59 lbs. No. 9, 12 to 71 (K), all nearly	8·0
Kilogramme, Modena Trans. t. xxv.	8·025
KLAPROTH's kilogramme, Berlin Trans. 1825.	8·055
G lb. No. 5 (M)	8·061
G lb. No. 16 (M)	8·073
Russian troy pound (K)	8·092
G lb. No. 25 (M)	8·101
G lb. No. 17 a (M)	8·117
G lb. No. 7 (M)	8·122
G lb. No. 28 (M)	8·126
Kew lb. marked 7000 (M)	8·128
Swedish pound (K)	8·132
Kilogramme, electro-gilt (M)	8·133
G lb. No. 24 (M)	8·142
Russian lb. No. 1 (K)	8·142
STEINHEIL's divided kilogramme, Munich Trans.	8·150
G lb. No. 23 (M)	8·151
Mr. VANDOME's troy pound V, 1758 (M)	8·151
G lb. No. 30 (M)	8·153
G lb. No. 26 (M)	8·153
G lb. No. 27 (M)	8·162
Russian lb. No. (2) (K)	8·162
G lb. No. 8 (M)	8·163
G lb. No. 29 (M)	8·186
Russian lb. No. 3 (K)	8·192
G lb. No. 22 (M)	8·198
Kew Standard lb., electro-gilt (M)	8·204
SCHUMACHER's troy pound Sb	8·228
Russian lb. No. 4 (K)	8·272
G lb. No. 10 (M)	8·283
G lb. No. 6 (M)	8·287
G lb. No. 18 (M)	8·303
G lb. No. 3 (M)	8·304
Mr. BARROW's lb., electro-gilt (M)	8·310
Bank troy pound B, 1758 (M)	8·317
G lb. No. 12 (M)	8·319
Kilogramme (M)	8·320
G lb. not numbered (M)	8·320
Exchequer kilogramme (M)	8·328
Russian divided lb. D (K)	8·332
G lb. No. 19 (M)	8·340
G lb. No. 2 (M)	8·341
Mint troy pound M, 1758 (M)	8·349
G lb. No. 14 (M)	8·349
Exchequer 10 lb., electro-gilt (M)	8·354
G lb. No. 15 (M)	8·361
G lb. No. 1 (M)	8·361
G lb. No. 11 (M)	8·363
G lb. No. 4 (M)	8·365
Weight of 6400 grains (M)	8·368
Russian lb. M (K)	8·373
ALCHORNE's troy pound O, 1758 (M)	8·403
Mint 10 troy ounces, electro-gilt (M)	8·414
G lb. No. 13 (M)	8·413
Exchequer 10 troy ounces, electro-gilt (M)	8·460
Electro-gilt lb. No. 35 (M)	8·470
Electro-gilt lb. No. 32 (M)	8·470
Electro-gilt lb. No. 33 (M)	8·479
An electro-gilt lb. (M)	8·481
Electro-gilt lb. No. 36 (M)	8·496
An electro-gilt lb. (M)	8·507
Russian lb. N (K)	8·508
Electro-gilt lb. No. 34 (M)	8·512
Electro-gilt lb. No. 31 (M)	8·514
G lb. No. 20 (M)	8·558
G lb. No. 17 b (M)	8·558

The Commissioners for the Restoration of the Standards of Weight and Measure, in their Report dated December 21, 1841, recommended that the avoirdupois pound of 7000 grains be adopted instead of the troy pound of 5760 grains, as the new Parliamentary Standard of weight, and that the new standard and four copies of it be constructed of platinum. In accordance with this recommendation, five platinum weights were made by Mr. BARROW, a little in excess of 7000 grains. The form of these pounds is that of a cylinder nearly 1·35 inch in height and 1·15 inch in diameter, with a groove round it, the middle of which is about 0·34 inch below the top of the cylinder, for insertion of the prongs of a forked lifter of ivory. They are marked PS 1844 1 lb.; PC No. 1 1844 1 lb.; PC No. 2 1844 1 lb.; PC No. 3 1844 1 lb.; PC No. 4 1844 1 lb., respectively.

The weight of 7000 grains might have been formed from one of 5760 grains, by the use of either a decimal or a binary system of weights. In either case, however, the number of the weights to be compared with one or the other or both of the weights of 7000 grains and 5760 grains would have been large, and the errors of the comparisons between themselves might by their accumulation sensibly affect the resulting weight of 7000 grains. Moreover, the repeated comparison of weights made up of the sum of several others, was a very troublesome process previous to the use of the method described in page 764, which had not been thought of at the time the weights were ordered. These two evils were in a great measure avoided by the use of a platinum weight T of about 5760 grains, or, more correctly, very nearly equal to Sp or RS, and of the following series of auxiliary weights, also of platinum, and all constructed by Mr. BARROW: A, B, C, D each of 1240 grains; F of 800 grains; G of 440 grains; H of 360 grains; K, L, M, N each of 80 grains; R, S each of 40 grains, nearly. The platinum of which the five lbs., T and the auxiliary weights were made, was prepared by Messrs. JOHNSON and COCK.

The numbers of the weights of each denomination, and their values, are given by the quotients and divisors obtained in the conversion of $\frac{7000}{5760}$ into a continued fraction. The errors of these weights are found by the following comparisons:—Sp and RS with T; T with A+B+C+D+F; each of the weights A, B, C, D with F+G; F with G+H; G with each of the weights H+K, H+L, H+M, H+N; H with K+L+M+N+R and K+L+M+N+S; each of the weights K, L, M, N with R+S. Sp and RS, instead of being true troy pounds, and consequently equal to U in a vacuum, had been adjusted so as to appear as heavy as U nearly, when weighed in air of ordinary density, and are therefore lighter than U by about 0·53 grain, the weight of the air contained in a space equal to the difference between the volume of U and that of Sp or RS. A space equal to the difference between the volume of 7000 grains of metal of the density of U and 7000 grains of platinum, contains about 0·645 grain of air. Calling this Q, PS may be compared with each of the weights T+A+Q, T+B+Q, T+C+Q, T+D+Q. In order to determine the error of the weight of 0·645 grain with the greatest precision, ten weights Q of 0·645 grain each, so accurately

adjusted that no sensible difference could be detected between them, a weight V of 6·451 grains, and a weight W of 12·901 grains, all of platinum, were obtained from Mr. BARROW. Then, Y and Z being platinum weights of 20 grains each belonging to the two ROBINSON's balances, the following comparisons became possible:—each of the weights R and S with Y+Z; each of the weights Y and Z with W+V+ each of the weights Q; W with V+ sum of ten weights Q; V with the sum of the ten weights Q. In comparing PS with each of the weights T+A+Q, T+B+Q, T+C+Q, T+D+Q, the weight Q was changed at the end of every four comparisons, and thus each of the ten weights Q used in turn in a series of forty comparisons.

The following comparisons of the auxiliary weights with Sp and RS, and with each other, were made for the purpose of finding their errors preparatory to a more accurate adjustment, and in order to obtain a series of weights to be used in finding the densities of T and of the five platinum lbs. of 7000 grains each, from June 4, 1844, to the end of the year. In the comparisons of A+B+C+D+F with Sp and RS, X and Y denote the weights of the two detached pans. The counterpoise is placed in the left-hand pan. The numbers in each column are the readings of the scale in the position of equilibrium of the balance, when the weight at the head of the column is in the right-hand pan. In these, and all the other comparisons of weights in air, the results of the alternate weighings are arranged in separate columns.

June 18, 1844. 100 parts $=0·2208$ grain.

A+B+C+D+F+Y.	Sp+X.	A+B+C+D+F+X.	Sp+Y.
27·80	25·30	22·90	22·50
27·00	26·10	22·90	21·90
25·20	24·05	21·40	22·05
25·30	24·60	26·45	26·50
24·60	24·40	29·50	27·15
24·40	23·50	28·65	28·50
23·55	22·75	27·30	28·50
23·00	23·00	29·30	28·90
22·55	22·20	30·10	29·65
22·40	21·90	30·40	29·75
245·80	237·80	268·90	265·40

$10(A+B+C+D+F+Y) \triangleq 10(Sp+X)+8·0$ parts. $10(A+B+C+D+F+X) \triangleq 10(Sp+Y)+3·5$ parts.

$A+B+C+D+F \triangleq Sp+0·00127$ grain.

June 12, 1844. 100 parts $=0·2293$ grain.

A+B+C+D+F+Y.	RS+X.	A+B+C+D+F+X.	RS+Y.
21·20	20·70	18·60	19·60
19·00	21·10	18·10	20·20
19·30	21·60	18·70	20·00
19·50	20·80	18·50	20·00
19·40	21·30	19·10	19·35
18·70	20·80	18·00	19·85
117·10	126·30	111·00	119·00

$6(A+B+C+D+F+Y) \triangleq 6(RS+X)-9·2$ parts. $6(A+B+C+D+F+X) \triangleq 6(RS+Y)-8·0$ parts.

$A+B+C+D+F \triangleq RS-0·00329$ grain.

For the platinum of which T and the auxiliary weights are made log $\Delta=1{\cdot}32566$ by SCHUMACHER's Tables. For Sp and RS log $\Delta=1{\cdot}32608$, by the same tables. The space vU$-v$Sp contains 0·53 grain of air nearly. Therefore the weight of Sp or RS is nearly 5759·47 grains. The weight of A+B+C+D+F is very nearly the same. Hence v(A+B+C+D+F)=272·09, vSp=271·84, vRS=271·84, the unit of volume being the volume of a grain of water at its maximum density. Therefore

v(A+B+C+D+F) is larger than vSp or vRS by the volume of 0·25 grain of water, or 0·00030 grain of air of ordinary density.

Hence A+B+C+D+F=Sp+0·00157 gr. But Sp=U−0·52956 gr. Therefore A+B+C+D+F=U−0·52799 gr. Also A+B+C+D+F=RS−0·00299 gr. But RS=U−0·52441 gr. Therefore A+B+C+D+F=U−0·52740 gr. Giving the former value twice the weight of the latter,

A+B+C+D+F=U−0·52779=5759·47221 grains.

In the following comparisons of the auxiliary weights with each other the 10½-inch balance by ROBINSON was used.

June 6, 1844. 100 parts =0·3593 grain.

A.	F+G.	B.	F+G.
1·80	1·75	2·70	2·75
2·50	2·35	2·65	3·05
2·50	2·85	2·55	2·90
2·90	3·25	2·60	3·05
3·20	3·15	2·80	2·70
3·00	3·05	2·50	3·25
3·25	3·15	2·70	3·05
3·15	3·05	2·50	3·25
2·90	2·85	2·80	3·20
3·00	2·90	2·95	3·30
2·75	2·55	3·00	3·35
2·85	2·85	2·75	3·30
33·80	33·75	32·50	37·15

12A=12(F+G)−0·05 part. 12B=12(F+G)+4·65 parts.

A=F+G−0·00001 grain. B=F+G+0·00138 grain.

June 7, 1844. 100 parts =0·3593 grain.

C.	F+G.	D.	F+G+0·004 gr.
3·10	3·00	3·10	3·20
2·75	3·10	3·15	3·15
3·10	2·85	3·20	3·00
2·80	3·00	3·20	3·15
2·75	2·90	2·90	3·10
3·65	3·65	2·95	2·95
3·65	3·65	3·00	3·05
3·40	2·90	3·30	3·55
2·90	2·90	3·00	3·30
2·60	3·10	2·80	2·90
2·75	2·95	3·30	3·20
2·90	3·00	3·00	3·15
36·35	37·00	36·90	37·70

12C=12(F+G)+0·65 part. 12D=12(F+G+0·004 gr.)+0·8 part.

C=F+G+0·00020 grain. D=F+G+0·00424 grain.

June 8, 1844. 100 parts = 0·2855 grain.

gr.	Scale.		Scale.
F+0·010	4·55	G+H	2·45
F+0·020	1·15	G+H	2·60
F+0·010	5·00	G+H	2·45
F+0·020	1·00	G+H	2·60
F+0·010	4·80	G+H	2·95
F+0·020	1·60	G+H	3·00
F+0·010	5·30	G+H	2·65
F+0·020	1·10	G+H	2·70
F+0·010	4·80	G+H	3·00
F+0·020	1·45	G+H	2·75
F+0·015	3·10	G+H	2·80
F+0·015	3·50	G+H	2·95
12F+0·18	37·35	12(G+H)	32·90

12F+0·18 gr.=12(G+H)−4·45 parts.
F=G+H−0·01606 grain.

June 8, 1844. 100 parts = 0·2708 grain.

	Scale.	gr.	Scale.		Scale.	gr.	Scale.
G	3·20	H+K+0·005	2·97	G	3·60	H+L+0·004	3·55
G	3·10	H+K+0·004	3·15	G	3·45	H+L+0·004	3·60
G	3·20	H+K+0·004	3·30	G	3·25	H+L+0·004	4·05
G	3·15	H+K+0·004	3·25	G	3·40	H+L+0·004	3·75
G	3·45	H+K+0·004	3·25	G	3·50	H+L+0·004	4·00
G	3·15	H+K+0·004	3·00	G	3·30	H+L+0·004	4·10
G	3·20	H+K+0·004	3·15	G	3·40	H+L+0·004	4·15
G	3·35	H+K+0·004	3·30	G	3·60	H+L+0·006	3·35
G	3·20	H+K+0·004	3·45	G	3·55	H+L+0·006	3·25
G	3·30	H+K+0·004	3·00	G	3·40	H+L+0·006	3·05
G	3·10	H+K+0·004	3·30	G	3·25	H+L+0·006	2·95
G	3·20	H+K+0·004	3·60	G	3·00	H+L+0·006	2·85
12G	38·60	12(H+K)+0·049	38·72	12G	40·70	12(H+L)+0·058	42·65

12G=12(H+K)+0·049 gr.+0·12 pt.
G=H+K+0·00411 grain.

12G=(H+L)12+0·058 gr.+1·95 pt.
G=H+L+0·00527 grain.

June 10, 1844. 100 parts = 0·2708 grain.

	Scale.	gr.	Scale.		Scale.	gr.	Scale.
G	2·85	H+M+0·003	3·35	G	2·65	H+N+0·003	2·65
G	2·90	H+M+0·006	2·30	G	2·70	H+N+0·003	3·25
G	2·85	H+M	4·70	G	3·20	H+N+0·003	3·25
G	3·00	H+M+0·003	3·45	G	3·30	H+N+0·003	3·20
G	2·95	H+M+0·003	3·35	G	2·85	H+N+0·003	3·20
G	2·70	H+M+0·003	3·10	G	3·50	H+N+0·003	3·00
G	2·60	H+M+0·004	2·90	G	3·30	H+N+0·003	3·65
G	3·15	H+M+0·004	3·10	G	3·55	H+N+0·003	3·65
G	3·00	H+M+0·004	3·25	G	3·10	H+N+0·003	3·25
G	3·10	H+M+0·004	3·05	G	3·60	H+N+0·003	3·60
G	3·05	H+M+0·004	2·95	G	3·30	H+N+0·003	3·20
G	3·05	H+M+0·004	3·15	G	3·30	H+N+0·003	3·30
12G	35·20	12(H+M)+0·042	38·65	12G	38·35	12(H+N)+0·036	39·20

12G=12(H+M)+0·042 gr.+3·45 pt.
G=H+M+0·00428 grain.

12G=12(H+N)+0·036 gr.+0·85 pt.
G=H+N+0·00319 grain.

June 11, 1844. 100 parts =0·2535 grain.

K..R=K+L+M+N+R.

	Scale.	gr.	Scale.
H	2·90	K..R	3·30
H	2·70	K..R	3·30
H	2·70	K..R	3·45
H	2·90	R..R	3·30
H	2·60	K..R	3·30
H	2·75	K..R	3·25
H	2·65	K..R	3·10
H	2·50	K..R	3·10
H	2·60	K..R +0·003	2·20
H	2·70	K..R +0·003	2·20
H	2·70	K..R +0·003	2·35
H	2·70	K..R +0·003	2·30
12H	32·40	12(K..R)+0·012	35·15

12H=12(K..R)+0·012 gr.+2·75 pt.
H=K+L+M+N+R+0·00158 grain.

K..S=K+L+M+N+S.

	Scale.	gr.	Scale.
H	2·70	K..S +0·003	3·00
H	2·90	K..S +0·005	2·00
H	2·85	K..S +0·004	2·40
H	2·80	K..S	4·05
H	2·75	K..S	3·90
H	2·35	K..S +0·006	1·50
H	2·30	K..S +0·003	2·50
H	2·70	K..S +0·003	2·60
H	2·80	K..S +0·003	3·00
H	3·05	K..S +0·003	2·70
H	3·00	K..S +0·003	2·80
H	2·95	K..S +0·003	2·60
12H	33·15	12(K..S)+0·036	33·05

12H=12K..S+0·036 gr.−0·10 pt.
H=K+L+M+N+S+0·00298 grain.

June 19, 1844. 100 parts =0·1198 grain.

gr.	Scale.		Scale.
K	3·20	R+S	2·15
K	3·80	R+S	2·80
K+0·002	2·45	R+S	2·90
K+0·002	2·15	R+S	2·60
K	3·90	R+S	2·90
K+0·002	2·00	R+S	2·60
K	3·85	R+S	3·10
K+0·002	2·45	R+S	2·85
K	3·80	R+S	3·00
K+0·002	2·35	R+S	2·70
K	4·20	R+S	3·05
K+0·002	2·45	R+S	2·80
12K+0·012	36·60	12(R+S)	33·45

12K+0·012 gr.=12(R+S)−3·15 pt.
K=R+S−0·00131 grain.

gr.	Scale.		Scale.
L	5·70	R+S	3·10
L+0·003	3·00	R+S	3·15
L+	5·20	R+S	2·65
L+0·002	2·85	R+S	2·20
L+0·002	3·20	R+S	2·80
L	4·70	R+S	2·70
L+0·004	1·40	R+S	2·40
L	4·60	R+S	2·30
L+0·002	3·00	R+S	2·60
L+0·004	1·45	R+S	2·20
L+0·004	2·50	R+S	3·55
L+0·004	2·50	R+S	3·30
12L+0·025	40·10	12(R+S)	32·95

12L+0·025 gr.=12(R+S)−7·15 pt.
L=R+S−0·00280 grain.

June 19, 1844. 100 parts=0·1198 grain.

gr.	Scale.		Scale.
M	4·55	R+S	3·65
M+0·002	2·75	R+S	3·60
M	4·65	R+S	4·00
M+0·002	2·60	R+S	2·90
M	4·00	R+S	3·20
M+0·002	2·40	R+S	2·90
M	4·10	R+S	3·10
M+0·003	1·60	R+S	2·95
M	4·00	R+S	2·90
M+0·003	1·40	R+S	2·85
M	4·00	R+S	2·90
M+0·003	1·60	R+S	2·70
12M+0·015	37·65	12(R+S)	37·65

12M+0·015 gr.=12(R+S)
M=R+S−0·00125 grain.

	Scale.		Scale.
N	3·15	R+S	3·00
N	3·30	R+S	3·00
N	3·50	R+S	3·10
N	3·10	R+S	3·10
N	3·55	R+S	3·20
N	3·20	R+S	2·80
N	3·40	R+S	3·30
N	2·80	R+S	3·20
N	3·30	R+S	2·80
N	2·95	R+S	2·95
N	2·90	R+S	2·85
N	3·10	R+S	2·80
12N	38·25	12(R+S)	36·10

12N=12(R+S)+2·15 pt.
N=R+S+0·00022 grain.

				gr.
T	=			5759·48815
A+B+C+D+F	=			5759·47221
A	=	F+G	−	0·00001
B	=	F+G	+	0·00138
C	=	F+G	+	0·00020
D	=	F+G	+	0·00424
F	=	G+H	−	0·01606
G	=	H+K	+	0·00411
G	=	H+L	+	0·00527
G	=	H+M	+	0·00428
G	=	H+N	+	0·00319
H	=	K+L+M+N+R	+	0·00158
H	=	K+L+M+N+S	+	0·00298
K	=	R+S	−	0·00131
L	=	R+S	−	0·00280
M	=	R+S	−	0·00125
N	=	R+S	+	0·00022
A+B+C+D	=	4F+4G	+	0·00581
4F	=	4G+4H	−	0·06424
4G	=	4H+K+L+M+N	+	0·01685
2H	=	2(K+L+M+N)+R+S	+	0·00456
K+L+M+N	=	4(R+S)	−	0·00514
4(K+L+M+N)	=	16(R+S)	−	0·02056
4H	=	4(K+L+M+N)+2(R+S)	+	0·00912
4G	=	4H+K+L+M+N	+	0·01685
4F	=	4G+4H	−	0·06424
5760 grains	=	A+B+C+D+F	+	0·52779
4H	=	18(R+S)	−	0·01144
4G	=	22(R+S)	+	0·00027
4F	=	40(R+S)	−	0·07541
4F+4G	=	62(R+S)	−	0·07514
A+B+C+D	=	62(R+S)	−	0·06933
A+B+C+D+F	=	72(R+S)	−	0·08818
72(R+S)	=			5759·56039
18(R+S)	=			1439·89010
40(R+S)	=			3199·75577
10(R+S)	=			799·93894
62(R+S)	=			4959·62143
22(R+S)	=			1759·86567
A+B+C+D	=			4959·55210
4F+4G	=			4959·54629
F+G	=			1239·88657

Values of the auxiliary weights from June 4, 1844, to the end of the year.

		gr.			gr.
A	=	1239·88656	K	=	79·99258
B	=	1239·88795	L	=	79·99109
C	=	1239·88677	M	=	79·99264
D	=	1239·89081	N	=	79·99411
F	=	799·92009	R	=	39·99624
G	=	439·96648	S	=	39·99764
H	=	359·96966			

In January 1845 the auxiliary weights were adjusted so as to reduce still further the differences between the sums of the weights compared with each other.

Observations for finding the density of T.

T weighed in air.

June 22, 1844. 100 parts =0·19917 grain.

Sp+0·02 gr.+Y.	T+X.	Sp.+0·02 gr.+X.	T+Y.
14·20	13·35	20·20	19·55
15·85	14·70	21·40	21·40
16·25	14·35	22·00	22·10
16·10	14·40	23·00	22·30
15·50	13·80	23·50	22·40
15·30	13·00	23·10	20·80
14·10	12·90	21·85	21·60
14·55	12·65	22·35	21·00
13·90	12·00	21·95	21·30
14·10	12·60	22·10	21·90
13·70	11·90	22·70	21·95
163·55	145·65	244·15	236·30

11(Sp+0·02 gr.+Y)≏11(T+X)+17·9 parts. 11(Sp+0·02 gr.+X)≏11(T+Y)+7·85 parts.

T≏Sp+0·01767 grain.

May 1, 1844. 100 parts =0·1456 grain.

T.	RS+0·01 gr.
22·10	22·20
23·10	24·85
26·00	24·55
29·45	26·70
29·20	28·10
28·90	27·70
30·10	26·90
28·60	27·80
29·70	27·65
30·20	29·07
277·35	265·52

10T≏10(RS+0·01 gr.)+11·83 parts.

T≏RS+0·01172 grain.

For T, by SCHUMACHER's Tables, $\log\Delta=1·32566$. For Sp and RS, $\log\Delta=1·32608$ by the same Tables. The space vU$-v$Sp contains 0·53 grain of air nearly. Therefore the weight of Sp is nearly 5759·47 grains, and that of T nearly 5759·49 grains. Hence vT=271·63, vSp=271·37, vRS=271·37. Therefore vT is larger than vSp or vRS, by the volume of 0·26 grain of water, or 0·00031 grain of air of ordinary density.

Hence	T =Sp +0·01798.	gr.
But	Sp =U −0·52956.	
Therefore	T =U −0·51158.	
Also	T =RS+0·01203.	
But	RS=U −0·52441.	
Therefore	T =U −0·51238.	

Giving the former value twice the weight of the latter,

T=U−0·51185 grain=5759·48815 grains.

In the observations for finding the apparent weight of T in water, the same pro-

cess was followed as in finding the density of V. The thermometers H, P were suspended in the water with their bulbs in a horizontal plane through the middle of T. t denotes the temperature of the air in centesimal degrees; b the height of the mercury in the barometer in millimètres, reduced to 0° C.; 100 parts =0·220 grain.

July 13, 1854. T and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
18·30	19·40	0·00	20·45
18·20	19·30	0·00	20·20
18·20	19·10	0·00	20·50
18·10	19·05	0·00	18·80
18·10	19·05	0·00	22·50
18·10	19·00	0·00	20·15
18·10	19·00	0·00	19·60
18·10	19·00	0·00	19·70
18·05	19·00	0·00	19·30
18·05	19·00	0·00	20·95
18·13	19·09	0·00	20·21

Hook in water. In right-hand pan.

	gr.	Scale.
	A+B+C+D+G+K+(8)+0·44	20·25
	+0·44	20·25
Air.	+0·48	48·40
t=18·2, b=760·21.	+0·40	12·20
	+0·40	10·05
	+0·40	3·00
	+0·4267	19·1

T in water (t=18·27)≏A+B+C+D+G+K+(8)+0·4291 grain in air (t=18·2, b=760·21).

July 15, 1844. T and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
17·90	18·65	0·04	21·10
18·00	18·90	0·04	24·90
18·02	18·95	0·04	25·15
18·00	19·00	0·03	23·55
18·05	19·00	0·02	22·25
18·00	19·00	0·02	22·00
18·15	19·10	0·01	19·80
18·20	19·10	0·01	19·90
18·20	19·10	0·01	19·95
18·06	18·98	0·244	22·07

Hook in water. In right-hand pan.

	gr.	Scale.
	A+B+C+D+G+K+(8)+0·42	15·30
	+0·43	16·65
	+0·44	18·70
	+0·44	18·85
Air.	+0·46	21·90
t=18·4, b=757·77.	+0·44	17·07
	+0·46	28·60
	+0·46	26·70
	+0·46	26·70
	+0·46	29·95
	+0·447	22·04

T in water (t=18·18)≏A+B+C+D+G+K+(8)+0·4227 grain in air (t=18·4, b=757·77).

July 29, 1844. T and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
19·00	19·95	0·86	19·5
		0·84	17·0
		0·88	23·0
19·00	19·95	0·84	16·5
		0·88	22·4
19·00	19·90	0·88	23·3
		0·84	15·5
		0·88	25·0
		0·84	16·2
		0·88	23·0
19·00	19·90	0·84	17·8
		0·88	24·1
19·00	19·92	0·8617	20·27

Hook in water. In right-hand pan.

	gr.	Scale.
A+B+C+D+G+K+(8)+	1·32	19·6
	1·32	20·2
Air.	1·32	21·5
t=20·4, b=761·17.	1·32	21·4
	1·32	21·0
	1·32	21·0
	1·32	20·78

T in water (t=19·11)≏A+B+C+D+G+K+(8)+0·4572 grain in air (t=20·4, b=761·17).

August 1, 1844. T and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
		0·86	20·8
		0·85	18·5
		0·86	19·5
		0·86	21·4
		0·85	20·4
18·00	18·80	0·85	17·3
		0·86	21·0
		0·85	15·1
		0·86	21·7
		0·86	22·6
		0·86	22·8
17·95	18·75	0·85	20·8
17·98	18·78	0·8567	20·16

Hook in water. In right-hand pan.

	gr.	Scale.
A+B+C+D+G+K+(8)+	1·28	19·5
	1·28	19·8
Air.	1·28	20·0
t=18·4, b=754·70.	1·28	20·5
	1·28	20·6
	1·28	19·6
	1·28	19·5
	1·28	19·93

T in water (t=18·04)≏A+B+C+D+G+K+8+0·4238 grain in air (t=18·4, b=754·70).

Apparent weight of T in water, weighed with platinum weights.

Water.	Apparent weight of T.	Air.	
t.	gr.	t.	b.
18·27	5487·9398	18·2	760·21
18·18	5487·9334	18·4	757·77
19·11	5487·9679	20·4	761·17
18·04	5487·9345	18·4	754·70

The resulting values of ΔT and log ΔT are:—

	ΔT.	log ΔT.
	21·1661	1·325642
	21·1661	1·325641
	21·1656	1·325631
	21·1667	1·325654
Mean	21·1661	1·325642
10 − log ΔT		8·674358
T (reduced)	5759·471	3·760383
vT	272·108	2·434741

Hence the volume of T at 0° is equal to the volume of 272·108 grains of water at its maximum density.

The resulting values of log ΔT by SCHUMACHER's Tables are:—

		log ΔT.
		1·325653
		1·325651
		1·325642
		1·325664
Mean log ΔT		1·325664
10 − log ΔT		8·674336
T (reduced)	5759·471	3·760383
vT	272·094	2·434719

Hence, using SCHUMACHER's Tables, the volume of T at 0° is equal to the volume of 272·094 grains of water at its maximum density.

Comparison of T *with* Sp *and* RS.

In December 1844 T was reduced till its weight was very nearly equal to that of Sp, and afterwards compared with Sp and RS. The weights of the two scale-pans are denoted by X and Y. The thermometers C, D were suspended in the balance case with their bulbs in a horizontal plane passing through the middle of the weights. F is the reading of ERNST's barometer, E that of the attached thermometer. The comparisons were made in a cellar under the north side of the Mineralogical Museum in Cambridge. The balance was placed upon a thick stone slab forming the sill of a recess containing a window so situated, that the lowest part of the window was considerably higher than the top of the balance case.

January 15, 1845. 100 parts =0·16949 grain.

T+X.	Sp+Y.		T+Y.	Sp+X.
17·14	15·95	D= 9·93	18·50	17·70
18·27	16·50	C= 9·90	19·15	18·31
18·82	17·97	F=754·8	19·00	18·50
19·44	17·56	E= 7·7	18·53	18·66
20·17	18·01		18·45	18·31
93·84	85·99		93·63	91·48

$5(T+X) \triangleq 5(Sp+Y)+7{\cdot}85$ parts. $5(T+Y) \triangleq 5(Sp+X)+2{\cdot}15$ parts.

$10T \triangleq 10Sp+0{\cdot}01695$ grain in air ($t=9{\cdot}91$, $b=754{\cdot}30$).

February 21, 1845. 100 parts =0·17396 grain.

Sp+X, T+Y.	T+Y, Sp+X.		Sp+Y, T+X.	T+X, Sp+Y.
24·76	25·00		23·77	21·20
25·05	25·21		22·87	20·22
25·32	25·26		22·61	18·76
24·28	22·67	D= 6·95	18·37	18·15
24·60	23·76	C= 7·0	20·40	17·82
21·35	20·68	F=759·5	17·37	17·40
19·43	22·91	E= 9·3	19·55	18·06
21·52	20·17		21·90	19·68
20·72	22·40		20·70	19·53
20·35	19·80		21·38	20·95
227·38	227·86		208·92	191·77

$20(T+Y) \bumpeq 20(Sp+X)-0·48$ part. $20(T+X) \bumpeq 20(Sp+Y)+17·15$ parts.

$40T \bumpeq 40Sp+0·02900$ grain in air ($t=6·94$, $b=758·80$).

February 26, 1845. 100 parts =0·17156 grain.

T+X.	Sp+Y.		T+Y.	Sp+X.
21·48	20·46		22·62	22·62
21·97	21·83		22·83	23·45
22·51	23·20	D= 6·30	23·18	23·52
23·00	21·98	C= 6·35	22·50	22·15
23·22	22·62	b=752·41	22·71	22·95
23·30	23·63		22·87	22·67
24·21	24·62		22·45	22·46
24·92	24·48		22·92	22·68
184·61	182·82		182·08	182·50

$8(T+X) \bumpeq 8(Sp+Y)+1·79$ part. $8(T+Y) \bumpeq 8(Sp+X)-0·42$ part.

$16T \bumpeq 16Sp+0·00235$ grain in air ($t=6·28$, $b=752·41$).

July 26, 1845. 100 parts =0·33860 grain.

T+Y.	Sp+X.		T+X.	Sp+Y.
19·88	18·16		18·25	18·41
19·32	18·97		18·32	18·56
19·27	19·06		18·31	17·90
19·70	18·55		18·26	18·48
19·51	19·16	D= 17·5	18·30	17·75
19·63	18·88	C= 17·5	17·70	18·10
19·36	20·03	F=761·5	17·75	17·96
19·00	19·31	E= 18	18·38	18·26
19·56	18·75		17·91	18·15
19·12	18·72		17·17	18·24
18·90	18·67		17·08	17·43
19·00	18·24		17·33	17·16
232·25	226·50		214·76	216·40

$12(T+Y) \bumpeq 12(Sp+X)+5·75$ parts. $12(T+X) \bumpeq 12(Sp+Y)-1·64$ part.

$24T \bumpeq 24Sp+0·01392$ grain in air ($t=17·53$, $b=759·79$).

August 16, 1845. 100 parts $=0{\cdot}27581$ grain.

Sp+Y, T+X.	T+X, Sp+Y.		Sp+X, T+Y.	T+Y, Sp+X.
19·22	20·30		17·87	19·50
19·16	20·15		20·02	18·36
20·67	20·52		20·04	17·82
21·27	22·02		20·35	17·14
19·11	20·51		20·30	17·51
21·20	20·54	D= 15·9	20·29	17·32
21·32	21·60	C= 15·9	19·34	17·77
20·27	19·69	F=760·2	19·65	17·61
18·31	20·31	E= 15·9	19·77	17·29
20·07	20·84		20·77	19·98
21·49	20·75		18·77	16·79
19·69	19·84		17·79	18·54
19·82	19·82		19·80	17·14
17·45	19·36		19·77	19·47
279·05	286·25		274·53	252·24

$28(T+X) \bumpeq 28(Sp+Y)-7{\cdot}20$ parts. $28(T+Y) \bumpeq 28(Sp+X)+22{\cdot}29$ parts.

$56T \bumpeq 56Sp+0{\cdot}04162$ grain in air ($t=15{\cdot}91$, $b=758{\cdot}70$).

August 18, 1845. 100 parts $=0{\cdot}27959$ grain.

Sp+X, T+Y.	T+Y, Sp+X.		Sp+Y, T+X.	T+X, Sp+Y.
18·85	18·34		19·94	19·42
19·90	20·49		18·16	19·39
20·05	18·17		19·17	21·21
20·27	20·67	D= 15·92	19·86	18·82
20·32	18·66	C= 15·9	18·99	19·36
20·35	20·12	F=756·5	18·50	20·49
20·61	19·95	E= 16	20·02	18·56
21·02	19·24		18·76	18·61
21·39	20·99		19·01	18·37
20·96	20·00		17·81	17·92
203·72	196·63		190·22	192·15

$20(T+Y) \bumpeq 20(Sp+X)+7{\cdot}09$ parts. $20(T+X) \bumpeq 20(Sp+Y)-1{\cdot}93$ part.

$40T \bumpeq 40Sp+0{\cdot}01443$ grain in air ($t=15{\cdot}93$, $b=754{\cdot}99$).

August 19, 1845. 100 parts $=0{\cdot}27204$ grain.

Sp+Y, T+X.	T+X, Sp+Y.		Sp+X, T+Y.	T+Y, Sp+X.
19·56	21·19		18·19	18·54
18·91	20·79	D= 15·55	20·95	19·95
20·93	20·25	C= 15·6	20·83	18·15
18·78	21·25	F=740·7	20·68	17·86
19·12	21·45	E= 15·5	20·60	18·50
97·30	104·93		101·25	93·00

$10(T+X) \bumpeq 10(Sp+Y)-7{\cdot}63$ parts. $10(T+Y) \bumpeq 10(Sp+X)+8{\cdot}25$ parts.

$20T \bumpeq 20Sp+0{\cdot}00169$ grain in air ($t=15{\cdot}6$, $b=739{\cdot}29$).

August 27, 1845. 100 parts $=0{\cdot}26861$ grain.

Sp+Y, T+X.	T+X, Sp+Y.		Sp+X, T+Y.	T+Y, Sp+X.
24·20	22·27		22·24	23·45
24·34	21·01		22·10	22·47
24·66	22·40		22·06	23·00
24·22	22·32	D= 14·93	21·96	22·37
24·15	22·32	C= 14·93	21·67	22·61
23·86	22·17	F=766·1	22·27	22·30
25·50	21·07	E= 15	22·57	22·82
23·40	21·67		22·02	22·40
23·21	21·32		21·69	23·12
23·51	21·16		22·17	21·94
241·05	217·71		220·75	226·48

$20(T+X) \bumpeq 20(Sp+Y)+23{\cdot}34$ parts. $20(T+Y) \bumpeq 20(Sp+X)-5{\cdot}73$ parts.

$40T \bumpeq 40Sp+0{\cdot}04730$ grain in air ($t=14{\cdot}95$, $b=764{\cdot}69$).

August 29, 1845. 100 parts = 0·26700 grain.

Sp+X, T+Y.	T+Y, Sp+X.		Sp+Y, T+X.	T+X, Sp+Y.
24·67	25·17		24·85	24·67
25·64	24·77		26·95	24·15
25·24	25·62		24·85	24·82
25·15	24·76	D= 14·55	26·62	23·97
25·19	24·80	C= 14·57	25·27	23·76
25·34	26·70	F=771·95	25·31	23·70
25·74	24·74	E= 15	23·99	23·51
25·25	26·29		25·86	22·35
25·35	25·70		24·27	23·52
25·10	24·54		25·44	23·35
252·67	253·09		253·41	237·80

20(T+Y)≏20(Sp+X)−0·42 part. 20(T+X)≏20(Sp+Y)+15·61 parts.

40T≏40Sp+0·04056 grain in air (t=14·58, b=770·53).

January 16, 1845. 100 parts = 0·16949 grain.

T+Y.	RS+X.		T+X.	RS+Y.
16·47	17·70		18·05	20·75
17·90	20·06	D= 9·03	18·46	20·62
17·31	21·00	C= 9·10	18·90	20·51
18·54	21·47	F=764·0	18·55	21·95
18·35	22·67	E= 8	19·41	21·99
88·57	102·90		93·37	105·82

5(T+Y)≏5(RS+X)−14·33 parts. 5(T+X)≏5(RS+Y)−12·45 parts.

10T≏10RS−0·04539 grain in air (t=9·05, b=763·46).

July 28, 1845. 100 parts = 0·33720 grain.

T+X.	RS+Y.		T+Y.	RS+X.
19·37	21·07		18·66	19·03
19·58	21·90		18·95	19·59
20·25	22·20		18·46	19·78
19·95	22·22		18·84	19·70
20·10	21·50	D= 16·95	18·67	19·65
19·66	22·05	C= 17·0	18·47	19·78
18·77	21·27	F=755·60	18·97	19·46
18·85	20·68	E= 17·2	18·20	18·70
19·08	19·51		18·90	19·20
18·71	20·52		18·55	19·82
18·92	20·52		18·33	19·60
19·26	20·40		18·72	19·60
232·50	253·84		223·72	233·91

12(T+X)≏12(RS+Y)−21·34 parts. 12(T+Y)≏12(RS+X)−10·19 parts.

24T≏24RS−0·10632 grain in air (t=17·01, b=753·95).

August 14, 1845. 100 parts = 0·29755 grain.

RS+X, T+Y.	T+Y, RS+X.		RS+Y, T+X.	T+X, RS+Y.
20·51	22·76		19·25	23·77
20·50	23·11		19·90	23·99
19·95	21·97		18·75	23·57
20·70	21·46		18·56	22·80
20·57	22·55	D= 15·95	18·50	22·27
19·70	21·47	C= 15·95	20·84	24·35
21·67	23·47	F=760·85	18·77	22·21
20·54	23·25	E= 15·9	18·17	23·31
20·95	23·74		17·80	22·99
21·10	22·84		18·37	22·71
21·34	23·75		17·91	23·35
227·53	250·37		206·82	255·32

22(T+Y)≏22(RS+X)−22·84 parts. 22(T+X)≏22(RS+Y)−48·5 parts.

44T≏44RS−0·21227 grain in air (t=15·96, b=759·34).

August 15, 1845. 100 parts =0·27921 grain.

RS+X, T+Y.	T+Y, RS+X.		RS+Y, T+X.	T+X, RS+Y.
18·05	20·30		15·70	18·89
18·64	21·87		14·89	20·05
18·67	21·75		16·06	19·79
16·41	19·47		16·39	20·05
16·96	20·75	D= 16·25	16·66	19·85
16·59	20·59	C= 16·30	17·00	19·72
16·46	20·04	F=756·9	18·81	21·60
17·17	20·10	E= 16	16·48	21·62
17·79	21·00		17·60	20·47
17·75	19·79		17·94	20·85
18·21	20·53		15·22	17·54
192·70	226·19		182·75	220·43

22(T+Y)≏22(RS+X)−33·49 parts. 22(T+X)≏22(RS+Y)−37·68 parts.

44T≏44RS−0·19871 grain in air (t=16·29, b=755·39).

1845.		t.	b.	gr.
January	15.	9·91	754·30	10T≏ 10Sp +0·01695
February	21.	6·94	758·80	40T≏ 40Sp +0·02900
February	26.	6·28	752·41	16T≏ 16Sp +0·00235
July	26.	17·53	759·79	24T≏ 24Sp +0·01392
August	16.	15·91	758·70	56T≏ 56Sp +0·04162
August	18.	15·93	754·99	40T≏ 40Sp +0·01443
August	19.	15·6	739·29	20T≏ 20Sp +0·00169
August	27.	14·95	764·69	40T≏ 40Sp +0·04730
August	29.	14·58	770·53	40T≏ 40Sp +0·04056
				286T≏286Sp +0·20782
		In air (t=13·74, b=758·91)		T≏ Sp +0·00073

		t.	b.	gr.
January	16.	9·05	763·46	10T≏ 10RS−0·04539
July	28.	17·01	753·95	24T≏ 24RS−0·10632
August	14.	15·96	759·34	44T≏ 44RS−0·21227
August	15.	16·29	755·39	44T≏ 44RS−0·19871
				122T≏122RS−0·56269
		In air (t=15·72, b=757·19)		T≏ RS−0·00461

Since ΔSp was computed by means of SCHUMACHER's Tables, we must employ the value of ΔT obtained by means of the same Tables in reducing the comparisons of T with Sp and RS. By these Tables log Δ=1·32566 for T, and log Δ=1·32608 for Sp and RS.

T≏Sp+0·00073 grain in air (t=13·74, b=758·91).

T displaces 0·33356 grain of air, Sp displaces 0·33324 grain of air. Therefore

T=Sp+0·00105 grain.

T≏RS−0·00461 grain in air (t=15·72, b=757·19).

T displaces 0·33037 grain of air, RS displaces 0·33005 grain of air. Therefore

T=RS−0·00429 grain.

If we suppose U to have had the same density as V, Sp=U−0·52956 grain, and RS=U−0·52441 grain. The former gives T=U−0·52851 grain, the latter T=U−0·52870. Mean, giving to the former twice the weight of the latter, because

the number of comparisons of Sp with U, and of T with Sp is about twice as large as the number of comparisons of RS with U, and T with RS,

$$T = U - 0{\cdot}52857 \text{ grain} = 5759{\cdot}47143 \text{ grains.}$$

During the comparisons of Sp and RS with U in Somerset House, the mean value of t was 18·7, and that of b was 755·64.

T displaced 0·32605 grain of air, U displaced 0·84717 grain of air.

Hence $T \triangleq U - 0{\cdot}00745$ grain in air ($t=18{\cdot}7$, $b=755{\cdot}64$) in Somerset House.

Comparisons of the auxiliary weights in January and February 1845.

The comparisons of the auxiliary weights among each other, after their second adjustment, were made with my 10½-inch ROBINSON. The comparisons of A+B+C+D+F with T were made with BARROW's balance.

February 22 and 24, 1845. 100 parts =0·17274 grain.

S=A+B+C+D+F.

T+X.	S+Y.	T+Y.	S+X.
22·00	20·06	19·16	20·65
21·05	29·37	21·52	21·50
22·16	20·50	23·17	22·73
21·46	20·52	23·86	21·00
22·73	21·21	20·88	20·47
22·71	21·31	21·60	20·67
23·21	21·32	20·93	21·10
23·53	21·68	21·20	21·67
23·68	21·50	22·50	21·40
24·05	22·00	21·61	21·77
23·27	21·23	21·65	21·26
249·85	231·70	238·08	234·22

11(T+X)=11(S+Y)+18·15 parts. 11(T+Y)=11(S+X)+3·86 parts.

T=A+B+C+D+F+0·00173 grain.

Jan. 29, Feb. 5 and 6, 1845. 100 parts =0·3063 grain.

A, F+G.	F+G, A.	B, F+G.	F+G, B.
1·80	2·29	3·35	3·15
1·95	2·28	3·46	3·01
2·45	2·57	3·20	2·65
2·75	3·75	3·35	2·85
3·00	3·59	3·90	3·05
3·39	3·35	4·14	2·85
2·97	2·50	3·66	1·69
2·05	2·30	2·73	1·80
2·65	2·50	3·41	1·76
1·94	2·51	2·50	2·13
2·21	1·93	2·60	1·28
2·29	2·86	3·00	2·09
2·20	2·69	2·88	1·95
2·55	2·73	3·00	1·75
2·71	3·44	3·18	2·80
2·71	3·56	3·08	2·15
2·66	2·86	3·16	2·65
42·28	47·71	54·60	39·61

A=F+G−0·00049 grain. B=F+G+0·00135 grain.

C, F+G.	F+G, C.	D, F+G.	F+G, D.
2·65	1·70	2·45	1·90
3·90	2·95	3·36	2·76
3·52	2·38	3·17	3·04
3·45	2·20	3·55	2·41
4·01	2·91	4·15	2·90
4·40	2·85	3·99	2·50
3·75	1·65	3·23	1·88
3·00	1·96	2·58	2·40
3·54	1·69	3·08	1·79
2·73	1·60	2·38	1·95
2·80	1·23	2·40	1·66
3·50	1·56	2·80	2·00
3·08	1·69	2·59	2·25
3·05	2·33	2·80	2·75
3·35	2·51	3·03	2·70
3·41	2·21	3·00	2·88
3·18	2·44	2·90	2·40
57·32	35·86	51·46	40·17

C=F+G+0·00193 grain. D=F+G+0·00102 grain.

Jan. 27 and 29, 1845. 100 parts =0·27082 grain.

F, G+H.	G+H, F.
2·64	2·85
2·69	2·97
2·30	2·86
2·55	2·82
2·36	2·91
2·51	3·07
2·27	2·77
2·16	2·59
2·72	2·75
2·52	3·46
2·41	3·39
2·59	2·87
29·72	35·31

F=G+H−0·00063 grain.

Jan. 25, 27. 100 parts =0·25016 grain.

G, H+K.	H+K, G.	G, H+L.	H+L, G.
4·65	3·00	5·10	2·60
4·67	3·34	5·22	2·67
3·30	1·94	3·72	1·34
3·85	2·76	4·40	2·17
3·87	2·85	4·52	2·32
3·99	3·01	4·66	2·44
3·47	2·15	3·85	1·82
3·51	2·45	3·89	1·77
3·35	2·05	3·89	1·66
3·49	2·45	3·87	1·86
38·15	26·00	43·12	20·65

G=H+K+0·00152 grain. G=H+L+0·00281 grain.

G, H+M.	H+M, G.	G, H+N.	H+N, G.
4·70	3·20	4·35	3·65
4·45	3·22	4·29	3·80
2·95	2·02	2·57	2·27
3·67	2·70	3·32	3·00
3·77	2·80	3·40	3·06
3·77	3·00	3·47	3·15
3·40	2·30	3·00	2·56
3·45	2·35	3·10	2·55
3·32	2·12	3·07	2·47
3·54	2·45	3·15	2·60
37·02	26·16	33·72	29·11

G=H+M+0·00136 grain. G=H+N+0·00058 grain.

Jan. 25, Feb. 7. 100 parts =0·251256 grain.

H, K+L+M+N+R.	K+L+M+N+R, H.	H, K+L+M+N+S.	K+L+M+N+S, H.
3·55	3·85	3·70	3·95
3·40	3·80	3·55	3·70
3·80	2·70	3·65	3·80
3·50	3·80	3·50	3·70
3·60	3·70	3·65	3·80
3·80	3·70	3·60	3·70
3·76	3·31	3·77	3·30
4·32	3·61	4·00	3·37
3·95	3·52	3·60	3·32
3·75	3·29	3·65	3·16
4·02	3·42	4·02	3·31
4·10	3·62	4·01	3·71
3·90	3·42	3·86	3·15
3·60	3·10	3·52	3·25
3·74	3·50	3·74	3·51
3·79	3·12	3·72	3·05
2·71	2·27	2·62	2·26
63·29	57·73	62·16	58·02

H=K+L+M+N+R+0·00034 grain. H=K+L+M+N+S+0·00030 grain.

January 25, and February 7. 100 parts =0·26560 grain.

K, R+S.	R+S, K.	L, R+S.	R+S, L.
3·45	3·80	2·80	4·00
3·25	3·30	2·70	3·70
3·20	3·30	2·75	3·80
3·10	2·90	2·70	3·30
3·00	3·30	2·75	3·70
3·20	3·15	2·70	3·70
2·00	2·05	1·45	2·62
2·75	2·95	2·30	3·37
3·22	3·20	2·55	3·81
2·90	3·00	2·32	3·60
2·99	3·17	2·37	3·65
2·99	3·22	2·40	3·77
36·05	37·34	29·79	43·02

K=R+S−0·00014 grain. L=R+S−0·00146 grain.

M, R+S.	R+S, M.	N, R+S.	R+S, N.
3·50	3·25	3·70	2·95
3·30	3·20	3·50	2·95
3·20	3·20	3·50	2·80
3·10	2·95	2·90	2·70
3·20	3·40	3·50	2·90
3·30	3·15	3·50	2·90
2·07	2·05	2·31	1·75
2·96	2·69	3·17	2·64
3·20	3·35	3·47	2·90
2·92	3·05	3·26	2·64
3·07	3·10	3·41	2·66
2·96	3·12	3·36	2·77
36·78	36·51	39·58	32·56

M=R+S+0·00003 grain. N=R+S+0·00078 grain.

T =A+B+C+D+F	+0·00173
A =F+G	−0·00049
B =F+G	+0·00135
C =F+G	+0·00193
D=F+G	+0·00102
F =G+H	−0·00063
G =H+K	+0·00152
G =H+L	+0·00281
G =H+M	+0·00136
G =H+N	+0·00058
H=K+L+M+N+R	+0·00034
H=K+L+M+N+S	+0·00030
K=R+S	−0·00014
L =R+S	−0·00146
M=R+S	+0·00003
N=R+S	+0·00078

	gr.
A+B+C+D=4F+4G	+0·00381
4G=4H+K+L+M+N	+0·00627
2H=2(K+L+M+N)+R+S	+0·00064
K+L+M+N=4(R+S)	−0·00079
4(K+L+M+N)=16(R+S)	−0·00316
4H=4(K+L+M+N)+2(R+S)	+0·00128
4G=4H+K+L+M+N	+0·00627
4F=4G+4H	−0·00252
A+B+C+D=4F+4G	+0·00381
T=A+B+C+D+F	+0·00173
4H=18(R+S)	−0·00188
4G=22(R+S)	+0·00360
4F=40(R+S)	−0·00080
4F+4G=62(R+S)	+0·00280
A+B+C+D=62(R+S)	+0·00661
T=72(R+S)	+0·00814

		grains.
T	=	5759·47143
72(R+S)	=	5759·46329
18(R+S)	=	1439·86582
40(R+S)	=	3199·70183
10(R+S)	=	799·92546
62(R+S)	=	4959·53783
22(R+S)	=	1759·83601
A+B+C+D	=	4959·54444
4F+4G	=	4959·54063
F+G	=	1239·88516

		grains.
A	=	1239·88467
B	=	1239·88651
C	=	1239·88709
D	=	1239·88618
F	=	799·92526
G	=	499·95990
H	=	359·96598
K	=	79·99241
L	=	79·99109
M	=	79·99258
N	=	79·99333
R	=	39·99625
S	=	39·99629

In the summer of 1845, after the adjustment and comparison of PS 200 times with T+0·645 grain together with each of the weights A, B, C, D in succession, the comparisons of the auxiliary weights with each other and with T presented some unaccountable discordances. By a most troublesome repetition of the weighings with different combinations of the weights, it became evident that A, C, F were subject to a very sensible fluctuation. This was at last found to be due to the circumstance that the platinum of which they were made had been very badly prepared, and contained cavities filled with a substance which attracted moisture from the air. In order to remove this injurious matter, they were digested in boiling water and then placed in a platinum capsule over a spirit-lamp, the heat of which caused a brown liquid to escape from the openings in their surfaces. After repeating this process several times till the coloured liquid ceased to appear, and till it was supposed that the whole of the deliquescent substance was removed, it was found that A, C and F had lost 0·04 grain, 0·031 grain, and 0·04 grain respectively. It now became obvious that all the weighings into which either A, B or F entered, must be repeated. This involved the rejection of the observations for determining the weight of PS, and the comparison of PS with the kilogramme, as well as those for comparing the auxiliary weights themselves. The weights lost by A, C, F were made up by the addition of bits of wire. In the following comparisons A denotes the weight marked A+0·04 grain, C the weight marked C+0·031 grain, F the weight marked F+0·04 grain.

June 26, 1846. 100 parts =0·26694 grain.

S=A+B+C+D+F.

S+Y, T+X.	T+X, S+Y.	S+X, T+Y.	T+Y, S+X.
19·97	19·80	18·00	18·24
20·04	19·40	17·44	18·30
20·57	18·66	17·60	18·54
20·41	18·89	17·95	18·61
19·75	18·65	17·90	17·96
19·96	19·55	17·82	18·60
19·99	18·80	17·27	18·16
18·92	18·52	17·17	18·41
18·41	18·59	18·27	18·14
18·94	17·22	17·60	18·29
196·96	188·08	177·02	183·25

20(T+X)=20(S+Y)+8·88 parts 20(T+Y)=20(S+X)−6·23 parts.

40T=40(A+B+C+D+F)+0·00708 grain.

July 1. 100 parts = 0·26965 grain.

S+Y, T+X.	T+X, S+Y.	S+X, T+Y.	T+Y, S+X.
17·61	18·09	15·67	16·70
17·67	17·87	16·17	16·99
16·95	17·81	16·01	15·85
17·09	17·47	15·55	16·96
17·04	17·49	16·19	16·89
17·06	17·59	16·15	16·07
16·82	17·36	15·52	16·00
16·79	17·70	15·47	16·46
17·15	17·19	15·40	16·80
16·81	17·01	15·90	15·99
170·99	175·58	158·03	164·71

$20(T+X)=20(S+Y)-4·59$ parts. $20(T+Y)=20(S+X)-6·68$ parts.

$40T=40(A+B+C+D+F)-0·03039$ grain.

$T=A+B+C+D+F-0·00029$ grain.

June 25, July 3, 4, 6, 1846. 100 parts = 0·49505 grain.

A, F+G.	F+G, A.	B, F+G.	F+G, B.
3·52	3·95	3·34	3·92
3·12	3·77	3·07	3·60
2·67	2·70	2·72	2·92
2·67	3·44	2·75	3·26
2·92	3·22	2·97	2·71
2·70	3·44	2·57	3·47
3·25	3·74	2·96	3·57
2·97	3·44	3·07	3·46
2·84	3·67	2·79	3·52
2·79	3·39	2·79	3·41
2·87	3·37	2·95	3·44
2·61	3·19	2·47	3·22
2·50	2·89	2·26	3·27
2·46	3·00	2·29	3·00
2·54	2·98	2·47	3·15
2·82	3·32	2·61	3·39
2·95	3·61	2·86	3·67
2·75	3·42	3·22	4·00
2·74	4·16	2·96	4·09
2·74	3·67	3·06	4·12
2·99	3·80	2·66	4·04
2·79	4·00	2·69	3·75
62·21	76·17	61·53	76·98

$A=F+G-0·00157$ grain. $B=F+G-0·00174$ grain.

June 25, July 3, 4, 6, 1846. 100 parts = 0·49505 grain.

C, F+G.	F+G, C.	D, F+G.	F+G, D.
3·21	3·94	3·05	3·77
3·05	3·87	3·02	3·56
2·70	2·90	2·72	2·89
2·80	2·97	2·69	2·92
3·12	3·29	3·10	3·21
2·52	3·46	2·85	3·51
2·89	3·70	3·02	3·97
2·92	3·20	2·91	3·16
2·55	3·44	2·75	3·12
2·70	3·42	2·90	3·47
2·82	3·60	2·84	3·57
2·35	3·59	2·10	3·31
2·67	3·05	2·27	3·35
2·16	3·24	2·65	3·26
2·50	3·12	2·32	3·35
2·50	3·11	2·54	3·55
2·77	3·61	2·67	4·01
2·97	3·84	2·62	3·75
2·62	3·70	2·35	3·70
2·51	3·67	2·56	3·91
2·86	3·80	2·46	3·71
2·86	3·75	2·45	3·49
60·05	76·27	58·84	76·54

C = F + G − 0·00182 grain. D = F + G − 0·00199 grain.

June 25, July 3, 4, 5, 6, 1846. 100 parts = 0·42553 grain.

F, G+H.	G+H, F.
4·72	4·07
4·60	4·02
5·05	4·19
4·05	3·96
3·95	3·32
3·99	3·37
3·82	3·45
3·20	3·17
3·55	3·32
3·45	3·04
4·01	3·60
4·10	3·57
4·14	3·55
4·17	3·39
4·17	3·77
4·46	3·56
4·24	3·52
4·19	3·29
4·15	3·60
4·07	3·44
4·09	3·31
4·37	3·46
4·44	3·67
4·49	3·31
4·40	3·02
4·30	3·80
4·40	3·50
4·45	3·34
4·11	3·64
4·54	3·32
4·17	3·12
129·84	108·69

F = G + H + 0·00145 grain.

The relations between G, H, K, L, M, N, R, S are given by the observations of 1845.

		gr.
T =A+B+C+D+F		−0·00029
A =F+G		−0·00157
B =F+G		−0·00174
C =F+G		−0·00182
D =F+G		−0·00199
F =G+H		+0·00145
G =H+K		+0·00152
G =H+L		+0·00281
G =H+M		+0·00136
G =H+N		+0·00058
H =K+L+M+N+R		+0·00034
H =K+L+M+N+S		+0·00030
K =R+S		−0·00014
L =R+S		−0·00146
M=R+S		+0·00003
N =R+S		+0·00078
A+B+C+D	=4F+4G	−0·00712
4G	=4H+K+L+M+N	+0·00627
2H	=2(K+L+M+N)+R+S	+0·00064
K+L+M+N	=4R+4S	−0·00079
4(K+L+M+N)	=	16(R+S)−0·00316
4H	=4(K+L+M+N)+	2(R+S)+0·00128
4H	=	18(R+S)−0·00188
K+L+M+N	=	4(R+S)−0·00079
4G	=4H+K+L+M+N	+0·00627
4G	=	22(R+S)+0·00360
4H	=	18(R+S)−0·00188
4G+4H	=	40(R+S)+0·00172
4F	=4G+4H	+0·00580
4F	=	40(R+S)+0·00752
4G	=	22(R+S)+0·00360
4F+4G	=	62(R+S)+0·01112
A+B+C+D	=	62(R+S)+0·00400
F	=	10(R+S)+0·00188
A+B+C+D+F	=	72(R+S)+0·00588
T	=	72(R+S)+0·00559

		grains.
T	=	5759·47143
72(R+S)	=	5759·46584
18(R+S)	=	1439·86646
40(R+S)	=	3199·70324
10(R+S)	=	799·92581
62(R+S)	=	4959·54003
22(R+S)	=	1759·83678
A+B+C+D	=	4959·54403
4F+4G	=	4959·55115
F+G	=	1239·88779
A	=	1239·88622
B	=	1239·88605
C	=	1239·88597
D	=	1239·88580
F	=	799·92769
G	=	439·96009
H	=	359·96614
K	=	79·99244
L	=	79·99112
M	=	79·99261
N	=	79·99336
R	=	39·99627
S	=	39·99631

In the observations for finding specific gravities made after June 1846, A, C, F were used without the bits of wire added to make up their loss of weight. Their values were, therefore,—

	grains.
A	=1239·84622
C	=1239·85497
F	= 799·88769

Y, Z are weights of 19·998 grains each, W a weight of 12·901 grains, V a weight of 6·451 grains, Q the mean of the ten weights of 0·645 grain nearly.

August 22, 1845, January 17, 1846. 100 parts =0·4065 grain.

Y+Z, R.	R, Y+Z.	Y+Z, S.	S, Y+Z.
3·69	3·49	3·61	3·54
3·50	3·40	3·47	3·45
3·57	3·42	3·50	3·37
3·50	3·44	3·52	3·39
3·56	3·42	3·56	3·40
3·55	3·34	3·47	3·36
21·37	20·51	21·13	20·51

Y+Z=R+0·00029 grain. Y+Z=S+0·00021 grain.

10(R+S)=799·92581 grains. Y+Z=39·99654 grains.

In the following comparisons of W+V+Q with Y and Z, the Q was changed at the end of each weighing, so that each Q was weighed eight times with W+V.

August 22, 1845, January 17, 1846. 100 parts =0·3992 grain.

W+V+Q, Y.	Y, W+V+Q.	W+V+Q, Z.	Z, W+V+Q.
3·02	3·91	3·17	3·85
3·10	3·92	3·32	3·70
3·10	3·96	3·27	3·76
3·11	3·94	3·35	3·75
3·16	4·07	3·35	3·67
3·05	4·05	3·31	3·79
3·20	4·01	3·31	3·62
3·10	4·06	3·22	3·67
3·17	4·00	2·82	3·57
3·20	3·97	3·25	3·67
3·01	3·80	3·19	3·71
2·88	3·86	3·22	3·69
2·91	3·85	3·23	3·62
2·97	3·81	3·20	3·59
3·01	3·81	3·25	3·61
3·08	3·87	3·20	3·56
2·97	3·84	3·22	3·67
2·98	3·86	3·23	3·64
3·04	3·83	3·27	3·59
3·07	3·80	3·28	3·70
61·13	78·22	64·66	73·43

W+V+Q=Y−0·001705 grain. W+V+Q=Z−0·000875 grain.

August 22, 1845, Jan. 17, 1846. 100 parts =0·4096 grain.

W, V+10Q.	V+10Q, W.	V, 10Q.	10Q, V.
3·29	3·65	3·45	3·62
3·30	3·56	3·42	3·47
3·29	3·56	3·37	3·41
3·32	3·61	3·44	3·38
3·25	3·60	3·45	3·46
3·30	3·59	3·37	3·50
19·75	21·57	20·50	20·84

$2V+20Q=2W+0{\cdot}00124$ grain. $40Q=4V+0{\cdot}00046$ grain.

$$
\begin{array}{lll}
 & & \text{gr.} \\
Y+Z & = & 39{\cdot}99654 \\
2W+2V+2Q & =Y+Z- & 0{\cdot}00258 \\
2V+20Q & =2W + & 0{\cdot}00124 \\
40Q & =4V + & 0{\cdot}00046 \\
62Q & = & 39{\cdot}99566 \\
Q & = & 0{\cdot}64509
\end{array}
$$

Let (64), (32), (16), (8), (4), (2), (1) denote platinum weights belonging to Barrow's balance, 2J the sum of two platinum grain weights belonging to Robinson's balances, G a brass grain weight.

April 11, 1845. 100 parts =0·29261 grain.

K, (64)+(16)	2·47	(64)+(16), K	4·82
L, (64)+(16)	1·88	(64)+(16), L	5·45
M, (64)+(16)	2·44	(64)+(16), M	4·80
N, (64)+(16)	2·70	(64)+(16), N	4·35
R+S, (64)+(16)	2·32	(64)+(16), R+S	4·85
K, (64)+(16)	2·30	(64)+(16), K	4·65
L, (64)+(16)	1·80	(64)+(16), L	5·32
M, (64)+(16)	2·42	(64)+(16), M	4·60
N, (64)+(16)	2·65	(64)+(16), N	4·29
R+S, (64)+(16)	2·35	(64)+(16), R+S	4·61
	23·33		47·74

$$
\begin{array}{lll}
 & & \text{gr.} \\
5[(64)+(16)] & =K+L+M+N+R+S+ & 0{\cdot}01786 \\
K+L+M+N & =4(R+S) \quad - & 0{\cdot}00079 \\
K+L+M+N+R+S & =5(R+S) \quad - & 0{\cdot}00079 \\
5(R+S) & = & 399{\cdot}96290 \\
K+L+M+N+R+S & = & 399{\cdot}96211 \\
5[(64)+(16)] & = & 399{\cdot}97997 \\
(64)+(16) & = & 79{\cdot}99599
\end{array}
$$

March 19, 1845. 100 parts =0·29485 grain.

	Scale.		Scale.
R, (32)+(8)	2·310	(32)+(8), R	3·675
S, (32)+(8)	2·640	(32)+(8), S	3·650
R, (32)+(8)	2·550	(32)+(8), R	3·675
S, (32)+(8)	2·675	(32)+(8), S	3·650
R, (32)+(8)	2·775	(32)+(8), R	3·725
S, (32)+(8)	2·860	(32)+(8), S	3·875
R, (32)+(8)	2·660	(32)+(8), R	3·790
S, (32)+(8)	2·825	(32)+(8), S	3·790
	21·295		29·830

$16[(32)+(8)]=8(R+S)+0{\cdot}02517$ grain.

January 19, 1846. 100 parts =0·40140 grain.

	Scale.		Scale.
R, (32)+(8)	3·300	(32)+(8), R	3·975
S, (32)+(8)	3·362	(32)+(8), S	3·875
R, (32)+(8)	3·300	(32)+(8), R	3·950
S, (32)+(8)	3·362	(32)+(8), S	3·875
R, (32)+(8)	3·200	(32)+(8), R	3·925
S, (32)+(8)	3·287	(32)+(8), S	3·825
R, (32)+(8)	3·175	(32)+(8), R	3·900
S, (32)+(8)	3·275	(32)+(8), S	3·862
R, (32)+(8)	3·212	(32)+(8), R	3·812
S, (32)+(8)	3·375	(32)+(8), S	3·800
	32·848		38·799

$$
\begin{aligned}
36[(32)+(8)] &= 18(R+S) + \overset{\text{gr.}}{0{\cdot}04906}\\
2[(32)+(8)] &= R+S + 0{\cdot}00273\\
R+S &= 79{\cdot}99258\\
2[(32)+(8)] &= 79{\cdot}99531\\
(32)+(8) &= 39{\cdot}99765
\end{aligned}
$$

January 19, 1846. 100 parts =0·40140 grain.

(64), (32)+(16)+(8)+(4)+(2)+2J.	(32)+(16)+(8)+(4)+(2)+2J, (64).
3·487	3·375
3·512	3·462
3·750	3·525
3·837	3·487
3·800	3·562
18·386	17·411

$(32)+(16)+(8)+(4)+(2)+2J=(64)-0{\cdot}00038$ grain.

(32), (16)+(8)+(4)+(2)+2J.	(16)+(8)+(4)+(2)+2J, (32).
3·325	3·450
3·437	3·537
3·512	3·537
3·625	3·550
3·587	3·637
17·486	17·711

$(16)+(8)+(4)+(2)+2J=(32)+0{\cdot}00009$ grain.

(16), (8)+(4)+(2)+2J.	(8)+(4)+(2)+2J, (16).	(8), (4)+(2)+2J.	(4)+(2)+2J, (8).
3·525	3·225	3·437	3·275
3·512	3·375	3·500	3·300
3·662	3·350	3·587	3·425
3·625	3·350	3·612	3·475
3·687	3·437	3·625	3·500
18·011	16·737	17·761	16·975

$(8)+(4)+(2)+2J=(16)-0{\cdot}00051$ grain. $(4)+(2)+2J=(8)-0{\cdot}00032$ grain.

January 16, 1845. 100 parts =0·4096 grain.

(4), (2)+2J.	(2)+2J, (4).	(2), 2J.	2J, (2).
3·312	3·212	3·287	3·387
3·412	3·425	3·337	3·450
3·612	3·425	3·325	3·487
3·587	3·412	3·312	3·537
3·562	3·500	3·387	3·512
17·485	16·974	16·648	17·373

$(2)+2J=(4)-0{\cdot}00021$ grain. $2J=(2)+0{\cdot}00030$ grain.

March 20, 1845. 100 parts $=0{\cdot}29485$ grain.

(2), (1)+G.	(1)+G, (2).	(1), G.	G, (1).
2·98	3·46	2·80	3·65
3·10	3·57	2·75	3·72
3·12	3·67	3·02	3·72
9·20	10·70	8·57	11·09

$(1)+G=(2)+0{\cdot}00074$ grain. $G=(1)+0{\cdot}00124$ grain.

Hence $2(1)=(2)-0{\cdot}00050$ grain.

		gr.
2J	= (2)	+ 0·00030
2J+(2)	= (4)	− 0·00021
2J+(2)+(4)	= (8)	− 0·00032
2J+(2)+(4)+(8)	=(16)	− 0·00051
2J+(2)+(4)+(8)+(16)	=(32)	+ 0·00009
2J+(2)+(4)+(8)+(16)+(32)	=(64)	− 0·00038

Hence

2J	= (2)	+ 0·00030
4J	= (4)	+ 0·00009
8J	= (8)	+ 0·00007
16J	=(16)	− 0·00005
32J	=(32)	+ 0·00050
64J	=(64)	+ 0·00053
80J	=(64)+(16)+0·00048	=79·99647
40J	=(32)+(8)+0·00057	=39·99822

Mean.........80J=79·99646

64J=63·99717	(64)	=63·99664
32J=31·99858	(32)	=31·99808
16J=15·99929	(16)	=15·99934
8J= 7·99965	(8)	= 7·99958
4J= 3·99982	(4)	= 3·99973
2J= 1·99991	(2)	= 1·99961
J= 0·99996	(1)	= 0·99956

Observations for finding the density of PC No. 1.

Weighing of PC No. 1 in air.

August 12, 1844. 100 parts $=0{\cdot}2564$ grain.

PC.	T+B+0·67 grain.
19·00	20·80
21·65	21·00
21·30	21·50
20·70	20·50
20·50	20·80
21·35	20·95
21·95	20·40
20·35	18·90
19·80	19·00
19·50	18·60
18·95	17·60
18·20	17·70
18·10	17·60
18·00	16·85
17·50	16·95
17·60	16·90
17·45	16·50
17·40	16·85
17·15	17·00
18·10	17·30
384·55	373·70

PC No. 1 $\bumpeq$ T+B+0·6714 gr. T=5759·4881 gr., B=1239·8880 gr. Therefore PC No. 1 $\bumpeq$ 7000·0475 grains.

For PC No. 1, log Δ=1·32566. For T+B+0·6714, log Δ=1·32560.

Hence *v*PC No. 1=330·70, *v*(T+B+0·6714 gr.)=330·72. *v*PC No. 1 is less than *v*(T+B+0·6714) by the volume of 0·02 grain of water, or of 0·00002 grain of air of ordinary density. Hence

PC No. 1=7000·0475 grains.

July 18, 1844. 100 parts =0·26 grain.

PC No. 1 and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
16·95	17·65	0·12	19·4
16·9	17·6	0·12	19·9
16·9	17·65	0·12	20·1
16·9	17·6	0·12	20·8
16·8	17·55	0·12	21·2
16·8	17·5	0·12	21·5
16·8	17·45	0·12	19·1
16·8	17·5	0·12	19·0
16·75	17·45	0·12	19·4
16·75	17·45	0·12	20·0
16·835	17·53	0·120	20·04

Hook in water. In right-hand pan.

	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·68	27·4
	0·66	20·4
	0·66	20·4
	0·66	19·3
	0·66	16·0
Air.	0·66	14·0
t=17·0, b=754·76.	0·68	22·0
	0·68	22·5
	0·66	19·2
	0·66	19·2
	0·66	16·5
	0·68	23·0
	0·68	26·2
	0·6677	20·47

PC No. 1 in water (t=16·95) $\bumpeq$ T+F+K+(16)+(8)+(4)+(2)+0·5466 gr. in air (t=17·0, b=754·76).

July 22, 1844. 100 parts =0·26 grain.

PC No. 1 and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
		0·04	22·0
		0·04	22·0
		0·08	41·6
		0·00	15·7
		0·04	22·5
19·4	20·4	0·00	13·0
		0·00	7·9
		0·00	15·5
		0·04	32·8
		0·04	21·5
		0·00	12·5
		0·00	12·5
19·4	20·4	0·0233	19·96

Hook in water. In right-hand pan.

	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·72	28·5
	0·68	7·5
	0·70	15·0
	0·71	21·6
Air.	0·70	18·0
t=20·0, b=765·83.	0·72	26·3
	0·68	13·5
	0·72	23·0
	0·72	33·0
	0·68	9·0
	0·703	19·54

PC No. 1 in water (t=19·54)≏T+F+K+(16)+(8)+(4)+(2)+0·6811 gr. in air (t=20·0, b=765·83).

July 25, 1844. 100 parts =0·26 grain.

PC No. 1 and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
21·03	22·15	0·20	36·5
21·0	22·10	0·16	24·8
		0·12	11·0
		0·16	22·5
		0·14	18·5
21·0	22·15	0·16	24·4
21·0	22·1	0·12	9·0
		0·14	18·0
21·0	22·1	0·12	13·0
		0·16	22·5
21·0	22·1	0·15	20·4
21·0	22·05	0·15	19·5
20·95	22·0	0·15	19·7
20·9	22·0	0·15	20·1
20·99	22·08	0·1486	19·99

Hook in water. In right-hand pan.

	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·98	28·3
	0·94	15·7
	0·96	18·0
	0·98	25·6
	0·94	15·5
	0·96	20·4
	0·98	23·4
	0·94	13·2
	0·98	25·5
	0·94	13·4
	0·96	19·5
	0·98	28·5
	0·94	13·1
	0·96	18·8
	0·960	99·99

PC No. 1 in water (t=21·15)≏T+F+K+(16)+(8)+(4)+(2)+0·8114 gr. in air (t=21·6, b=761·37).

August 2, 1844. 100 parts = 0·26 grain.

PC No. 1 and hook in water.		In right-hand pan.	
H.	P.	gr.	Scale.
		0·33	19·2
17·9	18·6	0·35	20·0
		0·35	20·2
17·8	18·5	0·35	20·5
		0·33	19·0
17·6	18·4	0·35	20·1
17·6	18·4	0·35	18·2
		0·37	20·6
17·5	18·35	0·37	21·0
		0·37	25·6
17·5	18·25	0·37	24·4
17·4	18·2	0·33	18·1
		0·33	14·0
		0·33	10·8
		0·33	8·2
		0·37	24·5
17·61	18·39	0·3487	19·02

Hook in water.	In right-hand pan.	
	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·94	23·1
	0·90	11·0
	0·94	16·5
	0·96	22·4
Air.	0·94	22·6
t=18·5, b=746·12.	0·94	21·2
	0·96	25·0
	0·92	17·5
	0·96	20·4
	0·96	20·0
	0·942	19·97

PC No. 1 in water (t=17·68) ≏ T+F+K+(16)+(8)+(4)+(2)+0·5908 gr. in air (t=18·5, b=746·12).

Apparent weight of PC No. 1 in water weighed with platinum weights.

Water. t.	Apparent weight of PC No. 1. gr.	Air. t.	b.
16·95	6669·9457	17·0	754·76
19·54	6670·0802	20·0	765·83
21·15	6670·2105	21·6	761·37
17·68	6669·9899	18·5	746·12

The resulting values of ΔPC No. 1 and log ΔPC No. 1 are,—

	ΔPC No. 1.	log ΔPC No. 1.
	21·1664	1·325647
	21·1662	1·325644
	21·1687	1·325695
	21·1673	1·325666
Mean	21·16715	1·325663
10 − log Δ............		8·674337
PC No. 1 (reduced)	7000·004	3·845098
vPC No. 1	330·701	2·519435

Observations for finding the density of PC No. 2.

Weighing of PC No. 2 in air.

August 13, 1844.

100 parts = 0·2432 grain.

PC.	T+B+0·653 gr.
20·20	19·90
21·30	20·80
21·40	21·50
22·30	22·40
22·80	24·40
22·85	22·45
22·95	22·85
23·80	23·15
24·00	23·10
24·35	24·35
21·47	19·85
21·37	20·65
21·57	20·95
21·52	20·90
21·00	20·55
20·32	20·10
20·22	19·87
20·62	19·72
20·52	19·62
19·95	19·77
434·51	424·88

PC No. 2 ≏ T+B+0·65417 gr. T=5759·4881 gr. B=1239·8880 gr. Therefore PC No. 2 ≏ 7000·0303 grains.

For PC No. 2, log Δ=1·32560. For T+B+0·6542, log Δ=1·32564.

Hence vPC No. 2=330·75, v(T+B+0·6542)=330·72. vPC No. 2 is greater than v(T+B+0·6542) by the volume of 0·03 grain of water, or of 0·00004 grain of air of ordinary density. Hence

PC No. 2=7000·0303 grains.

PC No. 2.

July 18, 1844.

100 parts = 0·26 grain.

PC No. 2 and hook in water. In right-hand pan.

H.	P.	gr.	Scale.
		0·20	20·6
		0·20	21·2
		0·20	21·1
		0·20	21·3
		0·20	21·5
		0·20	23·6
		0·18	23·2
		0·16	13·0
16·5	17·3	0·16	15·7
		0·18	22·0
		0·20	25·4
		0·24	38·5
		0·16	15·3
		0·14	12·5
		0·20	23·5
		0·16	16·1
		0·18	18·9
		0·20	21·3
16·5	17·3	0·1867	20·82

Hook in water.	In right-hand pan. gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·60	11·9
	0·68	27·4
	0·64	13·2
	0·66	20·9
Air.	0·70	44·5
t=17·0, b=753·94.	0·64	17·6
	0·60	12·2
	0·68	25·1
	0·60	9·4
	0·68	25·5
	0·648	20·77

PC No. 2 in water (t=16·58)≏T+F+K+(16)+(8)+(4)+(2)+0·4612 grain in air (t=17·0, b=753·94).

July 23, 1844. 100 parts =0·26 grain.

PC No. 2 and hook in water. H.	P.	In right-hand pan. gr.	Scale.
19·8	20·7	0·00	14·1
		0·04	22·1
19·9	20·85	0·04	30·0
		0·00	14·5
		0·04	34·0
19·95	20·95	0·00	15·2
		0·04	24·8
20·0	20·95	0·00	16·0
20·0	21·0	0·04	27·0
20·0	21·0	0·00	16·5
20·0	21·0	0·04	30·5
20·05	21·05	0·00	15·2
19·95	20·92	0·020	21·66

Hook in water.	In right-hand pan. gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·68	23·0
	0·64	14·5
Air.	0·68	23·5
t=21·5, b=761·72.	0·64	15·0
	0·68	26·5
	0·64	12·0
	0·660	19·08

PC No. 2 in water (t=20·07)≏T+F+K+(16)+(8)+(4)+(2)+0·6467 gr. in air (t=21·5, b=761·72).

July 27, 1844. 100 parts =0·26 grain.

PC No. 2 and hook in water. H.	P.	In right-hand pan. gr.	Scale.
20·5	21·6	0·23	15·5
		0·26	21·8
		0·26	23·4
		0·23	14·6
		0·26	23·5
20·5	21·5	0·26	23·7
		0·23	18·5
20·5	21·55	0·23	14·8
		0·26	21·6
		0·26	23·5
20·5	21·55	0·248	20·09

Hook in water.	In right-hand pan.	
	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·98	27·0
	0·98	25·5
	0·94	13·7
	0·98	24·8
	0·94	11·6
Air.	0·94	10·8
t=22·3, b=765·63.	0·98	22·9
	0·94	12·1
	0·98	22·5
	0·98	32·0
	0·94	12·8
	0·98	22·1
	0·98	24·5
	0·9646	20·18

PC No. 2 in water (t=20·65)≏T+F+K+(16)+(8)+(4)+(2)+0·7164 grain in air (t=22·3, b=765·63).

August 3, 1844. 100 parts =0·26 grain.

PC No. 2 and hook in water.		In right-hand pan.	
H.	P.	gr.	Scale.
18·0	18·9	0·41	21·5
17·95	18·8	0·40	23·0
		0·40	21·5
17·5	18·4	0·40	21·5
17·4	18·2	0·39	18·7
		0·40	11·5
17·1	17·95	0·40	17·4
16·95	17·6	0·44	20·5
		0·44	22·6
16·6	17·5	0·44	24·0
17·36	18·19	0·412	20·22

Hook in water.	In right-hand pan.	
	gr.	Scale.
T+F+K+(16)+(8)+(4)+(2)+	0·94	28·8
	0·94	28·0
	0·90	11·8
Air.	0·94	22·0
t=17·5, b=744·15.	0·90	12·0
	0·94	22·6
	0·94	25·0
	0·90	12·7
	0·94	23·0
	0·9267	20·31

PC No. 2 in water (t=17·5)≏T+F+K+(16)+(8)+(4)+(2)+0·5144 grain in air (t=17·5, b=744·15).

Apparent weight of PC No. 2 in water, weighed with platinum weights.

Water.	Apparent weight of PC No. 2.	Air.	
t.	gr.	t.	b.
16·58	6669·8603	17·0	753·94
20·07	6670·0458	21·5	761·72
20·65	6670·1155	22·3	765·63
17·50	6669·9135	17·5	744·15

Resulting values of ΔPC No. 2 and log ΔPC No. 2.

	ΔPC No. 2.	log ΔPC No. 2.
	21·1656	1·325631
	21·1640	1·325599
	21·1631	1·325579
	21·1633	1·325584
Mean	21·1640	1·325598
10− log Δ		8·674402
PC No. 2 (reduced)	7000·002	3·845098
*v*PC No. 2	330·750	2·519500

Observations for finding the density of PS.

Apparent weight in water of the hook and wire by which PS, PC No. 3 and PC No. 4 were suspended in water.

The weight of 123·6 inches of the platinum wire by which the weights were suspended was 38·815 grains. Hence one inch displaces 0·01365 grain of water. 100 parts of the micrometer scale correspond to a displacement of 0·01365 grain of water. With 110 grains in each pan, 100 parts =0·2559 grain. With the hook suspended in water, 100 parts =0·2559 grain +0·01365 grain =0·27 grain. The upper hook by which the wire was attached to the scale-pan was changed twice during the weighings.

March 27, 1845. Hook in water.

In right-hand pan.

gr.	Scale.
0·00	18·2
0·02	20·2
0·00	18·9
0·02	20·6
0·00	18·9
0·02	20·5
0·00	20·5
0·00	18·5
0·02	19·5
0·00	18·9
0·00	20·0
0·0073	19·52

Counterpoise balances hook in water + 0·0086 gr. in air.

In right-hand pan.

gr.	Scale.
(64)+(32)+(8)+(4)+(1)+0·88	21·75
+0·87	18·82
+0·88	22·12
+0·87	18·55
0·875	20·31

C.poise bal. (64)+(32)+(8)+(4)+(1)+0·8742 gr.
H. in w.≏(64)+(32)+(8)+(4)+(1)+0·8656 gr. in a.

April 1, 1845. Hook in water.

In right-hand pan.

gr.	Scale.
2·30	19·9
2·30	20·0
2·30	19·0
2·30	19·2
2·32	21·2
2·30	20·8
2·30	19·5
2·3029	19·94

Counterpoise balances hook in water + 2·3031 gr. in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·18	19·65
+0·18	19·72
0·180	19·68

Counterpoise balances K+(32)+0·1808 grain.
Hook in water ≏K+(32)−2·1223 grains in air.

April 5. Hook in water.

In right-hand pan.

gr.	Scale.
2·30	20·2
2·30	20·4
2·28	20·5
2·24	12·0
2·36	25·5
2·24	7·0
2·34	26·0
2·30	21·0
2·30	22·3
2·30	22·5
2·296	19·5

Counterpoise balances hook in water + 2·2972 gr. in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·18	17·15
0·19	20·55
0·20	23·87
0·190	20·64

Counterpoise balances K+(32)+0·1884 grain.

Hook in water ≏ K+(32)−2·1088 grains in air.

April 8. Hook in water.

In right-hand pan.

gr.	Scale.
2·30	18·0
2·30	20·5
2·30	18·8
2·30	19·3
2·30	19·5
2·30	18·5
2·30	23·1
2·30	18·9
2·30	20·2
2·30	19·2
2·300	19·6

Counterpoise balances hook in water+2·3011 gr. in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·18	18·05
0·19	21·7
0·18	18·0
0·1883	19·25

Counterpoise balances K+(32)+0·1902 grain.

Hook in water ≏ K+(32)−2·1109 grains in air.

April 16. Hook in water.

In right-hand pan.

gr.	Scale.
2·30	18·9
2·30	19·0
2·30	19·0
2·30	19·0
2·30	19·5
2·30	20·0
2·30	20·7
2·30	20·0
2·300	19·51

Counterpoise balances hook in water+2·3013 gr. in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·18	17·65
0·18	18·55
0·19	19·95
0·19	21·65
0·19	21·8
0·18	16·8
0·185	19·4

Counterpoise balances K+(32)+0·1865 grain.

Hook in water ≏ K+(32)−2·1143 grains in air.

Hook in water.

In right-hand pan.

gr.	Scale.
2·30	17·1
2·30	18·2
2·30	18·5
2·30	18·9
2·30	18·9
2·30	19·9
2·30	21·5
2·30	19·0
2·30	18·5
2·300	18·94

Counterpoise balances hook in water+2·3002 gr. in air.

April 17. In right-hand pan.

gr.	Scale.
K+(32)+0·18	15·9
0·19	17·6
0·22	29·05
0·18	15·3
0·19	15·7
0·20	21·05
0·1933	19·1

Counterpoise balances K+(32)+0·1956 grain.

Hook in water ≏ K+(32)−2·1046 grains in air.

Hook in water.

In right-hand pan.

gr.	Scale.
2·30	31·0
2·24	12·1
2·28	25·0
2·27	22·9
2·26	19·7
2·26	19·0
2·28	25·0
2·26	18·2
2·27	20·5
2·26	18·0
2·28	21·0
2·26	17·0
2·28	20·9
2·26	19·5
2·28	21·0
2·30	26·5
2·26	15·6
2·28	18·0
2·30	24·8
2·28	15·0
2·30	24·5
2·28	19·0
2·2745	20·65

Counterpoise balances hook in water+2·2727 gr. in air.

April 30. In right-hand pan.

gr.	Scale.
K+(32)+0·20	24·0
0·18	14·8
0·19	18·2
0·20	22·7

0·20	20·8
0·18	14·95
0·20	20·6
0·20	20·05
0·20	20·0
0·1944	19·57

Counterpoise balances K + (32) + 0·1955 grain.
Hook in water ≏ K + (32) − 2·0772 grains in air.

May 5. Hook in water.

In right-hand pan.

gr.	Scale.
2·27	21·1
2·27	21·0
2·26	22·9
2·25	16·5
2·26	19·2
2·27	22·5
2·25	18·5
2·26	20·6
2·26	20·9
2·25	15·2
2·26	11·2
2·29	20·0
2·29	20·6
2·29	20·5
2·2664	19·33

Counterpoise balances h. in w. + 2·2682 grains in air.

In right-hand pan.

gr.	Scale.
K + (32) + 0·20	19·0
0·21	20·0
0·21	22·3
0·21	23·9
0·20	16·7
0·20	18·5
0·21	19·5
0·21	19·5
0·21	19·0
0·21	19·1
0·207	19·75

Counterpoise balances K + (32) + 0·2076 grain.
Hook in water ≏ K + (32) − 2·0606 grains in air.

May 7. Hook in water.

In right-hand pan.

gr.	Scale.
2·26	11·2
2·29	20·0
2·29	20·6
2·29	20·5
2·29	25·5
2·26	15·6
2·26	15·0
2·28	20·5
2·28	20·7
2·28	19·5
2·28	19·9
2·28	20·8
2·28	22·4
2·28	22·5
2·24	15·0
2·24	15·0
2·28	25·0
2·28	26·0
2·26	20·0
2·26	20·6
2·26	19·8
2·26	19·5
2·2718	19·8

Counterpoise balances h. in w. + 2·2723 grains in air.

In right-hand pan.

gr.	Scale.
K + (32) + 0·20	20·8
0·20	20·2
0·20	21·3
0·20	24·0
0·20	23·4
0·18	19·2
0·18	20·9
0·18	17·1
0·18	17·2
0·1911	20·45

Counterpoise balances K + (32) + 0·1899 grain.
Hook in water ≏ K + (32) − 2·0824 grains in air.

Hook in water.

In right-hand pan.

gr.	Scale.
2·26	18·9
2·27	21·9
2·26	19·8
2·27	22·8
2·26	20·0
2·26	19·0
2·27	22·1
2·26	20·0
2·26	19·3
2·26	20·0
2·26	20·1
2·26	20·4
2·26	19·9
2·26	20·0
2·26	20·0
2·25	17·4
2·27	24·3
2·2618	20·35

Counterpoise balances h. in w. + 2·2609 grains in air.

May 10. In right-hand pan.

gr.	Scale.
K + (32) + 0·20	27·3
0·17	15·95
0·18	19·85
0·18	19·6
0·18	19·2
0·18	18·8
0·1817	20·1

Counterpoise balances K + (32) + 0·1814 grain.
Hook in water ≏ K + (32) − 2·0795 grains in air.

May 22. Hook in water.

In right-hand pan.

gr.	Scale.
2·26	23·7
2·24	16·5.
2·25	20·1
2·25	20·0
2·25	20·2
2·25	19·4
2·25	19·25
2·25	19·3
2·26	23·2
2·25	19·0
2·25	19·0
2·26	23·45
2·2517	20·26

Counterpoise balances h. in w. +2·2510 grains in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·20	27·6
0·16	12·4
0·18	20·05
0·18	19·95
0·18	20·0
0·18	19·5
0·180	19·92

Counterpoise balances K+(32)+0·1802 grain.

Hook in water ≏K+(32)−2·0708 grains in air.

Hook in water.

In right-hand pan.

gr.	Scale.
0·12	21·2
0·12	21·0
0·11	17·1
0·12	20·9
0·11	16·9
0·12	20·6
0·12	21·8
0·12	27·0
0·10	13·15
0·12	16·0
0·12	23·9
0·10	18·3
0·12	24·0
0·10	17·8
0·12	24·0
0·10	18·0
0·12	21·4
0·10	17·0
0·12	22·3
0·1137	20·12

Counterpoise balances h. in w. +0·1134 grain in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·30	28·6
0·28	20·4
0·27	20·7
0·26	10·3
0·28	20·0
0·30	27·65
0·28	19·45
0·26	16·25
0·28	17·3
0·30	25·8
0·28	19·4
0·281	20·53

Counterpoise balances K+(32)+0·2796 grain.

Hook in water ≏K+(32)+0·1662 grain in air.

May 31. Hook in water.

In right-hand pan.

gr.	Scale.
0·12	23·7
0·10	16·7
0·11	20·5
0·11	18·2
0·12	10·5
0·14	19·5
0·14	31·7
0·11	17·5
0·12	21·4
0·10	14·2
0·11	18·0
0·12	21·5
0·11	18·3
0·12	22·0
0·11	18·5
0·12	21·9
0·11	18·45
0·1159	19·57

Counterpoise balances h. in w. +0·1170 grain in air.

In right-hand pan.

gr.	Scale.
K+(32)+0·29	24·2
0·28	20·8
0·27	16·8
0·28	20·0
0·28	19·7
0·280	20·3

Counterpoise balances K+(32)+0·2792 grain.

Hook in water ≏K+(32)+0·1622 grain in air.

Apparent weight of the hook in water.

	gr.	Means.
March 27	109·8592	
April 1	109·8686	
April 5	109·8821	109·875
April 8	109·8800	
April 16	109·8761	
April 17	109·8863	
April 30	109·9137	
May 5	109·9303	
May 7	109·9085	109·917
May 10	109·9114	
May 22	109·9201	
May 30	112·1567	112·155
May 31	112·1527	

March 27 ... April 17. Hook in water ≏ 109·875 grains of platinum in air.
April 30 ... May 22. Hook in water ≏ 109·917 grains of platinum in air.
May 30 ... 31. Hook in water ≏ 112·155 grains of platinum in air.

With 6680 grains in the right-hand pan,—March 27...April 8, 100 parts =0·197 grain. April 29...May 3, 100 parts= 0·32 grain, liable to some uncertainty on account of the state of the knife-edge. May 9...May 30, 100 parts =0·207 grain. The dimensions of the wire by which the lbs. were suspended was such that a portion of the wire corresponding to 100 parts of the scale displaced 0·01365 grain of water. Hence, when a platinum lb. is suspended in water from the right-hand pan,—

March 27...April 8 . . . . 100 parts =0·211 grain.
April 29...May 3 . . . . . 100 parts =0·33 grain.
May 9...May 30 . . . . . 100 parts =0·221 grain.

The thermometer B was suspended in the water with its bulb in a horizontal plane through the middle of the lb.; the thermometer C was placed in the balance case. F denotes the reading of Ernst's barometer, E that of its attached thermometer.

The counterpoise in the left-hand pan is supposed to be in equilibrium with the weights in or suspended from the right-hand pan, when the reading of the scale is 20.

Weighing of PS in air.

June 4, 1845. 100 parts =0·2649 grain.

PS+X.	T+C+0·68 gr.+Y.		PS+Y.	T+C+0·68 gr.+X.
19·70	18·60		15·05	19·40
19·50	18·95		16·10	18·00
18·70	18·05		15·10	17·50
18·40	17·10	D= 13·3	15·70	18·00
17·00	18·00	C= 13·4	14·90	16·90
16·20	18·10	F=750·7	PS+Y.	T+C+0·67 gr.+X.
17·20	18·60	E= 14	16·40	14·30
17·80	17·75		14·65	15·80
18·10	19·05		16·25	13·80
16·90	18·70		14·05	13·70
179·50	183·70		15·50	12·75
			153·70	160·15

10(PS+X) ≏ 10(T+C+0·68 gr.+Y)−4·2 parts. 10(PS+Y) ≏ 10(T+C+0·675 gr.+X)−6·45 parts.

20PS=20(T+C)+13·52179 grains.

100 parts $=0\cdot3108$ grain.

PS+X.	T+C+0·68 gr.+Y.		PS+Y.	T+C+0·67 gr.+X.
15·50	16·55		21·10	19·30
15·90	15·20	D= 13·5	18·70	16·40
13·70	14·30	C= 13·55	17·20	16·00
14·95	14·10	F=752·4	16·50	18·90
18·00	15·90	E= 14·1	17·15	14·80
13·90	14·90		17·15	15·30
91·95	90·95		107·70	100·70

$6(\mathrm{PS}+\mathrm{X}) \triangleq 6(\mathrm{T}+\mathrm{C}+0\cdot68 \text{ gr.}+\mathrm{Y})+1$ part. $\quad 6(\mathrm{PS}+\mathrm{Y}) \triangleq 6(\mathrm{T}+\mathrm{C}+0\cdot67 \text{ gr.}+\mathrm{X})+7$ parts.

$$12\mathrm{PS} \triangleq 12(\mathrm{T}+\mathrm{C})+8\cdot12486 \text{ grains.}$$

$$32\mathrm{PS} \triangleq 32(\mathrm{T}+\mathrm{C})+21\cdot64665 \text{ grains.}$$

$$\mathrm{PS} \triangleq \mathrm{T}+\mathrm{C}+0\cdot67646 \text{ grain in air } (t=13\cdot45,\ b=750\cdot09).$$

$\mathrm{T}=5759\cdot47143$ grains. $\mathrm{C}=1239\cdot88708$ grains. $\mathrm{PS} \triangleq 7000\cdot03495$ grains in air.

For PS, $\log \Delta=1\cdot32545$. For $\mathrm{T}+\mathrm{C}+0\cdot676$ grain, $\log \Delta=1\cdot32566$. Hence $v\mathrm{PS}=330\cdot86$, $v(\mathrm{T}+\mathrm{C}+0\cdot676 \text{ gr.})=330\cdot70$. Therefore $v\mathrm{PS}$ is larger than $v(\mathrm{T}+\mathrm{C}+0\cdot676 \text{ gr.})$ by the volume of $0\cdot16$ grain of water, or of $0\cdot00019$ grain of air. Hence

$$\mathrm{PS}=7000\cdot03516 \text{ grains.}$$

Weighing of PS in water.

April 1, 1845. PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
9·05	7·85	769·8	8·1	1·00	14·5
				1·08	20·0
9				1·08	22·0
8·95	8·33	769·65	8·5	1·00	13·1
				1·08	24·0
8·9				1·04	19·5
8·87	8·4			1·08	27·2
				1·00	12·0
8·85				1·04	19·0
				1·04	18·5
8·94	8·19	769·72	8·3	1·044	18·98

Counterpoise in air ($\mathrm{C}=8\cdot19$, $\mathrm{F}=769\cdot4$, $\mathrm{E}=8\cdot3$) balances PS and hook in water ($\mathrm{B}=8\cdot94$) $+\,1\cdot0462$ grain in air.

In right-hand pan.

	gr.	Scale.
T+F+K+N+R+(16)+(4)+	1·10	20·67
+	1·10	20·7
	1·10	20·68

Counterpoise balances $\mathrm{T}+\mathrm{F}+\mathrm{K}+\mathrm{N}+\mathrm{R}+(16)+(4)+1\cdot0987$ grain in air.

PS and h. in w. $(\mathrm{B}=8\cdot94) \triangleq \mathrm{T}+\mathrm{F}+\mathrm{K}+\mathrm{N}+\mathrm{R}+(16)+(4)+0\cdot0525$ grain in air ($\mathrm{C}=8\cdot19$, $\mathrm{F}=769\cdot72$, $\mathrm{E}=8\cdot3$).

PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
13·05	8·85	762·25	9·6	1·00	27·4
				0·90	9·5
				0·98	21·9
12·85	8·93			0·96	19·0
12·5				0·94	18·0

				0·96	18·3
12·4		762·13	9·65	0·98	19·4
12·25				0·98	20·0
12·01	9·05			0·98	18·5
12				1·00	20·0
11·95				1·00	20·0
11·65	9·35	762·0	9·5	1·00	21·0
11·53				1·00	19·2
11·50				1·00	19·5
11·43				1·00	17·0
11·40	9·4			1·00	18·6
12·18	9·18	762·06	9·57	0·98	19·2

Counterpoise in air (C=9·18, F=762·06, E=9·57)≏PS and hook in water (B=12·18)+0·9817 grain in air.

In right-hand pan.

	gr.	Scale.
T+F+K+N+R+(16)+(4)	+1·10	20·02
	+1·10	20·25
	+1·10	21·2
	1·10	20·49

Counterpoise ≏T+F+K+N+R+(16)+(4)+1·0990 grain in air.

PS and h. in w.(B=12·18)≏T+F+K+N+R+(16)+(4)+0·1173gr. in air(C=9·18, F=762·06, E=9·57).

April 5. PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
7·5	8	764·3	8·4	1·00	10·0
				1·04	18·2
				1·06	19·1
7·5				1·08	21·8
				1·07	21·5
				1·04	19·4
7·55				1·04	20·7
				1·00	8·0
				1·10	28·5
				1·04	21·5
				1·02	16·5
				1·04	21·3
7·6				1·04	21·55
	8·5	763·9	8·95	1·02	20·6
7·65				1·00	4·5
				1·00	7·9
				1·10	23·5
				1·00	7·9
7·65				1·10	23·5
				1·10	25·4
				1·10	27·5
				1·10	29·4
	8·95			1·10	33·0
7·65				1·00	9·0
				1·04	18·0
7·65				1·04	21·0
				1·04	21·6
				1·04	21·4
				1·04	22·0
7·6	8·48	764·1	8·67	1·0479	19·47

Counterpoise in air (C=8·48, F=764·1, E=8·67)≏PS and hook in water (B=7·6)+1·0490 grain in air.

In right-hand pan.

	gr.	Scale.
T+F+K+N+R+(16)+(4)	+1·10	20·7
	+1·10	21·0
	1·100	20·85

Counterpoise ≏ T+F+K+N+R+(16)+(4)+1·0983 grain in air.

PS and h. in w. (B=7·6) ≏ T+F+K+N+R+(16)+(4)+0·0493 gr. in air (C=8·48, F=764·1, E=8·67).

May 3. PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
11·85				1·06	21·1
				1·06	22·3
11·7				1·06	20·0
				1·04	14·0
	11·5	759·4	12·4	1·06	19·5
				1·02	12·4
				1·06	23·0
				1·10	28·9
11·65				1·02	12·7
				1·06	23·7
11·6	12·2	759·1	12·2	1·02	15·4
				1·06	23·5
				1·04	21·6
				1·03	16·9
				1·04	20·5
				1·03	20·3
				1·00	12·5
				1·10	27·6
11·6				1·00	17·0
				1·10	29·4
				1·04	20·8
				1·02	16·8
				1·04	21·0
				1·03	18·2
11·5				1·04	20·5
				1·04	20·9
11·67	11·85	759·25	12·3	1·045	20·02

Counterpoise in air (C=11·85, F=759·25, E=12·3) ≏ PS and hook in water (B=11·67)+1·045 grain in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)	+1·18	26·9
	+1·14	15·0
	+1·16	20·3
	+1·16	21·8
	+1·16	17·5
	1·160	20·3

Counterpoise ≏ T+F+K+L+R+(16)+(4)+1·159 grain in air.

PS and h. in w. (B=11·67) ≏ T+F+K+L+R+(16)+(4)+0·114 gr. in air (C=11·85, F=759·25, E=12·3).

May 5. PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
10·05	10·4	759·1	11	1·04	18·0
				1·08	24·5
				1·04	19·4
				1·06	22·5
				1·05	21·5
				1·04	15·5
				1·04	16·7

				1·06	20·1
				1·06	20·4
				1·06	21·8
10·05				1·06	22·0
10·07	11·3	758·7	11·5	1·06	21·7
				1·04	17·7
				1·05	20·2
10·08				1·05	20·3
				1·05	20·05
				1·05	20·1
				1·05	21·1
10·09				1·06	21·15
				1·04	15·9
				1·04	16·8
				1·06	23·5
10·07	10·85	758·9	11·25	1·0518	20·04

Counterpoise in air (C=10·85, F=758·9, E=11·25) ≏ PS and hook in water (B=10·07) + 1·0517 grain in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·16	17·0
	1·17	15·7
	1·18	19·8
	1·18	17·0
	1·18	18·2
	1·21	22·1
	1·21	23·17
	1·20	23·4
	1·20	24·5
	1·1878	20·1

Counterpoise ≏ T+F+K+L+R+(16)+(4)+1·1875 grain in air.
PS and h. in w.(B=10·07) ≏ T+F+K+L+R+(16)+(4)+0·1358 gr. in a.(C=10·85, F=758·9, E=11·25).

May 28. PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
				0·21	24·0
		759·4	11	0·21	22·7
10·18				0·20	20·6
				0·20	22·5
				0·18	18·4
				0·20	21·4
10·2	11·1			0·18	20·3
				0·18	20·4
				0·16	9·0
				0·18	13·0
				0·20	22·5
10·4	11·6	758·9	11·4	0·18	20·1
				0·18	20·6
10·45				0·18	20·1
				0·17	19·4
				0·16	15·9
				0·20	25·9
10·31	11·35	759·15	11·2	0·1865	19·81

C.poise in air (C=11·35, F=759·15, E=11·2) balances PS and hook in water (B=10·31)+0·1869 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	2·52	18·6
	2·53	20·55
	2·54	29·5
	2·52	21·0
	2·54	30·7
	2·50	8·6
	2·52	18·9
	2·52	18·55
	2·53	21·65
	2·5244	20·89

Counterpoise balances T+F+K+L+R+(16)+(4)+(2)+0·5224 grain in air.
PS & h. in w. (B=10·31)≏T+F+K+L+R+(16)+(4)+(2)+0·3355 gr. (C=11·35, F=759·15, E=11·2).

PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
	10·5	756·0	11	0·18	22·0
				0·17	17·9
				0·18	20·9
				0·17	20·8
				0·18	24·8
10·3				0·17	21·2
10·4	10·6	755·6	11	0·15	14·1
				0·18	26·0
				0·15	15·2
				0·18	27·4
				0·15	12·8
				0·18	26·8
				0·15	12·6
10·35	10·55	755·8	11	0·1685	20·2

Counterpoise in air (C=10·55, F=755·8, E=11) balances PS and hook in water (B=10·35)+0·1681 gr. in air.
Counterpoise balances T+F+K+L+R+(16)+(4)+(2)+0·5211 grain in air. (Mean, May 28, 30.)
PS & h. in w.(B=10·35)≏T+F+K+L+R+(16)+(4)+(2)+0·3530 gr. in a.(C=10·55, F=755·8, E=11).

PS and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
10·12	10·65	758·85	11	0·18	26·0
				0·15	12·7
				0·18	27·5
10·2				0·16	16·1
				0·17	23·1
10·25				0·16	14·5
				0·17	20·5
				0·17	21·0
10·3		759·4	12	0·16	14·5
				0·17	20·5
10·55	12·1	760·05	12·1	0·18	27·8
				0·14	8·1
				0·16	20·0
				0·18	27·7
				0·16	19·5
				0·16	19·55
				0·14	9·6
10·6				0·18	31·5
				0·16	19·0
				0·16	20·7
				0·16	18·8
10·6				0·16	15·4
10·4	11·37	759·58	11·8	0·1641	19·73

Counterpoise in air (C=11·37, F=759·58, E=11·8) balances PS and hook in water (B=10·4)+0·1647 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	2·52	21·3
	2·52	22·4
	2·52	18·7
	2·50	9·5
	2·54	29·1
	2·52	19·7
	2·52	19·6
	2·520	20·04

Counterpoise balances T+F+K+L+R+(16)+(4)+(2)+0·5199 grain in air.

PS and h. in w. (B=10·4) ≏ T+F+K+L+R+(16)+(4)+(2)+0·3552 gr. (C=11·37, F=759·58, E=11·8).

Apparent weight of PS in water, weighed with platinum weights.

Water. *t.*	Apparent weight of PS. gr.	Air. *t.*	Air. *b.*
8·95	6669·555	8·18	769·13
12·15	6669·620	9·17	761·32
7·62	6669·552	8·47	763·47
11·63	6669·573	11·83	758·19
10·05	6669·595	10·84	757·97
10·29	6669·556	11·34	758·22
10·33	6669·573	10·54	754·90
10·38	6669·575	11·36	758·58

Resulting values of ΔPS and log ΔPS.

	ΔPS.	log ΔPS.
	21·1570	1·325455
	21·1572	1·325457
	21·1581	1·325477
	21·1553	1·325420
	21·1590	1·325496
	21·1563	1·325440
	21·1573	1·325461
	21·1574	1·325463
Mean	21·1572	1·325459
10 − log Δ		8·674541
PS (reduced)	7000·0009	3·845098
*v*PS	330·856	2·519639

Observations for finding the density of PC No. 3.

Weighing of PC No. 3 in air.

June 10, 1845. 100 parts = 0·273 grain.

PC+X.	T+C+0·67 gr.+Y.		PC+Y.	T+C+0·67 gr.+X.
16·75	18·25		24·2	22·4
16·6	17·8		24·6	22·4
16·7	13·4		PC+Y.	T+C+0·68 gr.+X.
16·9	14·7		20·5	21·0
15·9	14·4		19·55	21·1
15·5	15·55	D = 14·1	20·2	20·6
17·5	15·65	C = 14·3	20·4	20·7
17·2	14·4	F = 771·6	19·5	20·9
17·1	14·1	E = 14·9	20·0	20·5
16·3	15·0		19·2	20·7
15·25	15·6		19·1	21·7
15·1	14·4		19·85	21·4
14·8	13·9		18·65	21·4
			18·9	20·5
81·6	67·2		215·85	230·5

13(PC+X) ≏ 13(T+C)+8·71 gr.+14·4 pt. 13(PC+Y) ≏ 13(T+C)+8·82 gr.−10·65 pt.

26PC ≏ 26(T+C)+17·54024 grains.

June 11. 100 parts $=0{\cdot}29$ grain.

PC+Y.	T+C+0·68 gr.+X.		PC+X.	T+C+0·68 gr.+Y.
26·15	26·3	D= 14	22·3	24·0
23·45	25·0	C= 14	22·1	23·2
23·1	25·6	F =769·5	23·9	23·8
24·5	25·0	E= 15	22·62	24·15
97·2	101·9		90·95	95·15

$4(PC+Y) \triangleq 4(T+C+0{\cdot}68 \text{ gr.}+X)-4{\cdot}7$ pt. $\quad 4(PC+X) \triangleq 4(T+C+0{\cdot}68 \text{ gr.}+Y)-4{\cdot}2$ pt.

$$8PC \triangleq 8(T+C)+5{\cdot}41864 \text{ grains.}$$

$$34PC \triangleq 34(T+C)+22{\cdot}95888 \text{ grains.}$$

$$PC \triangleq T+C+0{\cdot}67526 \text{ grain in air } (t=14{\cdot}07,\ b=769{\cdot}66).$$

$$\text{PC No. } 3 \triangleq 7000{\cdot}03375 \text{ grains of platinum in air } (t=14{\cdot}07,\ b=769{\cdot}66).$$

For PC No. 3, $\log \Delta=1{\cdot}32555$. For $T+C+0{\cdot}675$ gr., $\log \Delta=1{\cdot}32566$. Hence vPC No. 3 $=330{\cdot}79$, $(vT+C+0{\cdot}675)$. Therefore vPC No. 3 is greater than $v(T+C+0{\cdot}675$ gr.) by the volume of $0{\cdot}09$ grain of water, or of $0{\cdot}00011$ grain of air. Hence

$$\text{PC No. } 3=7000{\cdot}03386 \text{ grains.}$$

Weighing of PC No. 3 in water.

March 31, 1845.

	PC and hook in water.			In right-hand pan.	
B.	C.	F.	E.	gr.	Scale.
9·18				1·00	27·8
				0·90	8·8
9·1				0·97	18·5
				0·98	20·0
9				0·98	21·55
		768·75	8·8	0·97	20·0
8·95	8·65			1·00	22·8
		768·83	9	0·97	20·6
				0·96	19·4
				0·96	19·3
8·93	8·84			0·97	19·7
				0·97	18·5
8·9		768·8	9	0·97	20·4
				0·97	21·0
9·01	8·74	768·79	8·93	0·9693	19·88

Counterpoise in air ($C=8{\cdot}74$, $F=768{\cdot}79$, $E=8{\cdot}93$) balances PC and hook in water ($B=9{\cdot}01$) $+0{\cdot}9696$ gr. in air.

	In right-hand pan. gr.	Scale.
T+F+K+L+R+(16)+(4)+(1)+	0·10	15·9
	0·11	22·75
	0·10	17·3
	0·10	17·9
	0·11	25·2
	0·104	19·81

Counterpoise balances $T+F+K+L+R+(16)+(4)+(1)+0{\cdot}1044$ grain.

PC and h. in w. ($B=9{\cdot}01$) $\triangleq T+F+K+L+R+(16)+(4)+0{\cdot}1348$ gr. in air ($C=8{\cdot}74$, $F=768{\cdot}79$, $E=8{\cdot}93$).

April 7. PC and hook in water. In right-hand pan.

	B.	C.	F.	E.	gr.	Scale.
					1·00	19·0
					1·00	29·5
					1·00	26·5
	8·45	8·4	759·8	8·9	1·00	20·2
					1·00	24·5
	8·43				0·96	13·6
					0·92	14·5
					0·92	9·5
					1·04	34·0
April 8.	7·4		749·2	8·4	1·00	19·0
					0·96	15·0
	7·37				1·00	20·0
					1·00	25·5
		8·4			0·96	14·5
					1·00	25·0
					0·96	17·0
					1·00	20·0
					1·00	27·5
					0·96	18·0
					1·00	21·6
					0·96	9·5
					1·00	21·7
	7·4				0·99	21·5
					0·95	14·5
		8·6	748·7	9·0	0·99	21·2
					0·95	7·5
					0·99	20·0
					0·99	21·6
					1·00	27·0
					1·00	24·6
					0·96	11·6
					1·00	23·7
	7·74	8·47	752·57	8·77	0·9831	20·27

Counterpoise in air (C=8·47, F=752·57, E=8·77) balances PC and h. in w. (B=7·74)+0·9825 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+N+R+(16)+(4)+	1·10	25·75
	1·08	14·45
	1·09	19·52
	1·09	19·91

Counterpoise balances T+F+K+N+R+(16)+(4)+1·0902 grain.

PC and h. in w. (B=7·74) $\triangleq$ T+F+K+N+R+(16)+(4)+0·1077 gr. in air (C=8·47, F=752·57, E=8·77).

April 17 PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
7·65	8·15	770·87	8·8	1·00	19·1
				1·04	33·5
				1·00	18·5
				1·02	20·7
7·66				1·00	20·5
				1·02	20·5
				0·96	18·6
				1·00	19·1
7·66				1·04	22·8
				1·00	19·5
				0·96	18
				1·00	19
				1·04	22·4

7·85	9·3	770·7	9·4	0·96	17·0
				1·00	19·3
				1·04	22·5
				1·00	18·6
				1·04	23·4
				0·96	18·9
				1·00	20·5
				0·96	18·0
				1·00	20·4
7·73	8·7	770·78	9·1	1·0018	20·49

Counterpoise in air (C=8·7, F=770·78, E=9·1) balances PC and h. in water (B=7·73)+1·0008 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·12	23·7
	1·10	12·15
	1·11	17·93

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1141 grain.

PC and h. in w. (B=7·73)≏T+F+K+L+R+(16)+(4)+0·1133 gr. in air (C=8·7, F=770·78, E=9·1).

May 9. PC and hook in water. In right-hand pan.

B.	C.	F.	F.	gr.	Scale.
10·4	10·3	750·5	10·6	0·98	23·5
				0·94	7·5
				0·98	23·5
				0·97	17·3
				0·98	21·5
10·35				0·97	16·0
				0·98	20·4
				0·98	22·0
				0·98	23·5
				0·97	18·3
10·33	10·7	750·4	10·9	0·98	24·0
				0·97	18·4
				0·98	21·2
10·25				0·97	12·5
				0·98	18·6
				0·99	31·5
				0·98	25·2
				0·97	23·5
10 25	10·9			0·97	11·0
10·3	10·63	750·45	10·75	0·9747	19·97

Counterpoise in air (C=10·63, F=750·45, E=10·75) balances PC and h. in w. (B=10·3)+0·9748 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·13	20·65
	1·13	20·55
	1·13	20·45
	1·13	20·55

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1289 grain.

PC & h. in w. (B=10·3)≏T+F+K+L+R+(16)+(4)+0·1541 gr. in air (C=10·63, F=750·45, E=10·75).

May 10. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
				0·97	12·4
9·5	9·8	749·6	10·4	1·01	39·4
				0·98	17·6
				0·99	22·2
				0·98	23·0
				0·97	13·2
				0·98	24·4
				0·97	12·9
9·5				0·98	19·4
				0·98	19·4
				0·98	19·5
				0·99	23·0
9·55	10·8	749·6	10·9	0·97	19·8
				0·97	19·0
				0·98	21·5
9·55				0·98	21·5
				0·97	18·7
				0·98	23·1
				0·97	21·0
9·6				0·98	25·0
				0·96	17·1
9·6				0·96	16·9
9·54	10·3	749·6	10·65	0·9773	20·45

Counterpoise in air (C=10·3, F=749·6, E=10·65) balances PC and hook in water (B=9·54)+0·9764 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·13	21·6
	1·13	22·3
	1·12	16·8
	1·13	22·5
	1·13	20·8
	1·128	20·8

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1264 grain.

PC and h. in w. (B=9·54) ≏ T+F+K+L+R+(16)+(4)+0·1500 gr. in air (C=10·3, F=749·6, E=10·65).

Apparent weight of PC No. 3 in water, weighed with platinum weights.

Water. *t.*	Apparent weight of PC No. 3. gr.	Air. *t.*	Air. *b.*
9·02	6669·635	8·73	768·13
7·76	6669·610	8·46	751·95
7·75	6669·614	8·69	770·09
10·28	6669·613	10·64	749·59
9·52	6669·609	10·3	748·76

Resulting values of ΔPC No. 3 and log ΔPC No. 3.

	ΔPC No. 3.	log ΔPC No. 3.
	21·1623	1·325563
	21·1622	1·325561
	21·1618	1·325555
	21·1602	1·325519
	21·1609	1·325534
	21·1615	1·325546
10 − log Δ..........		8·674454
PC No. 3 (reduced)	6999·999	3·845098
*v*PC No. 3	330·790	2·519552

Observations for finding the density of PC No. 4.

Weighing of PC No. 4 in air.

June 2, 3, 1845. 100 parts =0·26 grain.

PC+Y.	T+C+0·68 gr.+X.		PC+X.	T+C+068 gr.+Y.
25·25	25·25		24·9	25·8
25·65	26·75		25·5	26·2
25·8	26·6		25·3	26·0
24·9	25·7		27·1	27·1
24·5	25·5		28·8	26·5
24·3	25·0		27·8	26·8
23·8	23·7		28·3	27·9
23·0	23·35	D= 12·5	28·25	27·55
22·4	22·9	C= 12·48	20·25	20·7
21·5	22·8	F=752·93	21·5	19·5
20·9	21·3	E= 13·97	20·8	18·7
20·75	21·0		19·3	20·7
19·9	20·6		21·35	19·6
22·0	24·8		20·7	19·6
21·8	22·25		25·5	23·2
22·1	23·3		25·1	23·5
22·35	22·7		24·0	23·55
22·0	23·6		23·5	22·45
21·7	23·0		23·65	21·9
435·6	450·1		461·6	447·25

19(PC+Y)$\triangleq$19(T+C+0·68 gr.+X)−14·5 parts. 19(PC+X)$\triangleq$19(T+C+0·68 gr.+Y)+14·13 parts.

PC No. 4$\triangleq$T+C+0·67999 grain in air (t=12·51, b=751·68).

PC No. 4$\triangleq$7000·03848 grains of platinum in air (t=12·51, b=751·68).

For PC No. 4, log Δ=1·32543. For T+C+0·680 grain, log Δ=1·32566. vPC No. 4=330·88, v(T+C+0·68)=330·70. Therefore vPC No. 4 is greater than v(T+C+0·680 grain) by the volume of 0·18 grain of water, or of 0·00022 grain of air. Hence

PC No. 4=7000·03870 grains.

Weighing of PC No. 4 in water.

March 26, 1845. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
8·93	7·5	759·3	7·9	3·00	21·65
				2·96	12·5
8·92		795·15	8·0	2·99	18·5
				3·00	21·7
8·85				3·00	22·0
8·5	7·85	759·5	8·0	3·00	21·1
				2·98	19·5
				2·96	17·2
				3·00	21·8
8·7	7·67	759·36	7·97	2·9878	19·55

Counterpoise in air (C=7·67, F=759·36, E=7·97) balances PC and hook in water (B=8·7)+2·9887 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+N+K+S+(16)+(4)+	3·00	13·6
	3·04	35·8
	3·00	15·4
	3·04	35·6
	3·02	25·1

Counterpoise balances T+F+N+K+S+(16)+(4)+3·0103 grains.

PC and h. in w. (B=8·7) ≏ T+F+N+K+S+(16)+(4)+0·0216 gr. in air (C=7·67, F=759·36, E=7·97).

March 27. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
6·8	7·4	757·9	7·9	1·10	20·1
				1·10	20·8
				1·10	24·0
				1·10	23·6
				1·06	17·0
6·85				1·04	9·5
				1·10	20·0
		757·9	8	1·10	21·0
6·95				1·07	17·7
				1·08	18·2
6·97	8	757·9	8	1·10	19·1
				1·10	20·0
				1·10	21·2
6·89	7·7	757·9	7·97	1·0885	19·4

Counterpoise in air (C=7·7, F=757·9, E=7·97) balances PC and hook in water (B=6·89)+1·0898 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+N+S+(16)+(4)+	1·100	23·92
	1·080	14·45
	1·00	24·3
	1·09	20·89

Counterpoise balances T+F+K+N+S+(16)+(4)+1·0882 grain.

PC and h. in w. (B=6·89) ≏ T+F+K+N+S+(16)+(4)−0·0003 gr. in air (C=7·7, F=757·9, E=7·97).

April 16. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
7·87	8	771·3	8·5	1·10	20·9
				1·08	18·4
7·87				1·10	21·6
				1·08	19·8
7·85				1·09	19·8
				1·12	25·5
				1·04	16·0
7·85	8·35			1·08	18·6
				1·12	22·5
				1·08	18·5
		770·9	8·8	1·12	31·0
				1·08	19·0
				1·08	18·7
7·85	8·8	770·75	8·9	1·08	18·3
				1·12	23·0
				1·08	18·6
				1·08	19·5
				1·12	22·0
				1·08	19·7
				1·08	19·0

				1·12	21·0
				1·08	20·5
7·87	8·8			1·08	19·0
7·86	8·44	770·92	8·77	1·0909	20·47

Counterpoise in air (C=8·44, F=770·93, E=8·77) balances PC and h. in w. (B=7·86)+1·0899 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·10	15·95
	1·11	22·65
	1·10	16·55
	1·11	23·5
	1·105	19·66

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1057 grain.

PC and h. in w. (B=7·86)≏T+F+K+L+R+(16)+(4)+0·0158 gr. in air (C=8·44, F=770·92, E=8·77).

April 29. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
11·1	10·5	761·53	11	1·10	27·0
				1·06	14·7
				1·10	27·5
				1·06	17·9
				1·08	20·4
				1·08	25·0
				1·08	20·5
11·05				1·06	17·1
				1·08	22·0
				1·06	21·3
				1·06	21·3
11·03				1·04	19·0
11·01		761·57	11·5	1·06	22·6
				1·04	20·5
				1·00	12·3
				1·04	20·5
				1·02	16·7
11	11·55			1·04	20·0
11·04	10·84	761·55	11·2	1·0589	20·35

Counterpoise in air (C=10·84, F=761·55, E=11·25) balances PC and h. in w. (B=11·04)+1·0577 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·18	23·7
	1·16	20·7
	1·16	17·0
	1·16	15·0
	1·18	24·4
	1·168	20·18

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1674 grain.

PC and h. in w. (B=11·04)≏T+F+K+L+R+(16)+(4)+0·1097 gr. in a. (C=10·85, F=761·55, E=11·25).

April 30. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
10	10·7	763·2	11·1	1·08	25·0
				1·04	17·5
				1·08	23·5
				1·04	13·0
				1·08	22·9
10·05				1·04	12·5
				1·07	21·5

				1·04	11·4
				1·07	19·0
				1·08	20·5
10·07				1·07	19·5
				1·08	22·1
10·1	11·7	763·1	11·9	1·08	21·6
				1·07	20·5
10·13				1·06	18·0
				1·07	19·7
				1·06	18·4
10·2				1·07	21·4
				1·06	19·2
				1·07	20·4
				1·06	19·1
				1·07	21·6
10·3	11·9	762·7	11·9	1·07	22·0
10·12	11·25	763·05	11·63	1·0657	19·58

Counterpoise in air (C=11·25, F=763·05, E=11·63) balances PC and hook in water (B=10·12)+1·0671 gr.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·18	20·3
	1·18	23·6
	1·18	22·5
	1·17	17·6
	1·1775	21

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1743 grain.

PC and h. in w. (B=10·12) ≏ T+F+K+L+R+(16)+(4)+0·1072 gr. in air (C=11·25, F=763·05, E=11·63).

	PC and hook in water.				In right-hand pan.	
	B.	C.	F.	E.	gr.	Scale.
May 20.	10·5	9·9	759·3	10·4	1·02	14·2
					1·04	29·7
					1·04	19·0
					1·04	25·0
					1·04	26·6
					1·00	5·3
					1·04	30·1
					1·00	8·7
May 21.					1·04	17·8
					1·04	19·8
					1·04	21·5
					1·04	23·0
	9·35	10	754·5	10·2	1·03	17·2
					1·03	16·9
					1·04	21·2
					1·04	21·2
					1·03	17·4
			753·25	10·5	1·03	18·5
					1·04	20·4
					1·04	21·2
					1·03	18·1
					1·04	21·5
					1·03	19·0
	9·5	10·6			1·04	21·5
					1·03	19·8
					1·04	22·2
					1·03	19·7
	9·78	10·17	755·68	10·37	1·0333	19·5

Counterpoise in air (C=10·17, F=755·68, E=10·37) balances PC and h. in w. (B=9·78)+1·0344 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·11	21·1
	1·11	19·6
	1·11	17·8
	1·12	23·4
	1·11	18·1
	1·11	17·6
	1·11	17·0
	1·1114	19·23

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1130 grain.

PC & h. in w. (B=9·78) ≏ T+F+K+L+R+(16)+(4)+0·0786 gr. in air (C=10·17, F=755·68, E=10·37).

May 22. PC and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
9·4	9·7	756·5	10·5	1·03	16·3
				1·04	22·5
				1·03	17·0
				1·04	21·9
9·45				1·03	18·0
				1·04	22·5
				1·03	18·4
				1·04	22·8
				1·03	18·5
9·45				1·04	22·7
9·5	9·8	756·8	10·9	1·03	20·4
				1·03	21·1
				1·02	18·05
				1·03	21·05
				1·02	17·55
9·55				1·03	21·0
				1·02	19·6
				1·03	22·8
				1·02	18·7
9·6				1·03	21·55
9·5	9·75	756·65	10·7	1·0305	20·12

Counterpoise in air (C=9·75, F=756·65, E=10·7) balances PC and hook in water (B=9·5)+1·0302 gr. in air.

In right-hand pan.

	gr.	Scale.
T+F+K+L+R+(16)+(4)+	1·12	24·95
	1·11	14·9
	1·12	24·9
	1·10	14·6
	1·12	24·1
	1·11	14·4
	1·10	19·64

Counterpoise balances T+F+K+L+R+(16)+(4)+1·1108 grain.

PC and h. in w. (B=9·5) ≏ T+F+K+L+R+(16)+(4)+0·0806 gr. in air (C=9·75, F=756·65, E=10·7).

Apparent weight of PC No. 4 in water, weighed with platinum weights.

Water. t.	Apparent weight of PC No. 4. gr.	Air. t.	Air. b.
8·71	6669·524	7·65	758·82
6·92	6669·503	7·68	757·36
7·88	6669·516	8·43	770·27
11·01	6669·568	10·85	760·61
10·11	6669·566	11·26	762·06
9·76	6669·537	10·17	754·85
9·49	6669·539	9·75	755·79

Resulting values of ΔPC No. 4 and log ΔPC No. 4.

	ΔPC No. 4.	log ΔPC No. 4.
	21·1553	1·325420
	21·1552	1·325417
	21·1551	1·325416
	21·1556	1·325426
	21·1567	1·325449
	21·1554	1·325422
	21·1558	1·325430
	21·1556	1·325426
10 − log Δ		8·674574
PC No. 4 (reduced)	6999·998	3·845098
*v*PC No. 4............	330·881	2·519672

Comparison of PS *with each of the weights* T+Q+A, T+Q+B, T+Q+C, T+Q+D.

The weights Q were changed, so that each of the ten weights Q was used the same number of times in each series of weighings. The cistern of the barometer was 305 mm. above the weights. Therefore 0·03 mm. must be added to F.

June 13, 1846. 100 parts = 0·31041 grain.

PS+X, T+Q+A+Y.	T+Q+A+Y, PS+X.		PS+Y, T+Q+A+X.	T+Q+A+X, PS+Y.
18·60	15·56		17·97	15·91
17·15	15·29		17·42	15·99
18·35	15·74		17·44	15·84
17·75	15·31	D = 19·65	17·91	16·22
17·50	15·55	C = 19·6	16·77	15·92
17·86	14·96	F = 767·6	17·75	15·65
17·21	15·04	E = 20·9	17·41	16·01
17·54	15·55		18·19	15·80
18·00	15·39		17·34	16·00
17·50	14·81		18·02	15·87
177·46	153·20		176·22	159·21

20(PS+X) ≏ 20(T+Q+A+Y) − 24·26 parts. 20(PS+Y) ≏ 20(T+Q+A+X) − 17·01 parts.

40PS ≏ 40(T+Q+A) − 0·12811 grain in air (t=19·68, b=765·49).

June 24. 100 parts = 0·3006 grain.

PS+Y, T+Q+A+X.	T+Q+A+X, PS+Y.		PS+X, T+Q+A+Y.	T+Q+A+Y, PS+X.
15·71	14·25		14·49	13·35
15·87	14·06	D = 19·65	18·44	16·90
14·92	14·17	C = 19·65	18·72	16·74
15·29	13·86	F = 748·5	18·34	16·92
14·71	13·70	E = 20·3	17·74	16·85
76·50	70·04		87·73	80·76

10(PS+Y) ≏ 10(T+Q+A+X) − 6·46 parts. 10(PS+X) ≏ 10(T+Q+A+Y) − 6·97 parts.

20PS ≏ 20(T+Q+A) − 0·04038 grain in air (t ≏ 19·70, b=746·53).

June 29. 100 parts = 0·29539 grain.

PS+Y, T+Q+A+X.	T+Q+A+X, PS+Y.		PS+X, T+Q+A+Y.	T+Q+A+Y, PS+X.
16·95	15·70		19·02	15·49
16·92	15·82	D = 18·8	18·85	15·90
16·41	14·25	C = 18·8	18·39	15·86
16·35	14·17	F = 758·0	18·25	15·30
16·46	14·52	E = 20	18·55	16·65
83·09	74·46		93·06	79·20

10(PS+Y) ≏ 10(T+Q+A+X) − 8·63 parts. 10(PS+X) ≏ 10(T+Q+A+Y) − 13·86 parts.

20PS ≏ 20(T+Q+A) − 0·06643 grain in air (t=18·83, b=756·03).

June 12. 100 parts=0·30735 grain.

PS+Y, T+Q+B+X.	T+Q+B+X, PS+Y.		PS+X, T+Q+B+Y.	T+Q+B+Y, PS+X.
16·74	18·17		17·99	15·26
17·00	17·52		17·56	15·09
17·12	17·11		17·60	14·94
17·27	17·96	D= 19·05	17·65	15·32
17·00	17·10	C= 19·1	17·91	14·44
16·80	17·80	F=769·4	16·92	15·06
17·20	17·61	E= 20·4	16·97	14·67
16·86	17·82		17·10	14·87
16·66	17·21		17·77	15·44
16·16	16·97		17·64	14·75
168·81	175·27		175·11	149·84

$20(PS+Y) \bumpeq 20(T+Q+B+X)+6{\cdot}46$ parts. $20(PS+X) \bumpeq 20(T+Q+B+Y)-25{\cdot}27$ parts.

$40PS \bumpeq 40(T+Q+B)-0{\cdot}05781$ grain in air ($t=19{\cdot}12$, $b=767{\cdot}35$).

June 24. 100 parts=0·30065 grain.

PS+X, T+Q+B+Y.	T+Q+B+Y, PS+X.		PS+Y, T+Q+B+X.	T+Q+B+X, PS+Y.
18·46	17·21		18·06	15·52
17·94	17·27	D= 19·65	17·36	14·62
17·97	16·99	C= 19·68	17·42	15·06
18·46	16·97	F=748·5	17·95	15·29
17·60	17·02	E= 20·3	17·45	15·20
90·43	85·46		88·24	75·69

$10(PS+X) \bumpeq 10(T+Q+B+Y)-4{\cdot}97$ parts. $10(PS+Y) \bumpeq 10(T+Q+B+X)-12{\cdot}55$ parts.

$20PS \bumpeq 20(T+Q+B)-0{\cdot}05267$ grain in air ($t=19{\cdot}7$, $b=746{\cdot}53$).

June 29. 100 parts=0·29539 grain.

PS+X, T+Q+B+Y.	T+Q+B+Y, PS+X.		PS+Y, T+Q+B+X.	T+Q+B+X, PS+Y.
18·84	17·50		17·37	16·65
18·35	17·09	D= 18·8	17·36	17·19
18·31	16·96	C= 18·8	18·16	17·05
17·47	16·44	F =758·0	17·17	16·91
17·85	16·46	E= 20	17·54	16·69
90·82	84·45		87·60	84·49

$10(PS+X) \bumpeq 10(T+Q+B+Y)-6{\cdot}37$ parts. $10(PS+Y) \bumpeq 10(T+Q+B+X)-3{\cdot}11$ parts.

$20PS \bumpeq 20(T+Q+B)-0{\cdot}02800$ grain in air ($t=18{\cdot}83$, $b=756{\cdot}03$).

June 25. 100 parts=0·29161 grain.

PS+Y, T+Q+C+X.	T+Q+C+X, PS+Y.		PS+X, T+Q+C+Y.	T+Q+C+Y, PS+X.
18·00	17·97		18·47	17·44
17·70	18·51	D= 19·1	18·57	17·07
17·69	18·35	C= 19·1	18·42	16·99
17·70	17·97	F=756·85	18·31	17·47
17·72	18·44	E= 19·75	18·01	17·52
88·81	91·24		91·78	86·49

$10(PS+Y) \bumpeq 10(T+Q+C+X)+2{\cdot}43$ parts. $10(PS+X) \bumpeq 10(T+Q+C+Y)-5{\cdot}29$ parts.

$20PS \bumpeq 20(T+Q+C)-0{\cdot}00834$ grain in air ($t=19{\cdot}15$, $b=754{\cdot}92$).

June 27. 100 parts $=0{\cdot}29438$ grain.

PS+X, T+Q+C+Y.	T+Q+C+Y, PS+X.		PS+Y, T+Q+C+X.	T+Q+C+X, PS+Y.
19·94	18·05		17·40	15·52
19·57	18·30		17·90	16·57
19·32	17·91		16·84	16·40
18·59	17·70	D= 18·6	17·42	16·17
18·17	17·21	C= 18·6	16·94	15·92
17·77	16·49	F=755·7	16·80	14·70
18·07	16·71	E= 19·1	16·74	16·01
17·42	16·44		16·37	15·84
17·91	16·00		16·70	15·50
17·60	16·99		16·27	15·30
184·36	171·80		169·38	157·93

$20(PS+X) \bumpeq 20(T+Q+C+Y)-12{\cdot}56$ parts. $20(PS+Y) \bumpeq 20(T+Q+C+X)-11{\cdot}45$ parts.

$40PS \bumpeq 40(T+Q+C)-0{\cdot}07068$ grain in air ($t=18{\cdot}65$, $b=753{\cdot}85$).

June 30. 100 parts $=0{\cdot}29982$ grain.

PS+Y, T+Q+C+X.	T+Q+C+X, PS+Y.		PS+X, T+Q+C+Y.	T+Q+C+Y, PS+X.
20·09	18·96		18·64	16·27
19·51	18·75	D= 18·8	18·62	16·39
19·60	18·67	C= 18·85	18·30	15·62
19·06	17·85	F=761·2	17·69	15·55
18·89	18·44	E= 19·9	18·36	15·36
97·15	92·67		91·61	79·19

$10(PS+Y) \bumpeq 10(T+Q+C+X)-4{\cdot}48$ parts. $10(PS+X) \bumpeq 10(T+Q+C+Y)-12{\cdot}42$ parts.

$20PS \bumpeq 20(T+Q+C)-0{\cdot}05067$ grain in air ($t=18{\cdot}87$, $b=759{\cdot}23$).

June 5 and 6. 100 parts $=0{\cdot}29998$ grain.

PS+Y, T+Q+D+X.	T+Q+D+X, PS+Y.		PS+X, T+Q+D+Y.	T+Q+D+Y, PS+X.
15·24	14·16		21·59	20·85
14·37	14·25		21·17	20·04
13·86	12·47		20·70	19·89
13·21	12·17	D= 18·3	20·56	19·57
12·65	12·27	C= 18·27	20·51	19·16
11·60	12·56	F=766·8	18·30	18·77
18·90	19·90	E= 19·65	18·36	18·60
18·91	19·70		18·21	18·51
19·26	19·49		18·20	18·17
18·52	19·50		18·44	18·61
156·52	156·47		196·04	192·17

$20(PS+Y) \bumpeq 20(T+Q+D+X)-0{\cdot}05$ part. $20(PS+X) \bumpeq 20(T+Q+D+Y)-3{\cdot}87$ parts.

$40PS \bumpeq 40(T+Q+D)-0{\cdot}01176$ grain in air ($t=18{\cdot}32$, $b=764{\cdot}85$).

June 20. 100 parts $=0{\cdot}31320$ grain.

PS+X, T+Q+D+Y.	T+Q+D+Y, PS+X.		PS+Y, T+Q+D+X.	T+Q+D+X, PS+Y.
18·92	17·32		19·55	18·34
18·52	19·31		19·76	18·42
18·82	18·80		19·47	18·11
18·36	19·01	D= 21·25	18·89	17·61
18·42	18·42	C= 21·25	18·97	17·94
17·95	18·77	F=768·9	18·70	17·81
18·82	18·46	E= 22·4	18·44	17·95
18·76	18·30		18·96	17·85
20·22	19·59		19·16	18·11
19·71	18·84		19·09	17·85
188·50	186·82		190·99	179·99

$20(PS+X) \bumpeq 20(T+Q+D+Y)-1{\cdot}68$ part. $20(PS+Y) \bumpeq 20(T+Q+D+X)-11{\cdot}0$ parts.

$40PS \bumpeq 40(T+Q+D)-0{\cdot}03971$ grain in air ($t=21{\cdot}31$, $b=766{\cdot}60$).

June 30. 100 parts=0·29981 grain.

PS+X, T+Q+D+Y.	T+Q+D+Y, PS+X.		PS+Y, T+Q+D+X.	T+Q+D+X, PS+Y.
18·70	16·22		16·16	16·74
17·95	16·32	D= 18·8	16·96	16·92
17·75	16·00	C= 18·85	16·24	16·55
17·24	15·56	F=761·20	17·32	16·84
17·61	15·90	E= 19·9	16·34	16·56
89·25	80·00		83·02	83·61

$10(PS+X) \bumpeq 10(T+Q+D+Y)-9{\cdot}25$ parts. $10(PS+Y) \bumpeq 10(T+Q+D+X)+0{\cdot}59$ part.

$20PS \bumpeq 20(T+Q+D)-0{\cdot}02596$ grain in air ($t=18{\cdot}87$, $b=759{\cdot}23$).

	gr.	$t.$	$b.$
$40PS \bumpeq 40(T+Q+A)$	$-0{\cdot}12811$	19·68	765·49
$20PS \bumpeq 20(T+Q+A)$	$-0{\cdot}04038$	19·70	746·53
$20PS \bumpeq 20(T+Q+A)$	$-0{\cdot}06643$	18·83	756·03
$40PS \bumpeq 40(T+Q+B)$	$-0{\cdot}05781$	19·12	767·35
$20PS \bumpeq 20(T+Q+B)$	$-0{\cdot}05267$	19·7	746·53
$20PS \bumpeq 20(T+Q+B)$	$-0{\cdot}02800$	18·83	756·03
$20PS \bumpeq 20(T+Q+C)$	$-0{\cdot}00834$	19·15	750·60
$40PS \bumpeq 40(T+Q+C)$	$-0{\cdot}07068$	18·65	753·85
$20PS \bumpeq 20(T+Q+C)$	$-0{\cdot}05067$	18·87	759·23
$40PS \bumpeq 40(T+Q+D)$	$-0{\cdot}01176$	18·32	764·85
$40PS \bumpeq 40(T+Q+D)$	$-0{\cdot}03971$	21·31	766·60
$20PS \bumpeq 20(T+Q+D)$	$-0{\cdot}02596$	18·87	759·23

Means.

	gr.	$t.$	$b.$
$PS \bumpeq T+Q+A$	$-0{\cdot}002936$	19·47	758·38
$PS \bumpeq T+Q+B$	$-0{\cdot}001731$	19·19	759·31
$PS \bumpeq T+Q+C$	$-0{\cdot}001621$	18·83	754·38
$PS \bumpeq T+Q+D$	$-0{\cdot}000774$	19·63	764·43

Mean of all.

	gr.	$t.$	$b.$
$PS \bumpeq T+Q+\frac{1}{4}(A+B+C+D)$	$-0{\cdot}00177$	19·28	759·12

Or, since $A+B+C+D=4F+4G-0{\cdot}00712$ grain,

$$PS \bumpeq T+Q+F+G-0{\cdot}00355 \text{ grain in air } (t=19{\cdot}28,\ b=759{\cdot}12).$$

$$T+Q+F+G-0{\cdot}00355 \text{ grain } =7000{\cdot}00073 \text{ grains.}$$

PS displaces 0·39744 grain of air, $T+Q+F+G-0{\cdot}00355$ gr. displaces 0·39727 gr. of air. Hence

$$PS=T+Q+F+G-0{\cdot}00338 \text{ grain.}$$

Hence, supposing U to have had the same density as V, PS=7000·00093 grains, of which U contained 5760.

Apparent weight of the Commercial Standard.

During the comparison of U with Sp and RS in Somerset House, the mean temperature was 18°·7 C., and the mean height of the barometer, reduced to 0° C., was 755·64 mm. PS displaced 0·38035 grain of air, $T+Q+F+G$ displaced 0·38019 grain of air, when $t=18{\cdot}7$, $b=755{\cdot}64$. Hence $PS \bumpeq T+Q+F+G-0{\cdot}00354$ grain in air ($t=18{\cdot}7$, $b=755{\cdot}64$) in Somerset House.

Let W be a weight of 7000 grains of the same density as U. In air ($t=18{\cdot}7$, $b=755{\cdot}64$) $U \triangleq T+0{\cdot}00745$ grain. Therefore $W \triangleq \frac{7000}{5760}(T+0{\cdot}00745 \text{ grain})$.

$$\frac{7000}{5760}(T+0{\cdot}00745 \text{ gr.})=T+0{\cdot}00745 \text{ gr.}+\frac{124}{576}(T+0{\cdot}00745 \text{ gr.}).$$

$$T=72(R+S)+0{\cdot}00559 \text{ grain, and } 4F+4G=62(R+S)+0{\cdot}01112 \text{ grain.}$$

$$\frac{124}{576}(T+0{\cdot}00745 \text{ gr.})=\frac{62}{4{\cdot}72}[72(R+S)+0{\cdot}01303 \text{ gr.}]=\frac{62}{4}(R+S)+0{\cdot}00281 \text{ gr.}$$
$$=F+G+0{\cdot}00003 \text{ gr.}$$

Hence $$\frac{7000}{5760}[T+0{\cdot}00745 \text{ grain}]=T+F+G+0{\cdot}00748 \text{ grain.}$$

Therefore $$W \triangleq T+F+G+0{\cdot}00748 \text{ grain.}$$

Hence in Somerset House, $PS \triangleq W+Q-0{\cdot}01102$ gr. in air ($t=18{\cdot}7$, $b=755{\cdot}64$).

$Q-0{\cdot}01102$ grain $=0{\cdot}63407$ grain. The weight of 0·63407 grain was adjusted by PS. This, in air ($t=18{\cdot}7$, $b=755{\cdot}64$), is equivalent to a weight of 0·63413 grain adjusted by W.

The commercial standard lb. is a brass weight which in air ($t=18{\cdot}7$, $b=755{\cdot}64$), or at the temperature 65°·66 FAHRENHEIT, under the pressure of 29·750 inches of mercury at 32° FAHRENHEIT, in Somerset House, for which $\log \Delta=7{\cdot}078324-10$, appears to weigh as much as W, or as much as $PS-Q+0{\cdot}01102$ grain. For in air having the above-mentioned temperature and pressure, the apparent weight of such a lb. would be $\frac{7000}{5760}$ of that of the lost standard. This result is not affected by the uncertainty of the value of the specific gravity of the lost standard.

In the following comparisons of the platinum copies of the pound with PS, I, K, L, M, N will be used to denote PS, PC No. 1, PC No. 2, PC No. 3, PC No. 4 respectively. The zero of the scale of the barometer was 305 millimètres above the centre of gravity of the weights.

Comparison of PC No. 1 *with* PS.

February 16, 1846.	100 parts=0·26222 grain.		February 18.	100 parts=0·27683 grain.
K+Y, I+X.	I+X, K+Y.		K+X, I+Y.	I+Y, K+X.
20·14	20·30		15·32	17·76
20·44	20·61		15·29	17·40
21·25	20·94		15·51	17·41
20·64	19·61	D= 16·05	15·80	17·67
20·89	20·94	C= 16·1	16·96	17·75
20·01	20·85	F =766·8	16·46	17·24
20·57	19·46	E= 16·42	15·35	17·52
20·04	19·82		16·24	16·62
20·70	19·75		16·60	16·46
19·65	19·81		16·27	16·94
204·33	202·09		159·80	172·77

$20(K+Y) \triangleq 20(I+X)-2{\cdot}24$ parts. $\quad 20(K+X) \triangleq 20(I+Y)+12{\cdot}97$ parts.

$40K \triangleq 40I+0{\cdot}03031$ grain in air ($t=16{\cdot}09$, $b=765{\cdot}25$).

February 20. 100 parts=0·22624 grain.

K+Y, I+X.	I+X, K+Y.		K+X, I+Y.	I+Y, K+X.
21·70	23·17		18·09	19·79
23·27	22·27		18·55	19·02
22·22	22·86		19·11	20·25
22·29	23·07	D= 9·3	19·70	20·40
21·66	21·20	C= 9·5	18·54	20·74
21·26	20·77	F=766·8	19·16	20·39
20·95	21·24	E= 16·8	19·12	20·35
20·50	20·26		19·02	19·72
19·62	20·36		18·99	20·07
20·55	19·65		19·50	20·59
214·02	214·85		189·78	201·32

20(K+Y)≏20(I+X)+0·83 part. 20(K+X)≏20(I+Y)+11·54 parts.

40K≏40I+0·02799 grain in air (t=9·4, b=765·30).

February 25. 100 parts=0·29063 grain.

K+X, I+Y.	I+Y, K+X.		K+Y, I+X.	I+X, K+Y.
20·35	21·56		17·10	17·15
19·85	21·67		16·05	16·66
19·36	21·00		17·45	17·51
18·84	20·69	D= 18·05	16·81	17·70
18·62	19·92	C= 18·1	16·57	16·45
18·67	19·75	F=753·05	17·19	17·51
18·90	19·12	E= 18·5	16·69	17·77
18·62	18·97		16·50	17·42
18·37	19·06		16·92	16·42
18·21	18·22		16·76	16·49
189·79	199·96		168·04	171·08

20(K+X)≏20(I+Y)+10·17 parts. 20(K+Y)≏20(I+X)+3·04 parts.

40K≏40I+0·03839 grain in air (t=18·08, b=751·28).

March 6. 100 parts=0·29781 grain.

K+Y, I+X.	I+X, K+Y.		K+X, I+Y.	I+Y, K+X.
18·00	19·00		15·22	15·36
17·34	17·31		12·85	13·22
17·41	18·06		20·41	20·36
16·32	17·07	D= 17·85	20·05	19·50
16·07	17·57	C= 17·85	20·12	19·37
16·35	16·65	F=755·3	19·05	19·17
16·10	16·24	E= 18·35	18·07	18·42
15·80	16·37		17·47	17·42
15·54	16·46		16·54	16·55
16·04	15·97		15·80	15·62
164·97	170·70		175·58	174·99

20(K+Y)≏20(I+X)+5·73 parts. 20(K+X)≏20(I+Y)−0·59 part.

40K≏40I+0·01531 grain in air (t=17·88, b=753·54).

April 20. 100 parts=0·24107 grain.

K+X, I+Y.	I+Y, K+X.		K+Y, I+X.	I+X, K+Y.
20·09	21·27		18·71	19·11
20·65	21·30		18·42	18·47
20·62	21·25		18·52	18·75
20·71	21·17	D= 12·25	18·46	19·01
20·52	21·44	C= 12·3	17·65	18·89
19·99	20·00	F=766·2	17·76	18·00
19·34	20·79	E= 13	18·27	18·35
19·57	20·50		18·00	18·50
19·27	19·71		18·55	18·97
19·20	20·00		18·45	19·09
199·97	207·43		182·79	187·14

20(K+X)≏20(I+Y)+7·47 parts. 20(K+Y)≏20(I+X)+4·35 parts.

40K≏40I+0·02849 grain in air (t=12·3, b=765·07).

	gr.	t.	b.
40K≏	40I+0·03003	16·09	765·25
40K≏	40I+0·02799	9·4	765·30
40K≏	40I+0·03839	18·08	751·28
40K≏	40I+0·01531	17·88	753·54
40K≏	40I+0·02849	12·3	765·07
200K≏	200I+0·14021		

PC No. 1≏PS+0·00070 grain in air (t=14·75, b=760·09).

PC No. 1 displaces 0·40452 grain of air, PS displaces 0·40471 grain. Therefore

PC No. 1=PS+0·00051 grain.

Comparison of PC No. 2 *with* PS.

February 11, 12. 100 parts=0·25720 grain.

L+X, I+Y.	I+Y, L+X.		L+Y, I+X.	I+X, L+Y.
16·36	15·89		15·61	14·81
17·69	16·14		14·66	15·44
17·10	15·60		15·64	15·46
16·74	15·59	D= 14·95	14·41	13·54
16·94	15·04	C= 15	13·56	13·17
15·19	14·00	F=762·52	17·04	17·06
15·30	13·41	E= 15·42	16·26	17·01
14·95	13·37		15·61	16·16
14·61	13·10		15·87	15·77
14·81	13·24		14·75	15·77
159·69	145·38		153·41	154·19

20(L+X)≏20(I+Y)−14·31 parts. 20(L+Y)≏20(I+X)+0·78 part.

40L≏40I−0·03480 grain in air (t=15·0, b=761·1).

March 13, 14. 100 parts=0·29530 grain.

L+X, I+Y.	I+Y, L+X.		L+Y, I+X.	I+X, L+Y.
23·25	24·55		22·30	20·62
23·81	24·06		22·59	21·10
23·34	23·21		21·85	20·50
22·35	22·66	D= 17·55	21·49	20·00
23·16	22·99	C= 17·55	20·55	19·55
21·87	23·16	F=773·5	19·72	19·47
22·91	23·00	E= 18	21·14	19·54
22·11	21·62		20·89	18·91
22·36	23·81		19·67	18·32
22·22	23·72		21·04	19·66
21·86	22·85		20·85	19·17
249·24	255·63		232·09	216·84

22(L+X)≏22(I+Y)+6·39 parts. 22(L+Y)≏22(I+X)−15·25 parts.

44L≏44I−0·02616 grain in air (t=17·58, b=771·73).

March 17. 100 parts=0·26284 grain.

L+X, I+Y.	I+Y, L+X.		L+Y, I+X.	I+X, L+Y.
21·35	22·07		20·76	18·81
20·37	19·92		20·89	19·02
20·60	19·82		19·66	18·92
18·56	19·12	D= 16·22	20·44	18·47
19·42	19·26	C= 16·25	19·89	17·75
19·00	18·85	F=750·05	20·41	19·00
17·85	19·19	E= 17·1	21·19	19·36
18·02	18·74		20·42	18·36
17·34	17·94		20·51	19·84
16·50	16·86		20·62	19·84
14·59	15·55		20·52	19·40
203·60	207·31		225·31	208·77

22(L+X)≏22(I+Y)+3·71 parts. 22(L+Y)≏22(I+X)−17·54 parts.

44L≏44I−0·03635 grain in air (t=16·25, b=748·46).

April 1. 100 parts=0·27933 grain.

L+Y, I+X.	I+X, L+Y.		L+X, I+Y.	I+Y, L+X.
19·40	20·75		17·44	14·86
19·39	20·47		17·12	14·47
18·62	20·92		16·49	14·60
18·02	20·16	D= 16·2	16·54	14·96
17·64	19·16	C= 16·3	16·02	14·17
17·84	18·87	F=750·7	16·49	14·92
17·29	18·80	E= 18·6	16·52	14·65
17·37	18·51		16·72	15·29
16·56	17·41		16·42	15·15
16·29	17·64		16·62	16·00
16·04	17·36		16·72	15·42
194·49	210·05		183·10	164·49

22(L+Y)≏22(I+X)+15·56 parts. 22(L+X)≏22(I+Y)−18·61 parts.

44L≏44I−0·00852 grain in air (t=16·27, b=748·93).

April 8. 100 parts=0·24979 grain.

L+X, I+Y.	I+Y, L+X.		L+Y, I+X.	I+X, L+Y.
24·39	24·05		19·64	19·72
24·72	23·51		20·55	19·70
25·07	24·69		19·49	19·51
24·80	24·59	D= 15·02	19·84	19·04
24·52	21·70	C= 15·02	19·67	18·71
24·60	22·90	F=745·2	20·17	19·80
24·42	22·37	E= 15·9	19·94	19·46
24·25	22·19		20·47	19·17
24·12	22·55		20·79	19·45
23·46	22·84		20·16	19·42
24·00	21·16		20·24	19·34
268·35	252·55		220·96	213·32

$22(L+X) \bumpeq 22(I+Y) - 15{\cdot}80$ parts. $22(L+Y) \bumpeq 22(I+X) - 7{\cdot}64$ parts.

$44L \bumpeq 44I - 0{\cdot}05855$ grain in air ($t=15{\cdot}04$, $b=743{\cdot}77$).

gr.	t.	b.
40L ≏ 40I − 0·03480	15·0	761·1
44L ≏ 44I − 0·02616	17·58	771·73
44L ≏ 44I − 0·03635	16·25	748·46
44L ≏ 44I − 0·00852	16·27	748·93
44L ≏ 44I − 0·05855	15·04	743·77
216L ≏ 216I − 0·16438		

PC No. 2 ≏ PS − 0·00076 grain in air ($t=16{\cdot}12$, $b=754{\cdot}68$).

PC No. 2 displaces 0·39966 grain of air, PS displaces 0·39979 grain. Hence

PC No. 2 = PS − 0·00089 grain.

Comparison of PC No. 3 *with* PS.

February 9, 10. 100 parts=0·26146 grain.

M+Y, I+X.	I+X, M+Y.		M+X, I+Y.	I+Y, M+X.
17·65	17·54		17·64	16·26
17·15	17·52		17·86	16·07
16·31	16·42	D= 15·15	17·02	14·90
16·44	16·25	C= 15·25	17·85	15·22
16·04	15·56	F=771·1	17·79	15·79
15·47	15·46	E= 15·5	17·06	14·96
16·52	15·09		16·69	14·77
15·49	16·00		15·80	14·05
14·99	16·15		16·29	14·09
16·17	15·95		15·42	14·17
162·23	161·94		169·42	150·28

$20(M+Y) \bumpeq 20(I+X) - 0{\cdot}29$ part. $20(M+X) \bumpeq 20(I+Y) - 19{\cdot}14$ parts.

$40M \bumpeq 40I - 0{\cdot}05080$ grain in air ($t=15{\cdot}22$, $b=769{\cdot}64$).

February 13. 100 parts =0·26281 grain. February 14. 100 parts =0·26683 grain.

M+X, I+Y.	I+Y, M+X.		M+Y, I+X.	I+X, M+Y.
23·49	21·00		19·89	19·01
23·55	21·11		19·99	19·40
22·10	20·77		19·27	18·14
22·65	20·82	D= 15·73	19·26	17·47
23·57	20·27	C= 15·82	19·55	18·34
21·99	20·12	F=767·55	18·85	17·65
22·25	20·55	E= 16·05	18·12	17·70
22·41	20·54		17·95	16·41
22·01	21·27		18·12	17·24
21·59	21·30		17·91	17·14
225·61	207·75		188·91	178·50

20(M+X)≏20(I+Y)−17·86 parts. 20(M+Y)≏20(I+X)−10·41 parts.

40M≏40I−0·07472 grain in air (t=15·8, b=766·04).

March 18. 100 parts =0·24265 grain.

M+Y, I+X.	I+X, M+Y.		M+X, I+Y.	I+Y, M+X.
16·80	16·40		18·99	17·24
17·71	14·65		19·06	17·82
17·92	15·94		19·44	17·76
17·54	14·40	D= 15·13	19·39	17·96
16·66	15·16	C= 15·22	19·06	16·76
16·07	14·40	F=752·4	18·25	17·00
16·39	14·00	E= 15·9	18·24	17·05
15·52	13·36		18·64	16·85
14·92	13·42		17·25	17·01
15·35	13·75		17·57	16·15
164·88	145·48		185·89	171·60

20(M+Y)≏20(I+X)−19·4 parts. 20(M+X)≏20(I+Y)−14·29 parts.

40M≏40I−0·08176 grain in air (t=15·19, b=750·95).

March 19. 100 parts =0·24331 grain.

M+X, I+Y.	I+Y, M+X.		M+Y, I+X.	I+X, M+Y.
22·79	21·66		17·54	17·11
23·35	21·60		17·27	15·17
23·04	22·27		17·45	15·54
22·50	21·76	D= 14·0	16·77	14·84
23·12	21·65	C= 14·08	15·97	14·09
22·82	21·57	F=753·2	15·75	13·97
22·30	21·51	E= 14·45	15·74	14·64
22·21	21·42		15·92	13·57
21·40	20·65		15·24	13·40
20·15	20·01		15·64	14·50
223·68	214·10		163·29	146·83

20(M+X)≏20(I+Y)−9·58 parts. 20(M+Y)≏20(I+X)−16·46 parts.

40M≏40I−0·06336 grain in air (t=14·06, b=751·92).

April 11. 100 parts =0·23525 grain.

M+Y, I+X.	I+X, M+Y.		M+X, I+Y.	I+Y, M+X.
23·40	21·36		17·26	14·67
23·45	21·81		16·90	15·66
23·41	22·11		16·82	15·60
22·27	20·89		17·37	15·69
18·04	16·45	D= 14·0	17·47	15·57
18·21	16·52	C= 14·0	16·47	15·65
17·80	16·40	F=750·2	15·89	15·75
17·10	15·25	E= 14·7	15·02	15·42
17·20	14·94		16·79	15·62
17·47	16·00		16·41	15·12
17·62	15·20		17·55	15·99
215·97	196·93		183·95	170·74

22(M+Y)≏22(I+X)−19·04 parts. 22(M+X)≏22(I+Y)−13·21 parts.

44M≏44I−0·07587 grain in air (t=14·02, b=748·89).

	gr.	t.	b.
40M≏	40I−0·05080	15·22	769·64
40M≏	40I−0·07472	15·80	766·04
40M≏	40I−0·08176	15·19	750·95
40M≏	40I−0·06336	14·06	751·92
44M≏	44I−0·07587	14·02	748·89
204M≏	204I−0·34651		

PC No. 3≏PS−0·00170 grain in air (t=14·84, b=757·32).

PC No. 3 displaces 0·40302 grain of air, PS displaces 0·40310 grain. Therefore

PC No. 3=PS−0·00178 grain.

Comparison of PC No. 4 *with* PS.

February 7. 100 parts =0·29952 grain.

N+X, I+Y.	I+Y, N+X.		N+Y, I+X.	I+X, N+Y.
22·42	19·56		19·34	19·35
22·40	19·12		19·85	20·02
21·76	18·30		20·22	19·57
20·85	17·77	D= 17·4	19·67	20·04
21·14	17·10	C= 17·45	20·79	19·35
19·69	17·06	F=757·4	20·01	19·45
19·46	16·55	E= 17·65	20·20	18·89
20·20	16·97		19·84	19·04
20·00	15·72		20·09	19·24
19·89	16·01		20·25	19·85
207·81	174·16		200·26	194·80

20(N+X)≏20(I+Y)−33·65 parts. 20(N+Y)≏20(I+X)−5·46 parts.

40N≏40I−0·11715 grain in air (t=17·46, b=755·72).

March 23. 100 parts = 0·22348 grain.

N+Y, I+X.	I+X, N+Y.		N+X, I+Y.	I+Y, N+X.
20·49	17·81		16·41	13·50
20·61	17·35		17·21	14·20
21·05	18·07		17·49	14·31
21·17	18·19	D= 9·25	17·59	13·75
20·07	18·05	C= 9·4	17·47	13·31
20·04	17·49	F=743·0	17·72	14·10
19·15	16·44	E= 14·6	17·27	14·66
18·92	17·37		17·64	13·97
18·45	16·62		17·72	13·74
18·30	16·26		17·86	13·81
198·25	173·65		174·38	139·35

20(N+Y)≏20(I+X)−24·6 parts. 20(N+X)≏20(I+Y)−35·03 parts.

40N≏40I−0·13326 grain in air (t=9·32, b=741·72).

March 30. 100 parts = 0·27112 grain.

N+X, I+Y.	I+Y, N+X.		N+Y, I+X.	I+X, N+Y.
20·04	16·10		20·01	18·74
17·45	16·05		19·81	18·07
17·89	15·70		19·11	18·57
17·87	15·05	D= 16	19·95	18·56
22·11	18·50	C= 16	19·60	17·95
21·50	18·30	F=755·0	19·69	17·94
21·30	19·29	E= 17·9	19·70	18·45
20·31	18·49		18·75	18·35
20·91	18·66		17·80	16·31
20·80	17·50		17·20	15·86
200·18	173·64		191·62	178·80

20(N+X)≏20(I+Y)−26·54 parts. 20(N+Y)≏20(I+X)−12·82 parts.

40N≏40I−0·10671 grain in air (t=16·01, b=753·3).

April 13. 100 parts = 0·24070 grain.

N+X, I+Y.	I+Y, N+X.		N+Y, I+X.	I+X, N+Y.
18·27	15·11		16·69	13·36
17·77	15·90		16·79	13·12
19·05	16·07		16·57	12·60
18·59	15·52		16·09	12·35
18·46	15·89	D= 14·2	25·15	20·67
17·27	15·19	C= 14·25	20·99	17·45
16·65	14·22	F=753·45	21·47	18·71
16·85	15·17	E= 15·3	20·92	18·45
16·76	14·45		20·77	19·35
17·26	14·91		21·51	19·46
16·56	14·00		21·34	19·49
193·49	166·43		218·29	185·01

22(N+X)≏22(I+Y)−27·06 parts. 22(N+Y)≏22(I+X)−33·28 parts.

44N≏44S−0·14524 grain in air (t=14·24, b=752·06).

April 22. 100 parts $=0\cdot24294$ grain.

N+X, I+Y.	I+Y, N+X.		N+Y, I+X.	I+X, N+Y.
19·40	17·39		17·29	14·55
19·92	16·57		17·71	14·10
19·80	16·95		17·34	14·81
19·80	16·50	D= 12·75	17·67	14·41
19·75	17·01	C= 12·8	17·66	14·59
18·56	16·09	F=760·9	17·86	14·22
18·40	15·22	E= 13·95	17·60	13·70
18·29	15·69		17·34	14·81
18·42	15·26		17·64	15·81
17·36	14·80		17·64	13·71
189·70	161·48		175·75	144·71

$20(N+X) \triangleq 20(I+Y)-28\cdot22$ parts. $20(N+Y) \triangleq 20(I+X)-31\cdot04$ parts.

$40N \triangleq 40I-0\cdot14397$ grain in air ($t=12\cdot8$, $b=759\cdot67$).

	gr.	$t.$	$b.$
40N≏	40I−0·11715	17·46	755·72
40N≏	40I−0·13326	9·32	741·72
40N≏	40I−0·10671	16·01	753·30
44N≏	44I−0·14524	14·24	752·06
40N≏	40I−0·14397	12·80	759·67
204N≏	204I−0·64633		

PC No. $4 \triangleq PS-0\cdot00317$ grain in air ($t=13\cdot97$, $b=752\cdot49$).

PC No. 4 displaces 0·40184 grain of air, PS displaces 0·40181 grain. Therefore

PC No. $4=PS-0\cdot00314$ grain.

Determination of the volume of Professor SCHUMACHER'S *pound* Sp+V.

The troy pounds Sp, Sb, K were returned to Professor SCHUMACHER, accompanied by a platinum weight V of about 1240·53 grains which with Sp makes a pound of 7000 grains.

Weighing of V in air.

January 28. 100 parts $=0\cdot26743$ grain. $Z=F+H+R+S+Q$.

Z+0·06 gr. +X, V+Y.	V+Y, Z+X.	Z+0·06 gr. +Y, V+X.	V+X, Z+Y.
18·87	20·05	20·02	20·57
19·55	20·87	20·20	21·37
19·56	20·75	20·70	21·22
19·55	21·25	20·15	21·20
19·77	21·27	20·21	21·10
97·30	104·19	101·28	105·46

$10(V+Y) \triangleq 10(Z+0\cdot03\text{ gr.}+X)-6\cdot89$ parts. $10(V+X) \triangleq 10(Z+0\cdot03\text{ gr.}+Y)-4\cdot18$ parts.

$20V \triangleq 20Z+0\cdot6\text{ gr.}-0\cdot02960$ gr.

$V \triangleq 1240\cdot530$ grains of platinum in air.

$vV=59\cdot34$, $vZ=58\cdot61$, $vV-vZ=0\cdot73=$ volume of 0·0009 grain of air. Hence

$V=1240\cdot5309$ grains.

Weighing of V in water.

The wire by which V was suspended in water was so fine, that the effect of the displacement of the water by it, on the sensibility of the balance, may be neglected. The thermometer B was suspended in the water, with its bulb in a horizontal plane through V. The counterpoise in the left-hand pan is supposed to be in equilibrium with the weights suspended from the right-hand end of the beam, when the reading of the scale is 19.

January 24, 26, 1846. 100 parts =0·2626 grain.

V and hook in water. In right-hand pan.

B.	C.	F.	E.	gr.	Scale.
12·16				0·64	23·20
				0·62	14·80
				0·63	19·35
	11·1			0·63	18·50
				0·63	18·90
12·15				0·63	19·40
12·05				0·64	20·57
				0·63	18·15
		752·47	15·95	0·64	20·60
				0·63	18·30
12·0	11·35			0·63	18·20
				0·64	20·65
12·0				0·63	19·85
	11·37			0·63	19·50
				0·63	19·57
				0·63	20·10
				0·63	20·05
11·1				0·63	19·90
				0·63	19·85
				0·63	19·85
	11·35	742·2	16	0·63	19·65
11·1				0·63	19·52
11·76	11·29	747·33	16	0·6314	19·52

Counterpoise in air (C=11·29, F=747·33, E=16) balances V and hook in water (B=11·76)+0·630 gr. in air.

In right-hand pan.

gr.	Scale.
F+H+R+0·06	18·72
+0·06	18·82
+0·06	18·86
+0·06	18·92
+0·06	18·61
0·06	18·79

Counterpoise balances F+H+R+0·0606 grain.

V and hook in water (B=11·76)≏F+H+R−0·5694 grain in air (C=11·29, F=747·33, E=16).

January 26. 100 parts =0·292 grain.

Hook in water. In right-hand pan.

gr.	Scale.
1·61	20·00
1·61	20·05
1·61	20·20
1·61	19·40
1·61	20·70
1·61	21·45

1·61	23·05
1·61	22·55
1·61	22·25
1·61	22·75
1·61	21·24

Counterpoise balances hook in water +1·6045 grain in air.

In right-hand pan.

	gr.	Scale.
(16)+(2)+(1)+0·7 gr.+	0·00	20·92
	0·00	20·87
	0·00	20·95
	0·00	21·47
	0·00	21·05

Counterpoise balances (16)+(2)+(1)+0·694 grain in air.

Hook in water ≏(16)+(2)+(1)−0·9105 grain in air.

Hook in water ≏18·0880 grains.

V in water (t=11·67)≏1181·2028 grains of platinum in air (t=11·30, b=745·84).

Hence ΔV=20·8834, log ΔV=1·319797, vV=59·403.

Comparison of Sp+V *with* PS.

The zero of the scale of the barometer was 305 millimètres above the centre of gravity of the weights.

April 15, 1846. 100 parts =0·24256 grain.

PS+Y, Sp+V+X.	Sp+V+X, PS+Y.		PS+X, Sp+V+Y.	Sp+V+Y, PS+
19·35	19·92		18·72	19·61
20·30	20·21		18·90	19·75
19·85	20·51		18·35	19·69
20·10	20·32	D= 13·7	18·30	19·40
19·40	20·42	C= 13·8	19·05	19·67
19·35	19·40	F=756·4	18·20	19·74
19·44	19·67	E= 14·1	18·86	19·46
19·30	20·29		18·54	19·61
19·35	20·02		18·84	18·99
19·35	19·20		18·82	19·82
195·79	199·96		186·58	195·74

20(Sp+V+X)≏20(PS+Y)−4·17 parts. 20(Sp+V+Y)≏20(PS+X)−9·16 parts.

40(Sp+V)≏40PS−0·03233 grain in air (t=13·78, b=755·15).

April 17. 100 parts =0·24555 grain.

PS+Y, Sp+V+X.	Sp+V+X, PS+Y.		PS+X, Sp+V+Y.	Sp+V+Y, PS+X.
21·29	19·27		18·74	19·65
20·59	18·94		19·42	20·74
21·09	21·15		19·96	20·50
21·17	20·89	D= 13·55	19·66	20·17
20·35	20·57	C= 13·56	19·84	20·32
20·15	20·15	F=760·65	19·90	20·67
20·06	21·26	E= 14	20·09	20·37
20·35	19·20		20·00	20·60
20·17	18·91		20·05	20·34
19·87	18·89		19·81	20·19
205·09	199·23		197·47	203·55

20(Sp+V+X)≏20(PS+Y)+5·86 parts. 20(Sp+V+Y)≏20(PS+X)−6·08 parts.

40(Sp+V)≏40PS−0·00054 grain in air (t=13·59, b=759·41).

April 18. 100 parts =0·24021 grain.

PS+Y, Sp+V+X.	Sp+V+X, PS+Y.		PS+X, Sp+V+Y.	Sp+V+Y, PS+X.
24·30	24·69		16·20	17·36
24·76	25·35		16·25	17·30
18·77	19·39		17·47	17·50
19·10	19·74	D= 13·05	17·21	18·14
18·82	19·46	C= 13·1	18·52	19·60
19·10	18·96	F=761·8	19·35	20·62
18·20	19·36	E= 13·4	19·17	19·74
18·06	18·82		19·10	19·52
18·41	19·31		19·42	19·27
18·19	17·97		18·96	19·16
197·71	203·05		181·65	188·21

$20(Sp+V+X) \bumpeq 20(PS+Y)-5{\cdot}34$ parts. $20(Sp+V+Y) \bumpeq 20(PS+X)-6{\cdot}56$ parts.

$40(Sp+V) \bumpeq 40PS-0{\cdot}02858$ grain in air ($t=13{\cdot}1$, $b=760{\cdot}63$).

April 21. 100 parts =0·24160 grain.

PS+X, Sp+V+Y.	Sp+V+Y, PS+X.		PS+Y, Sp+V+X.	Sp+V+X, PS+Y.
21·57	20·41		14·97	14·65
20·97	20·02		14·60	15·47
20·21	19·12		14·40	14·84
19·66	18·70	D= 12·6	14·40	15·37
18·89	18·07	C= 12·63	14·02	15·86
19·29	17·60	F=764·03	14·52	15·39
19·02	17·52	E= 13·9	14·10	16·15
18·75	17·22		14·86	14·97
17·97	16·86		15·05	15·34
17·95	16·77		14·82	15·60
194·28	182·29		145·74	153·64

$20(Sp+V+Y) \bumpeq 20(PS+X)-11{\cdot}99$ parts. $20(Sp+V+X) \bumpeq 20(PS+Y)-7{\cdot}90$ parts.

$40(Sp+V) \bumpeq 40PS-0{\cdot}04805$ grain in air ($t=12{\cdot}64$, $b=762{\cdot}79$).

April 23. 100 parts =0·23834 grain.

PS+X, Sp+V+Y.	Sp+V+Y, PS+X.		PS+Y, Sp+V+X.	Sp+V+X, PS+Y.
21·50	21·50		19·66	19·64
20·85	21·80		19·72	20·14
21·14	20·90		19·12	20·04
20·92	21·96	D= 12·3	19·19	20·06
20·11	21·36	C= 12·35	19·39	20·56
20·24	21·11	F=759·0	19·52	20·36
20·12	21·12	E= 12·9	19·90	19·62
19·74	20·76		19·67	19·96
20·11	20·67		19·81	20·21
19·32	20·40		18·91	20·20
204·05	211·58		194·89	200·79

$20(Sp+V+Y) \bumpeq 20(PS+X)-7{\cdot}53$ parts. $20(Sp+V+X) \bumpeq 20(PS+Y)-5{\cdot}90$ parts.

$40(Sp+V) \bumpeq 40PS-0{\cdot}03200$ grain in air ($t=12{\cdot}35$, $b=757{\cdot}43$).

gr.	*t.*	*b.*
$40(Sp+V) \bumpeq 40PS-0{\cdot}03233$	13·78	755·15
$40(Sp+V) \bumpeq 40PS-0{\cdot}00054$	13·59	759·41
$40(Sp+V) \bumpeq 40PS-0{\cdot}02858$	13·10	760·63
$40(Sp+V) \bumpeq 40PS-0{\cdot}04805$	12·64	762·79
$40(Sp+V) \bumpeq 40PS-0{\cdot}03200$	12·35	757·43

Mean ...... $Sp+V \bumpeq PS-0{\cdot}00071$ grain in air ($t=13{\cdot}09$, $b=759{\cdot}08$).

vSp=271·832 as computed by means of SCHUMACHER's tables, but vT is 0·014 less as computed by SCHUMACHER's tables than as computed by the tables used in calculating vV and vPS; therefore probably vSp would be 271·846 if computed by the latter tables, vV=59·403, vPS=330·856. Hence v(Sp+V)=331·249, v(Sp+V)−vPS=0·393.

v(Sp+V)−vPS=volume of 0·00048 grain of air. Therefore

$$\mathrm{Sp}+\mathrm{V}=\mathrm{PS}-0{\cdot}00023 \text{ grain.}$$

Comparison of the Pound with the Kilogramme.

The necessity of a new comparison of the kilogramme with English weights appears by the following extract from a paper by Professor MOLL in the Journal of the Royal Institution, No. 4, 1831. "Each of the governments who sent Commissioners to the Committee of Weights and Measures in Paris were presented with a kilogramme and a mètre by the French government. A similar present was made to each of the Commissioners. These weights and measures were fabricated under the eyes of the united Commission and marked with their particular stamp, after being previously examined, it is said, with great care. The kilogramme and the mètre of the late M. VAN SWINDEN are now in my hands. I have compared VAN SWINDEN's kilogramme and several standards of the same weight with the English troy weights of ROBINSON, and I am very sorry to say that the result has left me in an entire darkness as to the real value of the kilogramme.

VAN SWINDEN's kilogramme	15432·295
A kilogramme modèle made by FORTIN and belonging to Government	15432·752
A kilogramme modèle made by GANDOLFI, Balancier de la Monnaye de Paris, and sent during the French occupation of this country by the Parisian Mint to the Mint at Utrecht	15432·730
A kilogramme by the same artist, also belonging to the Mint at Utrecht	15432·752
A kilogramme by NAGEL of Amsterdam, and serving as a standard at the Mint	15432·920
Another, made as a standard for the Royal Institute of Holland	15432·985
Another by the same, in my possession	15433·420
Another, also said to be a standard	15434·91
Annuaire du Bureau des Longitudes	15438·355
KELLY	15433."
According to HASSLER (Comparison of Weights and Measures reported to the Senate of the United States, 1832), the weight of an original brass kilogramme presented to M. TRALLES by the French Committee of Weights and Measures, in grains of which the United States Mint troy pound adjusted by Captain KATER, contains 5760	15433·159
Result of the comparison of a troy pound sent to Paris, according to a statement of the French Minister of the Interior, in a letter to the English Commissioners of Weights and Measures, dated Paris, February 28, 1821	15432·719

By the good offices of M. Arago, permission was obtained from the French Government to compare the pound with the standard kilogramme of platinum deposited in the Archives on the 22nd of June, 1799, known as the 'kilogramme des Archives.' The weights selected for comparison with the standard kilogramme, which henceforward will be designated by the letter 𝔄, were PC No. 1 and PC No. 2, together with the auxiliary weight B and a platinum weight V of about 192·4 grains, making altogether about 15432·35 grains. In order to obtain a second comparison perfectly independent of the former, a kilogramme of bronze was constructed by Mr. Barrow, with which, after comparison with 𝔄, it was my intention to compare PS together with each of the four platinum copies of the pound in turn, and other platinum weights sufficient to make up a kilogramme.

By some most unaccountable oversight 𝔄 had never been weighed in water previous to its final adjustment. Afterwards, on account of its legal importance, it was considered hazardous to immerse it in water, especially as, from the method of preparing platinum at that time in use (fusion with arsenious acid and subsequent ignition of the arsenide of platinum under a muffle till the arsenic was burnt away), there is reason to suppose that it is not entirely free from an admixture of arsenic which, on being wetted, might oxidize and then dissolve, and thus produce a very sensible alteration of weight. Its form is that of a cylinder of about 39·4 millimètres in diameter and 39·7 millimètres high, having its edges rounded by a surface 0·75 millimètre broad, and having a radius of about 3 millimètres. An approximate value of its density was obtained by Professor Schumacher and Olufsen in the following manner. In August and November 1831, the density of a kilogramme 𝔖 in Professor Schumacher's possession was found to be 21·212, by weighing it in water and in air. In March 1832, by measuring pairs of diameters at right angles to each other in planes cutting the axis in eight different points, and the distances between nine corresponding pairs of points in the ends of the cylinder, the volume of 𝔖 at 0° appeared to be 47114·4 cubic millimètres. In the autumn of 1834, Professor Olufsen measured two diameters at right angles to each other at the middle and at each end of 𝔄, and the distances between eight pairs of corresponding points in the circular ends. The volume of 𝔄 at 0°, deduced from these linear dimensions appeared to be 48615·4 cubic millimètres, and consequently its density 20·644*. These measurements, though made with the utmost care, appeared to be too few, and confined to too small a number of points, to determine the density of 𝔄 in this manner with sufficient accuracy. I therefore resolved to compare its volume by means of the stereometer, with that of a brass cylinder of nearly the same dimensions, the volume of which might afterwards be found by weighing in air and in water.

The representations of M. Arago procured for me the privilege of forwarding the balance and stereometer unexamined from Havre to the Douane in Paris, where I received them without being obliged to unpack the cases in which they were con-

* Schumacher's Jahrbuch für 1836, p. 237.

tained. From M. LETRONNE and M. LALLEMAND, Officers of the Archives, to the latter of whom the custody of the standards was confided, I received every possible assistance. The balance was mounted on a strong and heavy carpenter's bench in a room paved with brick, on the ground floor of the Archives. On unpacking the stereometer, the graduated tube was found to be broken, in consequence, as M. BUNTEN affirmed, of mere contact with a slender iron wire used in cleaning the tube with cotton wool, and left in it in ignorance of the peculiar action of iron wire on the interior of a glass tube, and not from any violent shock. M. BUNTEN replaced the broken tube, which had been divided into inches, by a tube divided into centimètres, and traced upon the slip of ivory a scale of 10 millimètres divided to every 0·2 of a millimètre. I procured one of ERNST's cistern barometers, which, after hanging all night by the side of the standard barometer of the observatory, was compared with it on the following day by one of the Assistants. M. GAMBEY was commissioned to construct a brass cylinder, either solid or, if hollow, air-tight, nearly of the dimensions of 𝔄, a cylindrical cup to receive the kilogramme or the model, fitting into the cup of the stereometer, and a second cylinder closed at both ends, to fill up as much as possible of the space left vacant in the cup of the stereometer. While with M. GAMBEY, I ascertained that he had some platinum kilogrammes finished, with the exception of the final reduction. This appeared to be a favourable opportunity for commencing the formation of a collection of accurate copies of foreign standards, which had been recommended in Art. 33 of the Report of the Committee, dated December 21, 1841. Also the comparison of 𝔄 with a copy having nearly the same density and expansion, and unalterable by mere exposure to the atmosphere, promised to be much more serviceable in finding the relation between the French and English standards of weight, than its comparison with a copy expanding nearly twice as much by heat, and having nearly three times its volume, and liable to become considerably heavier by oxidation in the course of a few years. I therefore applied to the Astronomer Royal for authority to purchase a platinum kilogramme for the use of the Committee. While waiting for his reply, I occupied myself with the comparison of 𝔄 with PC No. 1+PC No. 2+B+V.

The platinum kilogramme being a cylinder without a knob, does not admit of being lifted with a fork, consequently, in putting it into the scale-pan or taking it out, it must be held in the hand, a glove or a piece of silk being of course interposed between the fingers and the weight. The insertion of the hand into the balance case, and the communication of its warmth to the weight itself, are so prejudicial to the accuracy of a weighing, that it became necessary to seek for some method of obviating this source of error. Such a method was found in the employment of the detached scale-pans described in page 764, and answered so well, that I afterwards continued to use it in comparing weights of the usual form.

On the 16th of September, the Astronomer Royal having approved of the purchase of a platinum kilogramme for the use of the Committee, I procured one from

M. Gambey. In form it resembles 𝔄, except that it is not quite so large, in consequence of the greater density of the metal of which it is composed. The cylindrical surface and the ends are very accurately worked. The metal of which it is made is greatly superior to that of which the standard lb. and its copies are constructed, as not the slightest indication of any defective place can be observed on its surface. This kilogramme will be designated by the letter ℭ. The comparisons of 𝔄 with PC No. 1+PC No. 2+B+V were now discontinued, being of secondary importance after the acquisition of ℭ, and serving mainly to control the comparison of ℭ with 𝔄.

A considerable number of observations were made with the stereometer on the 25th of September, for the purpose of finding the volume of 𝔄 by comparing it with that of a hollow brass cylinder M of nearly the same dimensions. Towards the end of the series, M being in the cup, the mercury in the graduated tube was observed to descend perceptibly though very slowly. This was caused apparently by the passage of a small quantity of air into the upper part of the graduated tube, where the pressure was about half that of the external air. At first it was supposed that the air found an entrance either between the rim of the cup and the glass plate which closed it, or through the screw joint by which the cup was connected with the collar into which the graduated tube was cemented, or, lastly, through the cemented joint itself. The cement was then varnished with shell-lac dissolved in alcohol, and the screw and glass plate carefully smeared with lard, so as to render the passage of air through the joints all but impossible. The mercury in the graduated tube still continued to descend when M was in the cup, but not otherwise. It appeared therefore probable that the soldering of M had given way under the changes of pressure to which it had been exposed, so as to allow the air enclosed within it to escape slowly, when the pressure of the air in the cup was diminished. This conjecture was verified, on attempting to weigh M in water on the 22nd and 23rd of April, 1845, when M was found to have increased in weight after being left all night in the water in which it had been weighed; and on placing it in the receiver of an air-pump and partially exhausting the air, drops of water made their appearance at the junction of the plane and cylindrical surfaces at one end of the cylinder. It was evidently useless to continue the observations with M, on account of its presumed leakage; a second series of stereometer observations was therefore made on the 5th of October, in which the volume of 𝔄 was compared with that of ℭ.

Stereometer observations for finding the volume of 𝔄.

At 16° the mercury contained in the graduated tube between 163·2 mm. and 108·9 mm. weighed 1812·35 grains, and the mercury contained between 108·9 mm. and 0·0 mm. weighed 3614·99 grains. Hence the mercury contained between 0 mm. and 136 mm. weighed 4532·87 grains, and each mm. of the tube near 136 mm. contained 33·38 grains of mercury. The volume of PM is expressed in terms of the

volume of a grain of mercury at 16°. The temperature of the mercury in the stereometer was determined by a thermometer placed in the jar which received the mercury that escaped on opening the stopcock at the lower end of the tube. Let t denote this temperature. The barometer was suspended near the stereometer, and was presumed to have the same temperature. The height of the column of mercury in it, corrected for its constant error, but not reduced to 0°, will be denoted by h.

𝕮 in cup.

$t=16·92$, $h=750·24$.

PM.	PC.
136·43	518·10
136·60	518·80
136·40	519·10
136·40	519·20
136·35	518·85
136·40	519·10
136·25	518·20
136·45	518·85
136·41	518·77

CM= 382·36
h—CM= 376·89
Vol. PM=4546·56
Vol. cup—vol. 𝕮=4481·4

𝕬 in cup.

$t=17·1$, $h=759·07$.

PM.	PC.
136·35	529·55
136·15	530·50
136·00	531·00
136·20	530·65
136·45	531·50
136·30	531·30
136·00	530·50
135·90	530·15
136·10	530·70
136·20	531·10
136·65	531·95
136·40	531·70
136·22	530·88

CM= 394·66
h—CM= 364·41
Vol. PM=4540·2
Vol. cup—Vol. 𝕬=4192·2

𝕮 in cup.

$t=17·3$, $h=758·65$.

PM.	PC.
136·10	517·90
136·20	518·55
136·10	518·40
136·05	517·90
136·45	518·90
136·05	517·40
136·25	517·75
136·35	518·60
136·20	518·15
139·65	519·30
136·23	518·28

CM= 382·05
h—CM= 376·60
Vol. PM=4540·7
Vol. cup—Vol. 𝕮=4475·9

Hence, at 17°·1, vol. cup — vol. 𝕮 = volume of 4478·65 grains of mercury at 16°, and vol. cup — vol. 𝕬 = volume of 4192·2 grains of mercury at 16°. Therefore, at 17°·1, vol. 𝕬 — vol. 𝕮 = volume of 286·45 grains of mercury at 16°.

v𝕬—v𝕮 in vol. of a grain of mercury at 16° ...	286·45	2·45705
Δ mercury at 0° : Δ mercury at 16°	1·00287	0·00124
		2·45829
Δ mercury at 0° : Δ water at 4°	13·596	1·13341
v𝕬—v𝕮 at 17°·1		1·32488
Δ platinum at 0° : Δ platinum at 17°·1		0·00020
v𝕬—v𝕮 at 0°	21·119	1·32468

Weighing of 𝕮 in water.

The kilogramme, after having been washed with alcohol, was suspended from the right-hand pan of the balance, by a loop of platinum wire wound round it, so as to hang in the middle of the jar destined to receive the water in which it was to be weighed. In order to prevent the formation of air-bubbles, the water was poured through a funnel having a very small opening inserted into the upper end of a

glass tube the lower end of which reached to the bottom of the jar. The left-hand pan contained the counterpoise consisting of 14003 grains of bronze, for which $\log \Delta = 0{\cdot}92260$. The thermometers C, D were suspended in the water with their bulbs in a horizontal plane through the centre of gravity of ℭ. The counterpoise is supposed to be in equilibrium with the weights in or suspended from the right-hand pan, when the reading of the scale is 20.

Let K, L denote PC No. 1, PC No. 2 respectively, and P, Q, R the bronze weights marked 100, 200, 400. ℭ and the loop of wire being suspended in water, and the weight in the first column placed in right-hand pan, the second column gives the corresponding reading of the scale.

September 27, 1844. 100 parts =0·500 grain.

C.	D.	F.	E.	gr.	Scale.
14·97	15·0			0·0	20·1
				0·0	20·15
				0·0	20·15
				0·0	20·2
		764·65	16·1	0·0	18·9
				0·0	18·9
				0·0	19·9
				0·0	20·0
				0·0	20·05
				0·0	19·9
14·97	15·0			0·0	20·25
				0·0	20·3
		764·65	16·1	0·0	20·3
				0·0	20·35
				0·0	20·4
				0·0	20·4
14·98	15·0			0·0	18·8
				0·0	18·8
		764·55	16·2	0·0	18·8
				0·0	20·5
				0·0	20·45
				0·0	20·45
				0·0	20·55
				0·0	20·45
				0·0	20·45
14·97	15·0	764·62	16·13	0·0	19·98

Counterpoise in air ($t=16{\cdot}05$, $b=763{\cdot}08$) balances ℭ and wire loop in water ($t=15{\cdot}0$) $+0{\cdot}0001$ grain in air.

The wire loop alone being suspended in the water contained in the jar, to which a volume of water equal to that of ℭ had been added, and the weights $K+L+(2)+(1)+P+Q+R$, together with the weight given in the first column, being placed in the right-hand pan, the second column gives the corresponding reading of the scale.

D.	C.	F.	E.	gr.	Scale.
				0·60	45·00
				0·50	22·80
				0·50	22·35
				0·44	11·85
				0·48	15·00
				0·48	15·80
				0·50	20·80

				0·50	20·80
				0·40	4·25
				0·40	4·20
16·05	16·05	764·23	16·2	0·50	20·80
				0·50	20·80
				0·50	20·85
				0·50	23·20
				0·50	23·20
				0·50	21·00
				0·50	21·00
				0·50	21·10
				0·50	21·15
				0·48	20·25
				0·48	20·40
				0·46	18·45
				0·46	18·40
				0·48	20·15
				0·48	20·40
16·4	16·4			0·48	20·45
				0·48	20·50
				0·48	20·50
				0·48	20·50
16·22	16·22	764·23	6·2	0·4848	19·86

In a.(t=16·24, b=762·68) counterpoise balances the wire loop in w.+K+L+(2)+(1)+0·4855 gr.+P+Q+R.

A correction must be applied for the change of density of the air displaced by the counterpoise after 𝕮 was taken out of the water. When 𝕮 and the wire loop were in the water, t=15·07, b=763·08. When the wire loop alone was in the water and weights in the right-hand pan, t=16·24, b=762·68. The counterpoise displaces 2·0453 grs. of air (t=16·07, b=76·308), and 2·0429 grs. of air (t=16·24, b=762·68). 14003−2·0453 grs. balance 𝕮 and wire loop in water (t=15·0)+0·0010 grain in air. 14003−2·0429 grains balance the wire loop in water and K+L+(2)+(1)+0·4855 grain+P+Q+R in air (t=16·22, b=762·68).

Hence 𝕮 in water (t=15·0) ≏ K+L+(2)+(1)+0·4830 grain +P+Q+R in air (t=16·24, b=762·68).

Comparison of P+Q+R with the sum of the auxiliary platinum weights G, K, L, M, (16), (4).

March 5, 1845. 100 parts = 0·244 grain.

pt.
P+Q+R≏G+K+L+M+(16)+(4)+0·70
+0·95
+0·82
+1·50

P+Q+R≏G+K+L+M+(16)+(4)+0·00242 grain in air (t=4·88, b=763·86).
P+Q+R=700·003 grains nearly.
G+K+L+M+(16)+(4)+0·00242=699·93747 grains.

The weights of the air displaced by the bronze and platinum weights are 0·10668 gr. and 0·04217 gr. respectively. Hence P+Q+R=700·00197 grains.

The weight of air displaced by the platinum and bronze weights used in weighing 𝕮 in water =0·40355 gr.+0·40361 g. +0·00020 gr.+0·10212 gr.=0·90948 grain.

K+L+(2)+(1)+0·4830 grain	14003·48359	
P+Q+R	700·00197	
Apparent weight of $\mathfrak{C}$ in water	14703·48556	
Weight of air displaced by the weights	0·90948	
Weight of $\mathfrak{C}$ in water	14702·57608	
Weight of $\mathfrak{C}$	15432·32653	
Weight of water at 15·0 displaced by $\mathfrak{C}$	729·75045	2·8631744
Max. density of water: density of water at 15·0		0·0003708
Δ pt. at 15·0 : Δ pt. at 0°		9·9998241
$v\mathfrak{C}$	730·078	2·8633693
$\mathfrak{C}$	15432·32653	4·1884314
$\Delta\mathfrak{C}$	21·13791	1·3250621

The stereometer observations gave $v\mathfrak{A}-v\mathfrak{C}=21·119$. Hence $v\mathfrak{A}=751·197$.

In 1845 the Committee received from Professor SCHUMACHER an account in manuscript of fourteen newer and more accurate determinations of the density of $\mathfrak{S}$, and of several hundred measurements of its linear dimensions, together with a copy of Professor STEINHEIL's paper entitled "Ueber das Bergkrystall-Kilogramm*," containing numerous determinations of the linear dimensions of $\mathfrak{A}$ by himself and GAMBEY in May 1837, in addition to those made by OLUFSEN in 1834. The linear dimensions of $\mathfrak{S}$ and $\mathfrak{A}$ were all measured with the same instrument, which was made by GAMBEY.

Assuming the linear expansion of platinum to be 0·0000085655 for 1° C., $\Delta\mathfrak{S}=21·2037$, by fourteen observations made February 3...24, 1837. (The value $\Delta\mathfrak{S}=21·2047$ given in STEINHEIL's memoir is the mean of four sets of observations of rather doubtful accuracy made in September 1831.) By the comparisons of $\mathfrak{S}$ with $\mathfrak{A}$, April 9...14, 1835, $\mathfrak{S}=999·9993$ grammes. Previous to February 9, 1837, $\mathfrak{S}$ had lost about 1·6 milligramme, in consequence, as was supposed, of being washed with alcohol. In May 1837, therefore, its weight may be taken =999·998 grammes nearly. The volumes of $\mathfrak{S}$ and $\mathfrak{A}$ at 0°, deduced from their linear dimensions, appeared to be 47147 and 48650 cubic millimètres respectively.

$\Delta\mathfrak{S}$	21·2037	1·3264117
$v\mathfrak{S}$ (in cubic millimètres)	47147	4·6734541
$\mathfrak{A}:\mathfrak{S}$		0·0000009
		5·9998667
$v\mathfrak{A}$ (in cubic millimètres)	48650	4·6870828
$\Delta\mathfrak{A}$	20·5487	1·3127839
$\mathfrak{A}$ (in grains nearly)	15432·35	4·1884321
$v\mathfrak{A}$	751·014	2·8756482

Hence the volume of $\mathfrak{A}$ at 0° is equal to the volume of 751·014 grains, or 48·665 grammes of water at its maximum density.

The observations by which this value of $v\mathfrak{A}$ was obtained are so numerous, and made with such extreme care, that I have no hesitation in adopting it in preference to the value 751·197 resulting from the observations made with the stereometer.

The value of $v\mathfrak{A}$ just found depends upon $\Delta\mathfrak{S}$; therefore, in computing $v\mathfrak{A}-v\mathfrak{C}$, for reducing the weighings by which $\mathfrak{C}$ is compared with $\mathfrak{A}$, the density of $\mathfrak{C}$ must

* Abhandlungen der K. Akademie der Wissenschaften zu München, B. 4, S. 165.

be calculated with the same data that were used in calculating the density of $\mathfrak{S}$, subtracting 0·00018 from log β on account of the difference between the force of gravity in Altona and in Paris. The same expansions of platinum and bronze are to be used in calculating the weight of the air displaced by the weights used in weighing $\mathfrak{C}$ in water, as were used in calculating their volumes. 16°·24 C.=61°·23 F., 762·68 mm. =30·027 in., 15°·0 C.=59°·0 F.

Weight of air displaced by the weights used in weighing $\mathfrak{C}$ in water.

30·027	1·47751	30·027	1·47751
61·23	5·61139	61·23	5·61139
vK at 61·23	2·51963	vL at 61·23	2·51969
0·40600	9·60853	0·40606	9·60859
30·027	1·47751	30·027	1·47751
61·23	5·61139	61·23	5·61139
v[(2)+(1)+0·483 gr.] at 61·23	9·21641	v[P+Q+R] at 61·23	1·92286
0·00020	6·30531	0·10275	9·01176

Weight of air displaced by the weights =0·40600 gr.+0·40606 gr.+0·00020 gr.+0·10275 gr.=0·91501 gr.

Calculation of the volume and density of $\mathfrak{C}$.

Apparent weight of $\mathfrak{C}$ in water	14703·48556	
Weight of air displaced by the weights	0·91501	
Weight of $\mathfrak{C}$ in water	14702·57055	
Weight of $\mathfrak{C}$	15432·32653	
Weight of water (t=59·0) displaced	729·75598	2·8631777
Max. density of water : Δ water at 59°·0 F.		0·0003606
Δ pt. at 59°·0 F. : Δ pt. at 38° F.		9·9998326
$v\mathfrak{C}$	730·081	2·8633709
$\mathfrak{C}$	15432·327	4·1884315
$\Delta\mathfrak{C}$	21·13783	1·3250606

$v\mathfrak{A}$=751·014. Hence the volume of $\mathfrak{A}$—the volume of $\mathfrak{C}$, at the freezing-point, is equal to the volume of 20·933 grains of water at its maximum density.

Comparison of $\mathfrak{C}$ with $\mathfrak{A}$.

The zero of the scale of the barometer was 180 mm. below the middle of the weights, consequently 0·02 mm. must be subtracted from the height of the mercury in the barometer. The left-hand pan contained a counterpoise, and $\mathfrak{A}$ and $\mathfrak{C}$ in the detached scale-pans, the weights of which are denoted by X and Y, were suspended alternately from the agate-plane which rested on the right-hand knife-edge.

September 28, 1844. 100 parts=0·49579 grain.

$\mathfrak{C}$+Y.	$\mathfrak{A}$+X.		$\mathfrak{C}$+X.	$\mathfrak{A}$+Y.	
22·71	21·00	C= 17·0	21·13	23·22	C= 17·25
23·36	22·40	F=761·67	23·37	23·29	F=760·3
25·20	23·63	E= 16·55	23·58	23·21	E= 17·0
25·62	23·96		23·67	24·18	
24·77	24·24		24·10	23·66	
24·76	24·77		23·25	23·17	
24·51	24·02		23·50	24·88	
24·46	24·46		24·61	24·17	

24·90	24·04		23·98	23·67	
24·68	25·35		23·49	23·37	
23·82	24·45		24·07	23·73	
24·92	23·93		23·16	22·81	
24·97	24·48		22·50	24·21	
23·87	24·10		22·75	22·90	
24·76	24·71		24·05	24·00	
24·24	24·59		23·46	24·32	
24·62	24·35		23·48	23·22	
24·26	24·09	C= 17·25	23·27	24·22	C= 17·4
23·97	23·91	F=760·2	23·93	24·30	F=759·5
23·50	23·84	E= 17·0	24·48	23·99	E= 17·15
480·32	487·90		472·83	474·52	

$20(\mathfrak{C}+\mathrm{Y})\bumpeq 20(\mathfrak{A}+\mathrm{X})+7{\cdot}58$ parts. $20(\mathfrak{C}+\mathrm{X})\bumpeq 20(\mathfrak{A}+\mathrm{Y})-1{\cdot}69$ part.

$\mathfrak{C}\bumpeq\mathfrak{A}+0{\cdot}0007251$ grain in air ($t=17{\cdot}27$, $b=758{\cdot}79$).

October 1. 100 parts=0·47393 grain.

$\mathfrak{C}$+Y.	$\mathfrak{A}$+X.		$\mathfrak{C}$+X.	$\mathfrak{A}$+Y.	
19·55	19·50	C= 14·9	21·54	21·15	C= 15·8
19·87	19·06	F=767·44	20·39	21·49	F=764·82
20·11	19·44	E= 15·05	20·54	20·42	E= 15·1
20·07	18·71		20·82	22·15	
19·40	19·44		21·49	21·02	
20·62	19·78		22·97	22·20	
19·54	18·78		23·35	22·72	
19·31	18·52		22·19	22·70	
19·55	18·25		23·47	23·06	
19·87	19·53		22·66	22·24	
20·09	18·58		22·70	22·66	
19·57	18·48		22·25	23·01	
19·06	18·90		22·05	22·09	
18·80	18·76		22·71	22·67	
19·00	17·84		22·87	22·93	
18·84	19·09		22·37	22·37	
18·79	17·45		22·51	22·82	
18·50	18·46	C= 15·8	24·03	23·42	C= 15·75
18·46	18·52	F=764·82	24·00	23·27	F=763·85
18·25	18·15	E= 15·1	23·85	24·01	E= 15·2
387·25	375·24		448·76	448·40	

$20(\mathfrak{C}+\mathrm{Y})\bumpeq 20(\mathfrak{A}+\mathrm{X})+12{\cdot}01$ parts. $20(\mathfrak{C}+\mathrm{X})\bumpeq 20(\mathfrak{A}+\mathrm{Y})+0{\cdot}36$ part.

$\mathfrak{C}\bumpeq\mathfrak{A}+0{\cdot}001466$ grain in air ($t=15{\cdot}59$, $b=763{\cdot}79$).

October 3. 100 parts =0·48591 grain.

$\mathfrak{C}$+Y.	$\mathfrak{A}$+X.		$\mathfrak{C}$+X.	$\mathfrak{A}$+Y.	
21·07	19·62	C= 15·65	17·40	18·34	C= 16·4
20·56	20·42	F=761·57	18·57	17·83	F=760·73
20·86	20·27	E= 16·0	18·50	18·59	E= 16·05
20·51	20·49		18·75	19·22	
19·60	19·96		18·89	18·95	
19·98	19·12		18·96	18·80	
20·34	20·01		18·55	18·79	
20·01	18·99		18·51	18·47	
19·61	19·27		18·30	18·46	
19·92	19·60		18·06	17·75	
19·55	19·91		18·01	17·62	
20·03	19·69		18·38	17·40	
19·59	19·98		18·27	18·36	
19·55	19·29		18·12	18·05	
20·11	19·01		18·19	18·06	
19·59	19·37		18·39	18·45	
19·14	19·62		18·54	18·16	

19·64	18·70	C= 16·4	18·37	17·45	C= 16·42
18·99	19·32	F=760·73	18·26	17·99	F=760·5
17·79	19·27	E= 16·05	18·09	17·36	E= 16·0
398·44	391·91		367·11	364·10	

$20(\mathfrak{C}+\mathrm{Y}) \bumpeq 20(\mathfrak{A}+\mathrm{X})+6{\cdot}53$ parts. $20(\mathfrak{C}+\mathrm{X}) \bumpeq 20(\mathfrak{A}+\mathrm{Y})+3{\cdot}01$ parts.

$\mathfrak{C} \bumpeq \mathfrak{A}+0{\cdot}001159$ grain in air ($t=16{\cdot}25$, $b=759{\cdot}31$).

October 8. 100 parts $=0{\cdot}48077$ grain.

$\mathfrak{C}$+Y.	$\mathfrak{A}$+X.		$\mathfrak{C}$+X.	$\mathfrak{A}$+Y.	
21·10	20·59	C= 14·7	18·62	18·69	C= 15·65
21·57	20·95	F=760·9	18·80	18·40	F=759·1
21·97	20·50	E= 14·6	19·30	19·05	E= 14·8
22·37	20·94		18·96	18·33	
22·14	21·97		19·19	19·09	
22·35	21·87		18·95	18·51	
21·61	20·54		18·81	18·62	
21·10	20·90		20·24	19·68	
20·97	20·42		19·06	18·75	
21·55	21·20		19·64	18·64	
21·19	21·05		19·37	18·61	
21·82	20·59		19·40	18·10	
20·57	20·54		18·96	19·03	
21·74	20·54		19·30	18·50	
20·84	20·12		19·10	18·95	
20·56	19·96		19·59	18·54	
21·02	20·90		18·80	18·60	
20·54	19·57	C= 15·65	18·86	18·84	C= 15·35
20·35	19·60	F=759·1	19·47	18·87	F=756·6
19·52	20·07	E= 14·8	19·10	19·47	E= 14·5
424·88	412·82		383·52	375·27	

$20(\mathfrak{C}+\mathrm{Y}) \bumpeq 20(\mathfrak{A}+\mathrm{X})+12{\cdot}06$ parts. $20(\mathfrak{C}+\mathrm{X}) \bumpeq 20(\mathfrak{A}+\mathrm{Y})+8{\cdot}25$ parts.

$\mathfrak{C} \bumpeq \mathfrak{A}+0{\cdot}002441$ grain in air ($t=15{\cdot}37$, $b=757{\cdot}80$).

October 9. 100 parts $=0{\cdot}48757$ grain.

$\mathfrak{C}$+Y.	$\mathfrak{A}$+X.		$\mathfrak{C}$+X.	$\mathfrak{A}$+Y.	
20·97	20·67	C= 14·35	19·26	19·24	C= 15·45
21·85	21·64	F=746·68	19·48	19·41	F=745·15
22·06	22·22	E= 14·1	19·46	19·47	E= 14·6
22·19	22·24		19·56	19·40	
22·05	22·58		19·24	19·75	
22·35	22·22		19·47	20·25	
22·57	22·51		19·34	19·37	
22·22	22·17		19·88	19·49	
21·55	22·22		19·65	19·58	
22·09	21·64		19·50	19·91	
21·81	21·53		19·19	19·52	
22·04	21·42		19·62	19·49	
21·79	21·57		19·70	19·89	
21·34	20·87		19·57	19·57	
21·05	21·11		19·67	19·54	
21·00	20·66		20·24	19·46	
20·75	20·51		19·66	19·50	
20·31	19·86	C= 15·45	20·12	19·60	C= 15·47
20·05	19·65	F=745·15	19·42	19·16	F=743·9
20·29	19·84	E= 14·6	19·39	19·45	E= 15·05
430·33	427·13		391·42	391·05	

$20(\mathfrak{C}+\mathrm{Y}) \bumpeq 20(\mathfrak{A}+\mathrm{X})+3{\cdot}2$ parts. $20(\mathfrak{C}+\mathrm{X}) \bumpeq 20(\mathfrak{A}+\mathrm{Y})+0{\cdot}37$ part.

$\mathfrak{C} \bumpeq \mathfrak{A}+0{\cdot}000435$ grain in air ($t=15{\cdot}21$, $b=743{\cdot}89$).

	gr.	$t.$	$b.$
September 28.	$\mathfrak{C} \bumpeq \mathfrak{A}+0{\cdot}000725$	17·27	758·79
October 1.	$\mathfrak{C} \bumpeq \mathfrak{A}+0{\cdot}001466$	15·59	763·79
October 3.	$\mathfrak{C} \bumpeq \mathfrak{A}+0{\cdot}001159$	16·25	759·31
October 8.	$\mathfrak{C} \bumpeq \mathfrak{A}+0{\cdot}002441$	15·37	757·8
October 9.	$\mathfrak{C} \bumpeq \mathfrak{A}+0{\cdot}000435$	15·21	743·89

If $v\mathfrak{A}-v\mathfrak{C}=20{\cdot}933$, as given by the observations of SCHUMACHER combined with those of OLUFSEN, STEINHEIL and GAMBEY, the true weight of $\mathfrak{A}$ compared with $\mathfrak{C}$ will be larger than its apparent weight by the weight of air displaced by a mass of platinum the volume of which at 0° is equal to the volume of 20·933 grains of water at its maximum density.

Weight of air displaced by $\mathfrak{A}$ — weight of air displaced by $\mathfrak{C}$.

September 28.	0·025316
October 1.	0·025643
October 3.	0·025430
October 8.	0·025462
October 9.	0·025007

Therefore

	gr.
September 28.	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02459$
October 1.	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02417$
October 3.	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02427$
October 8.	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02302$
October 9.	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02457$
Mean	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02412$ grain.

If $v\mathfrak{A}-v\mathfrak{C}=21{\cdot}119$, as given by the stereometer observations, the differences of the weights of air displaced by $\mathfrak{A}$ and $\mathfrak{C}$ will be

September 28.	0·025541
October 1.	0·025871
October 3.	0·025655
October 8.	0·025688
October 9.	0·025230

Therefore

	gr.
September 28.	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02482$
October 1.	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02439$
October 3.	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02450$
October 8.	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02325$
October 9.	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02480$
Mean	$\mathfrak{C}=\mathfrak{A}-0{\cdot}02435$ grain.

Comparison of $\mathfrak{C}$ with each of the weights I+K+A, I+L+B, I+M+Γ, I+N+Δ.

The use of auxiliary weights corresponding to the divisors obtained in the process of expressing in a continued fraction the ratio of one to the other of the two weights to be compared, answered so well in deducing the pound of 7000 grains from the pound troy of 5760 grains, that I resolved to employ it in comparing the kilogramme with the pound. The weight of $\mathfrak{C}$ is nearly 15432·325 grains, and that of each of the weights I, K, L, M, N, the new standard lb. and its four copies respectively, is 7000 grains. Then, each of the weights A, B, Γ, Δ being 1432·324 grains, Z being 1270·707 grains, and Θ being 161·629 grains, the following comparisons became possible:—

$\mathfrak{C}$ with each of the weights I+K+A, I+L+B, I+M+Γ, I+N+Δ; each of the weights K, L, M, N with I; I with A+B+Γ+Δ+Z; each of the weights A, B, Γ, Δ with Z+Θ. The value of Θ in terms of I was known from comparisons made for a different purpose. The weights A, Γ, Δ consisted of the auxiliary platinum weights A, C, D, used in forming the lb. of 7000 grains, with the addition to each of a three rouble platinum coin and a little more than 33 grains of platinum wire weights; B of the auxiliary platinum weight B together with the platinum weight V+0·03 gr. which had been used with K+L in weighing the kilogramme des Archives. Z consisted of the platinum weights F, G and 30·85 grains of platinum wire weights. Θ was made up of the platinum weights L and M of nearly 80 grains each, one of the 1-grain weights of the two ROBINSON's balances, used by turns, and one of the ten weights Q of 0·64509 grains each, also used by turns.

The zero of the scale of the barometer was 305 mm. above the middle of the weights, 0·03 mm. must therefore be added to the observed value of F.

Let $\mathfrak{K}$=I+K+A, $\mathfrak{L}$=I+L+B, $\mathfrak{M}$=I+M+Γ, $\mathfrak{N}$=I+N+Δ.

February 27, 1846. 100 parts=0·65509 gr.

$\mathfrak{K}$+Y, $\mathfrak{C}$+X.	$\mathfrak{C}$+X, $\mathfrak{K}$+Y.	
19·26	18·80	
19·05	19·04	
19·00	18·50	
18·01	17·96	
18·00	18·15	
18·82	17·02	
17·57	17·21	
17·67	17·02	D= 18·9
17·17	16·60	C= 18·9
16·82	16·54	F=755·9
17·30	16·25	E= 19·0
17·07	16·69	
17·11	16·69	
17·26	16·86	
17·51	17·46	
17·72	17·40	
17·55	17·34	
302·89	295·53	

$34(\mathfrak{C}+X) \bumpeq 30(\mathfrak{K}+Y)+7·36$ parts.

March 2. 100 parts=0·67362 grain.

$\mathfrak{K}$+X, $\mathfrak{C}$+Y.	$\mathfrak{C}$+Y, $\mathfrak{K}$+X.	
16·46	15·25	
16·94	16·84	
17·22	17·29	
16·39	17·15	
15·89	15·80	
16·15	16·35	
16·77	16·26	
16·04	16·60	D= 19·6
16·24	15·96	C= 19·6
15·91	15·75	F=763·2
15·99	14·76	E= 22·0
15·97	14·66	
15·65	14·56	
15·29	14·32	
16·20	14·79	
15·69	14·75	
15·56	13·95	
274·36	265·04	

$34(\mathfrak{C}+Y) \bumpeq 34(\mathfrak{K}+X)+9·32$ parts.

$68\mathfrak{C} \bumpeq 68\mathfrak{K}+0·11100$ grain in air (t=19·3, b=757·52).

January 31. 100 parts=0·56899 grain.

$\mathfrak{L}$+X, $\mathfrak{C}$+Y.	$\mathfrak{C}$+Y, $\mathfrak{L}$+X.	
18·90	17·64	
18·82	18·61	
18·79	18·14	
19·00	17·72	D= 11
18·51	17·85	C= 11·05
18·81	17·82	F=760·4
17·84	16·92	E= 17·5
18·36	17·52	
18·72	18·17	
18·64	17·30	
186·39	177·69	

$20(\mathfrak{C}+Y) \bumpeq 20(\mathfrak{L}+X)+8·7$ parts.

February 2. 100 parts=0·61069 grain.

$\mathfrak{L}$+Y, $\mathfrak{C}$+X.	$\mathfrak{C}$+X, $\mathfrak{L}$+Y.	
18·01	17·70	
16·51	16·84	
16·51	17·06	
16·04	16·07	D= 15·7
15·75	16·22	C= 15·75
15·74	16·02	F=759·0
15·46	16·15	E= 16·0
16·06	16·64	
16·12	16·07	
15·97	16·54	
162·17	165·31	

$20(\mathfrak{C}+X) \bumpeq 20(\mathfrak{L}+Y)-3·14$ parts.

$40\mathfrak{C} \bumpeq 40\mathfrak{L}+0·03032$ grain in air (t=13·37, b=758·12).

March 24. 100 parts=0·57337 grain.

$\mathfrak{L}$+X, $\mathfrak{C}$+Y.	$\mathfrak{C}$+Y, $\mathfrak{L}$+X.		$\mathfrak{L}$+Y, $\mathfrak{C}$+X.	$\mathfrak{C}$+X, $\mathfrak{L}$+Y.	
17·57	17·45		16·56	15·92	
16·34	15·97		17·35	16·30	
16·02	15·54		16·62	16·54	
16·54	15·04	D= 9·55	16·20	16·07	D= 9·55
16·60	16·34	C= 9·7	16·22	16·12	C= 9·7
16·02	15·05	F=746·3	15·67	15·46	F=746·3
15·92	15·45	E= 11·6	16·45	16·05	E= 11·6
16·04	15·25		16·35	15·40	
15·50	15·25		16·10	16·36	
15·49	15·05		16·11	16·24	
162·04	156·39		163·63	160·46	

$20(\mathfrak{C}+Y) \bumpeq 20(\mathfrak{L}+X)+5·65$ parts. $20(\mathfrak{C}+X) \bumpeq 20(\mathfrak{L}+Y)+3·17$ parts.

$40\mathfrak{C} \bumpeq 40\mathfrak{L}+0·05057$ grain in air (t=9·62, b=745·38).

February 3. 100 parts=0·64809 grain. February 4. 100 parts=0·64288 grain.

$\mathfrak{M}$+Y, $\mathfrak{C}$+X.	$\mathfrak{C}$+X, $\mathfrak{M}$+Y.		$\mathfrak{M}$+X, $\mathfrak{C}$+Y.	$\mathfrak{C}$+Y, $\mathfrak{M}$+X.	
18·67	18·06		18·92	17·37	
18·29	18·17		18·37	17·39	
17·90	17·70		16·84	17·45	
17·64	16·97	D= 16·05	18·57	17·26	D= 16·35
17·61	17·26	C= 16·15	17·87	16·71	C= 16·4
17·00	16·16	F=759·1	17·24	15·82	F=765·2
16·35	16·24	E= 16·6	17·01	15·00	E= 17·1
16·97	16·19		16·87	14·45	
16·61	16·37		17·40	14·71	
16·72	16·25		16·65	14·37	
173·76	169·37		175·74	160·53	

$20(\mathfrak{C}+X) \bumpeq 20(\mathfrak{M}+Y)+4·39$ parts. $20(\mathfrak{C}+Y) \bumpeq 20(\mathfrak{M}+X)+15·21$ parts.

$40\mathfrak{C} \bumpeq 40\mathfrak{M}+0·12623$ grain in air (t=16·24, b=760·56).

March 25. 100 parts=0·57820 grain.

$\mathfrak{M}$+Y, $\mathfrak{C}$+X.	$\mathfrak{C}$+X, $\mathfrak{M}$+Y.		$\mathfrak{M}$+X, $\mathfrak{C}$+Y.	$\mathfrak{C}$+Y, $\mathfrak{M}$+X.	
19·22	17·11		16·56	15·82	
19·01	17·34		16·62	14·89	
18·90	17·52		17·37	15·62	
18·80	17·10	D= 9·5	17·17	15·16	D= 9·5
18·55	17·72	C= 9·5	17·67	14·71	C= 9·5
18·30	16·57	F=747·3	17·07	15·27	F=747·3
17·84	17·02	E= 15·7	17·54	15·52	E= 15·7
17·57	16·37		17·06	15·99	
17·85	16·02		17·66	14·77	
17·62	15·95		17·46	14·95	
183·66	168·72		172·18	152·70	

$20(\mathfrak{C}+X) \bumpeq 20(\mathfrak{M}+Y)+14·94$ parts. $20(\mathfrak{C}+Y) \bumpeq 20(\mathfrak{M}+X)+19·48$ parts.

$40\mathfrak{C} \bumpeq 40\mathfrak{M}+0·19902$ grain in air (t=9·55, b=745·88).

February 5. 100 parts=0·63391 grain.

$\mathfrak{N}$+X, $\mathfrak{E}$+Y.	$\mathfrak{E}$+Y, $\mathfrak{N}$+X.	
17·50	16·19	
16·92	15·71	
17·09	17·00	
16·70	16·66	D= 16·45
16·80	15·76	C= 16·45
15·79	15·32	F=759·65
15·97	14·71	E= 16·9
16·00	14·74	
15·69	14·49	
15·80	14·89	
164·26	155·47	

$20(\mathfrak{E}+\mathrm{Y}) \bumpeq 20(\mathfrak{N}+\mathrm{X})+8{\cdot}79$ parts.

February 6. 100 parts =0·64246 grain.

$\mathfrak{N}$+Y, $\mathfrak{E}$+X.	$\mathfrak{E}$+X, $\mathfrak{N}$+Y.	
15·74	16·17	
14·97	15·79	
13·44	13·62	
18·19	17·74	D= 17·5
16·87	17·70	C= 17·5
16·52	17·07	F=763·8
16·02	17·20	E= 18·6
16·54	17·69	
17·79	18·37	
17·55	18·57	
163·63	169·92	

$20(\mathfrak{E}+\mathrm{X}) \bumpeq 20(\mathfrak{N}+\mathrm{Y})-6{\cdot}29$ parts.

$40\mathfrak{E} \bumpeq 40\mathfrak{N}+0{\cdot}01531$ grain in air ($t=17{\cdot}0$, $b=760{\cdot}02$).

March 26. 100 parts=0·60994 grain.

$\mathfrak{N}$+X, $\mathfrak{E}$+Y.	$\mathfrak{E}$+Y, $\mathfrak{N}$+X.	
18·02	17·90	
18·00	17·84	
17·09	18·06	
18·37	17·66	D= 15·6
17·94	18·45	C= 15·65
17·41	17·81	F=752·05
17·72	18·19	E= 17·4
17·21	17·46	
16·84	17·91	
17·29	17·51	
16·79	16·65	
192·68	195·44	

$22(\mathfrak{E}+\mathrm{Y}) \bumpeq 22(\mathfrak{N}+\mathrm{X})-2{\cdot}76$ parts.

$\mathfrak{N}$+Y, $\mathfrak{E}$+X.	$\mathfrak{E}$+X, $\mathfrak{N}$+Y.	
14·89	15·54	
15·94	15·45	
16·14	15·91	
16·26	15·75	D= 16·4
15·54	15·97	C= 16·4
15·46	15·12	F=752·05
15·61	15·35	E= 17·4
15·56	15·81	
15·01	15·24	
15·82	15·66	
15·25	15·40	
171·48	171·20	

$22(\mathfrak{E}+\mathrm{X}) \bumpeq 22(\mathfrak{N}+\mathrm{Y})+0{\cdot}28$ part.

$44\mathfrak{E} \bumpeq 44\mathfrak{N}-0{\cdot}01513$ grain in air ($t=16{\cdot}03$, $b=750{\cdot}42$).

gr.	t.	b.	No. of observations.
$\mathfrak{E} \bumpeq \mathfrak{K}$ +0·00163	19·3	757·52	68
$\mathfrak{E} \bumpeq \mathfrak{L}$ +0·00101	11·5	756·25	80
$\mathfrak{E} \bumpeq \mathfrak{M}$ +0·00407	12·9	753·22	80
$\mathfrak{E} \bumpeq \mathfrak{N}$ +0·00000	16·51	754·99	84

$v\mathrm{I}=330{\cdot}856$, $v\mathrm{K}=330{\cdot}701$, $v\mathrm{L}=330{\cdot}750$, $v\mathrm{M}=330{\cdot}790$, $v\mathrm{N}=330{\cdot}881$.

For the weights A, B, Γ, Δ of 1432·324 grains each, $\log \Delta=1{\cdot}325642$. Therefore the volume of either of them at 0° will be 67·6704. Hence $v\mathfrak{K}=729{\cdot}227$, $v\mathfrak{L}=729{\cdot}276$, $v\mathfrak{M}=729{\cdot}316$, $v\mathfrak{N}=729{\cdot}407$. $v\mathfrak{E}=730{\cdot}078$, and, therefore, $v\mathfrak{E}-v\mathfrak{K}=0{\cdot}851$, $v\mathfrak{E}-v\mathfrak{L}=0{\cdot}802$, $v\mathfrak{E}-v\mathfrak{M}=0{\cdot}762$, $v\mathfrak{E}-v\mathfrak{N}=0{\cdot}671$.

The true weight of $\mathfrak{E}$ compared with $\mathfrak{K}$, $\mathfrak{L}$, $\mathfrak{M}$, $\mathfrak{N}$ will be larger than its apparent weights, by the weights of air displaced by masses of platinum the volumes of which at 0° are respectively equal to the volumes of 0·851 gr., 0·802 gr., 0·762 gr., 0·671 gr. of water at its maximum density. These are 0·00102 gr., 0·00099 gr., 0·00093 gr., 0·00081 gr. respectively.

Hence

$$
\begin{aligned}
&\qquad\qquad\quad \text{gr.}\\
\mathfrak{C} &= \mathrm{I}+\mathrm{K}+\mathrm{A}+0{\cdot}00265\\
\mathfrak{C} &= \mathrm{I}+\mathrm{L}+\mathrm{B}+0{\cdot}00200\\
\mathfrak{C} &= \mathrm{I}+\mathrm{M}+\Gamma+0{\cdot}00500\\
\mathfrak{C} &= \mathrm{I}+\mathrm{N}+\Delta+0{\cdot}00081
\end{aligned}
$$

March 29. 100 parts=0·24964 grain.

H=A+B+Γ+Δ+Z.

H+Y, I+X.	I+X, H+Y.		H+X, I+Y.	I+Y, H+X.	
26·62	26·31		17·00	17·99	
26·76	26·52		17·57	17·90	
21·99	22·50		17·14	17·75	
21·11	21·99		16·61	18·75	
20·12	21·90	D= 14·8	16·96	17·97	D= 15·1
20·80	21·26	C= 14·8	16·57	17·52	C= 15·2
19·46	19·44	F=766·9	17·57	18·74	F=766·9
19·65	20·74	E= 16·4	17·46	18·80	E= 16·4
19·77	20·24		17·00	17·42	
19·49	20·06		17·46	17·60	
18·94	19·42		17·32	18·50	
18·32	19·49		17·05	18·54	
253·03	259·87		205·71	217·48	

24(I+X)≏24(H+Y)−6·84 parts. 24(I+Y)≏24(H+X)−11·77 parts.

I≏H−0·00097 grain in air (t=15·0, b=765·35).

Log ΔH=1·325642, therefore vH=330·717, vI=330·856. Hence vI−vH=0·139. The true weight of I is greater than its apparent weight when compared with H, by 0·00017 grain, the weight of air displaced by a mass of platinum the volume of which at 0° is equal to the volume of 0·139 gr. of water of maximum density. Hence

$$\mathrm{I}=\mathrm{A}+\mathrm{B}+\Gamma+\Delta+\mathrm{Z}-0{\cdot}00080 \text{ grain.}$$

March 27. 100 parts=0·25617 grain.

Z+Θ+X, A+0·01 gr.+Y.	A+0·01 gr.+Y, Z+Θ+X.	Z+Θ+Y, A+0·01 gr.+X.	A+0·01 gr.+X, Z+Θ+Y.
16·49	18·69	15·44	17·70
15·92	18·27	15·40	18·24
16·37	18·24	15·10	17·61
15·69	18·15	15·17	17·89
15·51	18·57	15·34	16·86
79·98	91·92	76·45	88·30

10(A+0·01 gr.+Y)=10(Z+Θ+X)−11·94 parts. 10(A+0·01 gr.+X)=10(Z+Θ+Y)−11·85 parts.

A=Z+Θ−0·01305 grain.

Z+Θ+Y, B+0·01 gr.+X.	B+0·01 gr.+X, Z+Θ+Y.	Z+Θ+X, B+0·01 gr.+Y.	B+0·01 gr.+Y, Z+Θ+X.
18·77	19·85	16·64	18·47
18·35	20·47	16·82	18·70
17·47	19·92	16·81	18·40
17·25	20·14	16·60	18·35
17·37	19·79	16·35	17·85
89·21	100·17	83·22	91·77

10(B+0·01 gr.+X)=10(Z+Θ+Y)−10·96 parts. 10(B+0·01 gr.+Y)=10(Z+Θ+X).

B=Z+Θ−0·01250 grain.

Z+Θ+Y, Γ+0·02 gr.+X.	Γ+0·01 gr.+X, Z+Θ+Y.	Z+Θ+X, Γ+0·02 gr.+Y.	Γ+0·01 gr.+Y, Z+Θ+X.
18·27	18·47	17·92	18·76
18·67	18·86	18·27	18·45
17·44	18·62	21·92	23·45
18·22	18·41	22·20	23·57
17·90	18·35	21·92	22·99
90·50	92·71	102·23	107·22

$10(\Gamma+0·015\text{ gr.}+X)=10(Z+\Theta+Y)-2·21$ parts. $10(\Gamma+0·015\text{ gr.}+Y)=10(Z+\Theta+X)-4·99$ parts.

$$\Gamma=Z+\Theta-0·01592 \text{ grain.}$$

Z+Θ+X, Δ+0·01 gr.+Y.	Δ+0·01 gr.+Y, Z+Θ+X.	Z+Θ+Y, Δ+0·01 gr.+X.	Δ+0·01 gr.+X, Z+Θ+Y.
19·51	20·30	18·57	19·11
20·04	20·22	17·76	18·80
20·05	19·57	18·61	18·75
19·40	19·15	18·64	18·00
18·81	19·14	17·69	18·21
97·81	98·38	91·27	92·87

$10(\Delta+0·01\text{ gr.}+Y)=10(Z+\Theta+X)-0·57$ part. $10(\Delta+0·01\text{ gr.}+X)=10(Z+\Theta+Y)-1·60$ part.

$$\Delta=Z+\Theta-0·01028 \text{ grain.}$$

$$\begin{aligned} & \qquad\quad \text{gr.} \\ L &= 79·99112 \\ M &= 79·99261 \\ J &= 0·99996 \\ Q &= 0·64509 \\ \Theta=L+M+J+Q &= 161·62878 \end{aligned}$$

gr.	gr.	gr.
$\mathfrak{C}=I+K+A+0·00265$	$K=I+0·00051$	$A=Z+\Theta-0·01305$
$\mathfrak{C}=I+L+B+0·00200$	$L=I-0·00089$	$B=Z+\Theta-0·01250$
$\mathfrak{C}=I+M+\Gamma+0·00500$	$M=I-0·00178$	$\Gamma=Z+\Theta-0·01592$
$\mathfrak{C}=I+N+\Delta+0·00081$	$N=I-0·00316$	$\Delta=Z+\Theta-0·01028$

Whence

$$\begin{aligned} & \qquad\qquad \text{gr.} \\ \mathfrak{C} &= 2I+Z+\Theta-0·00989 \\ \mathfrak{C} &= 2I+Z+\Theta-0·01139 \\ \mathfrak{C} &= 2I+Z+\Theta-0·01270 \\ \mathfrak{C} &= 2I+Z+\Theta-0·01263 \\ \text{Mean......}\ \mathfrak{C} &= 2I+Z+\Theta-0·01165 \end{aligned}$$

$$\begin{aligned} & \qquad\qquad\qquad \text{gr.} \\ I &= A+B+\Gamma+\Delta+Z-0·00080 \\ A+B+\Gamma+\Delta &= 4Z+4\Theta \quad -0·05175 \\ I &= 5Z+4\Theta \quad -0·05255 \\ 5(Z+\Theta) &= I+\Theta \quad +0·05255 \end{aligned}$$

$I=7000·00000$ grains of the new standard, $\Theta=161·62878$ grains. Therefore $5(Z+\Theta)=7161·68133$ grs., $Z+\Theta=1432·33627$ grs. Therefore $\mathfrak{C}=15432·32462$ grs.

According to the observations of SCHUMACHER, OLUFSEN, STEINHEIL and GAMBEY, $v\mathfrak{A}-v\mathfrak{C}=20·933$, whence $\mathfrak{A}=\mathfrak{C}+0·02412$ grain. Therefore the kilogramme des Archives $=15432·34874$ grains, of which I, the new standard pound, contains $7000·00000$. If we adopt for $v\mathfrak{A}-v\mathfrak{C}$ the value $21·119$ given by the observations with the stereometer, $\mathfrak{A}=\mathfrak{C}+0·02435$ grain, and therefore $\mathfrak{A}=15432·34897$ grains. Of these two values of $\mathfrak{A}$ the former is probably the more accurate.

Comparison of $\mathfrak{A}$ *with* PC No. 1+PC No. 2+B+V+0·03 grain.

Let K, L denote PC No. 1, PC No. 2 respectively; $\mathfrak{P}$=K+L+B+V+0·03 grain. The zero of the scale of the barometer was 180 millimètres below the middle of the weights, consequently 0·02 mm. must be subtracted from the height of the mercury in the barometer.

September 16, 1844. 100 parts=0·565 grain.

$\mathfrak{A}$+X.	$\mathfrak{P}$+Y.		$\mathfrak{A}$+Y.	$\mathfrak{P}$+X.	
22·62	22·25	C= 20·4	20·46	20·69	C= 20·55
22·52	22·16	F=762·1	20·65	20·85	F=761·7
21·90	22·40	E= 20·1	20·51	21·16	E= 20·4
22·43	22·33		21·05	21·30	
22·65	22·53		20·71	20·71	
22·37	21·97		21·54	21·50	
21·48	21·81	C= 20·55	22·49	21·91	C= 20·55
21·23	21·19	F=761·7	21·80	21·55	F=761·35
21·25	20·86	E= 20·4	21·86	21·93	E= 20·55
198·45	197·50		191·07	191·60	

9($\mathfrak{A}$+X)$\bumpeq$9($\mathfrak{P}$+Y)+0·95 part. 9($\mathfrak{A}$+Y)$\bumpeq$9($\mathfrak{P}$+X)−0·53 part.

18$\mathfrak{A}\bumpeq$18$\mathfrak{P}$+0·00237 grain in air (t=20·61, b=759·64).

September 17. 100 parts=0·5746 grain.

$\mathfrak{A}$+X.	$\mathfrak{P}$+Y.		$\mathfrak{A}$+Y.	$\mathfrak{P}$+X.	
20·02	19·91	C= 19·8	19·05	19·37	C= 20·76
19·70	20·00	F=758·53	19·36	19·37	F=758·2
19·70	20·55	E= 20·4	19·51	19·24	E= 20·9
19·31	20·22		18·49	19·90	
19·57	20·32		19·45	19·56	
19·29	20·01		18·44	19·91	
20·09	20·47		19·39	19·37	
20·32	20·21	C= 20·76	18·82	19·80	C= 21
19·71	20·01	F=758·2	19·40	19·71	F=758·04
20·09	19·76	E= 20·9	19·03	19·15	E= 21
179·80	201·46		199·94	195·38	

10($\mathfrak{A}$+X)$\bumpeq$10($\mathfrak{P}$+Y)−3·66 parts. 10($\mathfrak{A}$+Y)$\bumpeq$10($\mathfrak{P}$+X)−4·44 parts.

20$\mathfrak{A}\bumpeq$20$\mathfrak{P}$−0·046543 grain in air (t=20·68, b=756·12).

September 19. 100 parts=0·5322 grain.

$\mathfrak{A}$+X.	$\mathfrak{P}$+Y.		$\mathfrak{A}$+Y.	$\mathfrak{P}$+X.	
25·73	25·59	C= 18·5	14·25	14·19	C= 18·8
25·32	26·00	F=758·23	18·94	19·61	F=758·12
24·73	26·46	E= 18	19·50	19·87	E= 17·6
24·52	25·41		19·42	20·00	
25·77	26·16		19·50	19·72	
25·66	25·44		19·89	20·05	
25·15	25·65		19·42	20·02	
25·93	25·59		20·10	20·19	
25·61	25·65	C= 18·8	19·49	20·04	C= 18·85
24·79	25·59	F=758·12	19·80	19·96	F=758·2
25·20	25·31	E= 17·6	19·23	19·22	E= 17·9
278·41	282·85		209·54	212·87	

11($\mathfrak{A}$+X)$\bumpeq$11($\mathfrak{P}$+Y)−4·44 parts. 11($\mathfrak{A}$+Y)$\bumpeq$11($\mathfrak{P}$+X)−3·33 parts.

22$\mathfrak{A}\bumpeq$22$\mathfrak{P}$−0·041356 grain in air (t=18·8, b=756·39).

By a mean of the 60 comparisons,

$$\mathfrak{A} \doteq \mathfrak{P} - 0{\cdot}00143 \text{ grain in air } (t=19{\cdot}97,\ b=757{\cdot}28).$$

The value of these observations is considerably diminished by the fact that B contained a small cavity filled with some hygroscopic substance, which renders its weight slightly variable.

The following comparisons of $V+0{\cdot}03$ grain with the sum of the auxiliary weights M, N, (32) were made with ROBINSON's $10\frac{1}{2}$-inch balance.

$$\Xi = M+N+(32)+0{\cdot}45 \text{ grain.}$$

March 29, 1845. 100 parts=0·290 grain.

$V+0{\cdot}03$ gr., Ξ.	Ξ, $V+0{\cdot}03$ gr.
4·27	3·05
4·42	3·05
4·57	3·20
4·55	3·25
17·81	12·55

$$8V+0{\cdot}03 \text{ gr.} = 8\Xi + 5{\cdot}26 \text{ parts.}$$

$$V+0{\cdot}03 \text{ gr.} = M+N+(32)+0{\cdot}45191 \text{ grain} = 192{\cdot}43590 \text{ grains.}$$

$v(B+V+0{\cdot}03 \text{ gr.})=67{\cdot}670$, $vK=330{\cdot}701$, $vL=330{\cdot}750$. Hence the volume of $\mathfrak{P}$ at 0° is equal to the volume of 729·121 grains of water of maximum density. But $v\mathfrak{A}=751{\cdot}014$, hence $v\mathfrak{A}-v\mathfrak{P}=$ volume of 21·893 grains of water at its maximum density = volume of 0·02617 grain of air ($t=19{\cdot}97$, $b=757{\cdot}28$). Therefore $\mathfrak{A}=K+L+B+V+0{\cdot}03 \text{ gr.}+0{\cdot}02474$ gr. $K=7000{\cdot}00051$ grains, $L=6999{\cdot}99911$ grains, of which I contains 7000·00000. The best determination of the weight of $B+V+0{\cdot}03$ gr. is that which was obtained in March 1846, in the process of comparing $\mathfrak{C}$ with I. This gave $B+V+0{\cdot}03 \text{ gr.}=Z+\Theta-0{\cdot}01250$ gr., $Z+\Theta=1432{\cdot}33627$ grs. Therefore $B+V+0{\cdot}03 \text{ gr.}=1432{\cdot}32377$ grains. Hence $\mathfrak{A}=15432{\cdot}34813$ grains.

A similar comparison made in November and December 1845, but afterwards rejected on account of the quantity of hygroscopic matter contained in the auxiliary weights A, C, F, gave,—$B+V+0{\cdot}03 \text{gr.}=\Phi+\Theta+0{\cdot}01055$gr., $\Phi+\Theta=1432{\cdot}31313$grs. Therefore $B+V+0{\cdot}03 \text{ gr.}=1432{\cdot}32368$ grains. Whence $\mathfrak{A}=15432{\cdot}34804$ grains.

On the 29th of March, 1845, $V+0{\cdot}03$ grain was compared eight times with $M+N+(32)+0{\cdot}45$ grain, for the purpose of obtaining an approximate value of $B+V+0{\cdot}03$ gr., to be used in the reduction of the weighings by which $\mathfrak{A}$ was compared with $K+L+B+V+0{\cdot}03$ gr. These comparisons gave $V+0{\cdot}03 \text{ gr.}=M+N+(32)+0{\cdot}45191$ grain. Whence $V+0{\cdot}03 \text{ gr.}=192{\cdot}43590$ grains. By numerous comparisons of the auxiliary weights with each other and with T after its final reduction, in January and February 1845, $B=1239{\cdot}88650$ grains, of which T contains 5759·47141 and I contains 7000·00093. Hence $B=1239{\cdot}88634$ grains, of which T contains 5759·47067 and I contains 7000·00000. Therefore $B+V+0{\cdot}03$ gr. $=1432{\cdot}32224$ grains, and $\mathfrak{A}=15432{\cdot}34660$ grains.

By a mean of 286 comparisons of T with Sp in January ... August, 1845, T $\stackrel{\frown}{=}$ Sp+0·00073 grain in air (t=13·74, b=758·91). vT−vSp=volume of 0·26 grain of water = volume of 0·00032 grain of air. Therefore T=Sp+0·00105 grain. If I=7000·00000 grains, T=5759·47067 grains, and therefore Sp=5759·46962 grains. According to the account in manuscript of Professor SCHUMACHER's comparisons of his kilogramme $\mathfrak{S}$ with his troy pound Sp, by the observations of December 1831 and October 1835, $\mathfrak{S}$=15433·77226 grains, of which Sp contains 5760. After these comparisons $\mathfrak{S}$ lost a portion of its substance, and became 0·02557 grain lighter; for by a mean of forty comparisons by PETERSEN in March 1837, and thirty-six comparisons by Captain v. NEHUS in February and March 1837, $\mathfrak{S}$=15433·74669 grains, of which Sp contains 5760. Hence, in 1837, $\mathfrak{S}$=15432·32555 grains, of which I contains 7000·00000. In STEINHEIL's Memoir, "Ueber das Bergkrystall-Kilogramm," (p. 168), the following passage occurs: "Conferenzrath SCHUMACHER erhielt eine Platina-Copie des Archivkilogrammes, welche, zufolge sorgfältiger Abwägungen des Professor OLUFSEN aus Koppenhagen im April 1835 (siehe Jahrbuch für 1856 von SCHUMACHER, p. 250) leichter war als das Archivkilogramm um 0·41 Milligrammen. Aus Unvorsichtigkeit eines Mechanikus wurde diese Copie später abgewaschen und hatte dadurch an Gewicht verloren. Um zu bestimmen, wie viel dieser Verlust betrug, wiederholte ich im Jahr 1837 die Vergleichung der Schumacher'schen Copie mit dem Normalkilogramm der Archive und es ergab sich, dass es nun 1·59 Milligrammen leichter war, also 1·09 Milligrammen an Gewicht durch die Operation des Abwaschens verloren hatte." The most probable explanation that can be offered of the discrepancy contained in these words, is that by an error of the press, 1·59 was substituted for 1·50. According to KUPFFER, however (Travaux de la Commission pour fixer les mesures et les poids de l'Empire de Russie, t. ii. p. 413), the value of $\mathfrak{A}-\mathfrak{S}$ is 1·592 milligramme. Supposing that in 1837, $\mathfrak{A}-\mathfrak{S}$=1·50 milligramme =0·02315 gr., then $\mathfrak{A}$=15432·34870 grs. If $\mathfrak{A}-\mathfrak{S}$=1·59 milligramme =0·02454 gr., $\mathfrak{A}$=15432·35009 grains.

By the observations of 1831 and 1835, $\mathfrak{S}$=15432·35112 grains, of which I contains 7000·00000. If in April 1835, $\mathfrak{A}-\mathfrak{S}$=0·41 milligramme =0·00633 grain, $\mathfrak{A}$=15432·35745 grains. This difference of 0·00875 grain in the weight of $\mathfrak{A}$ between 1835 and 1837, is difficult to account for, unless due to an accumulation of dust since wiped off; for it cannot be supposed that a weight so carefully preserved as the kilogramme des Archives, could have lost nearly 0·01 grain of its substance in the course of two years.

All the other comparisons, in which the number of observations is large, yield results sufficiently accordant with the value of $\mathfrak{A}$ deduced from the comparisons of $\mathfrak{A}$ with $\mathfrak{C}$, and of $\mathfrak{C}$ with I+K+A, I+L+B, I+M+Γ, I+N+Δ, which gave,—

Kilogramme des Archives =15432·34874 grains, of which the new platinum standard pound contains 7000. Or, kilogramme des Archives=(2·20462125) standard platinum lb. Standard platinum lb. =453·5926525 grammes, of which the kilogramme des Archives contains 1000.

Weight of the Kilogramme type laiton.

This kilogramme of brass, which will be designated by the letter $\mathfrak{L}$, is deposited in the Ministère de l'Intérieure in Paris, and serves as a standard for the purpose of adjusting the kilogrammes used in commerce. In a paper in the 'Memorie di Matematica e di Fisica della Società Italiana delle Scienze residente in Modena,' t. xxv. p. 1, it is stated that in 1850, $\mathfrak{A} \stackrel{\frown}{=} L+89{\cdot}5$ milligrammes in air ($t=18{\cdot}90$, $b=763{\cdot}80$). A brass kilogramme marked No. IV., the volume of which is assumed to be equal to that of $\mathfrak{L}$ (the grounds for this assumption are not stated), was weighed in water at 16·97. The absolute weight of the water displaced by it was 124·5536 grammes.

Hence $v\mathfrak{L}$ assumed $=v$Kilo. No. IV.$=$ volume of 124·590 grammes $=$ volume of 1922·720 grains of water at its maximum density. $v\mathfrak{C}=730{\cdot}078$, $v\mathfrak{A}-v\mathfrak{C}=20{\cdot}933$. Therefore $v\mathfrak{A}=751{\cdot}011$. The volume of 89·5 milligrammes or 1·38120 grain of platinum $=$ volume of 0·065 grain of water. Therefore $v(\mathfrak{A}-89{\cdot}5 \text{ milligr.})=750{\cdot}946$. During the comparison, $\mathfrak{L}$ displaced 2·32768 grains of air, and $\mathfrak{A}$ displaced 0·90868 grain. Therefore $\mathfrak{L}=\mathfrak{A}+0{\cdot}03780$ grain. $\mathfrak{L}$ displaces 2·30493 grains, and $\mathfrak{A}$ displaces 0·89981 grain of air ($\log\Delta=7{\cdot}07832-10$). Therefore $\mathfrak{L} \stackrel{\frown}{=} \mathfrak{A}-1{\cdot}36732$ grain in air ($\log\Delta=7{\cdot}07832-10$).

The commercial standard lb. is a weight of 7000 grains, of the same density as the lost standard troy pound U. Denoting the commercial standard lb. by W, and the platinum standard by I, in air ($\log\Delta=7{\cdot}07832-10$) $W \stackrel{\frown}{=} I-0{\cdot}63407$ grain. Therefore $\frac{15432{\cdot}34874}{7000}W \stackrel{\frown}{=} \frac{15432{\cdot}34874}{7000}I-1{\cdot}39788$ grain. $v\mathfrak{A}-v\frac{15432{\cdot}349}{7000}I=$ volume of 29·599 grains of water of maximum density $=$ volume of 0·03547 grain of air. Therefore $\mathfrak{A}+0{\cdot}03547$ grain $\stackrel{\frown}{=} \frac{15432{\cdot}34874}{7000}I$. Hence $\mathfrak{L} \stackrel{\frown}{=} \frac{15432{\cdot}34386}{7000}W$, in air, for which $\log\Delta=7{\cdot}07832-10$. $\mathfrak{L}$ appears to weigh 15432·344 grains, of which W contains 7000. W appears to weigh 453·59278 grammes, of which $\mathfrak{L}$ contains 1000.

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The Quartz Pound.

The hardness of quartz, its capability of receiving a high polish, the absence of any hygroscopic properties, and its indestructibility at the ordinary temperature of the atmosphere by any chemical agent except hydrofluoric acid, are such valuable qualities in a substance used for the construction of weights, that Professor STEINHEIL was induced to adopt it as the material for a copy of the kilogramme. The only objection to the use of a weight made of quartz is, that on account of the large amount of air displaced, the barometer and thermometer must be observed with extreme care during its comparison with a weight of any ordinary metal. Notwithstanding this disadvantage, it appeared desirable to the Committee that a weight of quartz should be constructed sufficiently near 7000 grains to admit of readily deducing the pound from it. They accordingly commissioned Mr. BARROW to construct a weight of

quartz of the form of a cube of about 2·2 inches, having its edges and angles rounded. Its apparent weight in air is intermediate between that of a lb. of platinum and a lb. of brass, approaching more nearly to the latter than to the former.

Weighing of the quartz weight in water.

The quartz weight, which will be designated by the letter Q, was suspended in water, from the right-hand pan of the balance, by a fine copper wire, one end of which was fastened to a thicker wire bent round Q in a plane through its centre, parallel to a pair of faces of the cube. 7·23 inches of the suspending wire weighed 0·96 grain. Hence a portion of the wire corresponding to 100 parts of the scale displaced 0·016 grain of water. When the counterpoise in the left-hand pan was in equilibrium with the weight A+B+D+G+K+M+S+(16)+(2)+0·1 grain in the right-hand pan, 100 parts =0·268 grain. Hence when Q is suspended in water 100 parts =0·284 grain. In the observations of July 15 the suspending wire alone immersed to the same depth as when Q was suspended in water ≏ 12·7029 grains of platinum in air. At the close of these observations the fine wire broke, and was replaced. In the observations which followed, the wire alone in water ≏ 12·5641 grains of platinum in air.

July 15, 1846. Q and wire in water. In right-hand pan.

G.	L.	gr.	Scale.
118·05	62·85	3·04	15·30
		3·05	21·30
		3·04	21·45
		3·02	11·00
		3·02	17·00
		3·02	14·00
118·20	63·05	3·04	20·45
		3·04	20·40
		3·00	16·00
		3·00	13·60
118·25	63·30	3·03	20·45
		3·02	19·45
		3·02	15·50
		3·02	19·10
		3·02	19·40
		3·02	20·30
118·40	63·50	3·00	17·25
		3·00	15·70
		3·00	17·30
118·22	63·17	3·020	17·71

Counterpoise balances Q and wire in water (G=118·40, L=63·17)+3·0237 grains in air.

In right-hand pan.

	gr.	Scale.
A+B+D+G+K+M+S+(16)+	2·68	24·6
	2·64	12·6
D=18·35, C=18·4, F=761·8, E=20.	2·67	23·2
	2·66	19·2
	2·6625	19·9

C.poise bal. A+B+D+G+K+M+S+(16)+2·6601 grs. in air (D=18·35, C=18·4, F=761·8, E=20).

	Q and wire in water.		In right-hand pan.	
	G.	L.	gr.	Scale.
	118·7	64·1	2·94	7·45
			2·94	11·00
			2·98	19·45
			2·98	18·35
	119·0	64·5	2·98	21·50
			2·94	17·50
			2·96	21·70
	118·85	64·3	2·9644	17·62

Counterpoise balances Q and wire in water (G=118·85, L=64·3)+2·9683 grains in air.
Q and wire in water (t=17·30) ≏ A+B+D+G+K+M+S+(16)−0·3359 gr. in air (t=18·41, b=759·79).

July 15, 1846.	Q and wire in water.		In right-hand pan.	
	G.	L.	gr.	Scale.
			2·94	14·80
			2·96	19·70
	119·25	64·90	2·96	22·90
			2·94	18·25
			2·94	20·75
			2·94	19·65
	119·40	65·10	2·94	21·10
			2·94	21·15
	119·32	65·00	2·945	19·79

Counterpoise balances Q and wire in water (G=119·32, L=65·0)+2·9428 grains in air.

	In right-hand pan.	
	gr.	Scale.
A+B+D+G+K+M+S+(16)	+2·66	17·60
	+2·67	19·85
D=18·5, C=18·5, F=761·8, E=20.	+2·67	19·97
	2·6667	19·14

C.poise bal. A+B+D+G+K+M+S+(16)+2·6663 grs. in air (D=18·5, C=18·5, F=761·8, E=20).

July 15, 1846.	Q and wire in water.		In right-hand pan.	
	G.	L.	gr.	Scale.
	119·60	65·50	2·94	26·10
			2·90	20·10
			2·90	12·10
			2·90	15·30
			2·94	21·50
			2·94	26·50
	119·80	65·80	2·90	18·35
			2·90	16·25
	119·7	65·65	2·915	19·52

Counterpoise balances Q and wire in water (G=119·7, L=65·15)+2·9135 grains in air.
Q and wire in water (t=17·44) ≏ A+B+D+G+K+M+S+(16)−0·2618 grain in air (t=18·54, b=759·79).

July 28.	Q and wire in water.		In right-hand pan.	
	G.	L.	gr.	Scale.
		71·05	2·24	18·20
			2·24	18·00
			2·24	18·30
		71·10	2·24	17·50
			2·24	17·60

		2·24	17·10
		2·24	17·00
	71·05	2·24	18·00
		2·24	17·30
		2·24	17·90
	71·07	2·240	17·69

Counterpoise balances Q and wire in water (t=18·15)+2·2437 grains in air.

	In right-hand pan. gr.	Scale.
	A+B+D+G+K+M+S+(16)+2·10	23·80
	+2·08	16·27
D=18·45, C=18·5, F=768·9, E=19·7.	+2·09	20·00
	+2·08	15·15
	+2·09	19·30
	2·088	18·90

C.poise bal. A+B+D+G+K+M+S+(16)+2·0883 grs. in air (D=18·45, C=18·5, F=768·9, E=19·7).

Q and wire in water. G.	L.	In right-hand pan. gr.	Scale.
122·9	70·93	2·24	16·4
		2·28	28·0
		2·24	10·6
		2·24	17·0
		2·24	18·5
		2·24	18·0
		2·24	14·7
122·87	70·8	2·24	17·1
		2·28	23·2
		2·28	24·9
		2·24	14·0
		2·28	22·5
		2·28	30·5
		2·24	14·5
122·9	70·90	2·24	16·5
122·89	70·88	2·2533	19·09

Counterpoise balances Q and wire in water (t=18·13)+2·2530 grains in air.

Q and wire in water (t=18·14)≏A+B+D+G+K+M+S+(16)−0·1603 gr. in air (t=18·51, b=766·91).

July 29.

Q and wire in water. G.	L.	In right-hand pan. gr.	Scale.
119·10	64·60	2·54	17·0
		2·58	28·9
		2·50	10·5
		2·54	22·6
		2·50	13·1
		2·54	29·5
119·30	64·90	2·50	12·5
		2·50	9·9
		2·54	28·8
		2·52	23·1
119·45	65·13	2·50	20·0
		2·50	20·8
		2·48	13·5
		2·50	19·2
		2·50	23·5
		2·48	18·8
119·28	64·88	2·5137	19·48

Counterpoise balances Q and wire in water (G=119·28, L=64·88)+2·5123 grains in air.

In right-hand pan.

		gr.	Scale.
	A+B+D+G+K+M+S+(16)	+2·08	20·6
		+2·07	16·3
D=18·4, C=18·4, F=766·1, E=20·7.		+2·08	19·6
		2·0767	18·83

C.poise balances A+B+D+G+K+M+S+(16)+2·0772 grs. in air (D=18·4, C=18·4, F=766·1, E=20·7).

Q and wire in water. G.	L.	In right-hand pan. gr.	Scale.
119·8	65·7	2·48	24·2
		2·44	7·5
		2·48	27·6
		2·44	11·8
		2·48	28·0
120·0	66·0	2·46	20·2
		2·44	8·5
		2·46	19·0
		2·46	21·8
		2·44	13·5
		2·46	22·2
120·15	66·3	2·44	18·5
		2·44	20·5
119·98	66·0	2·4554	18·71

Counterpoise balances Q and wire in water (G=119·98, L=66·0)+2·4562 grains in air.

Q and wire in water (t=17·46)≏A+B+D+G+K+M+S+(16)−0·4070 gr. in air (t=18·44, b=763·99).

July 29.

Q and wire in water. G.	L.	In right-hand pan. gr.	Scale.
120·30	66·57	2·44	18·9
		2·44	20·5
		2·44	24·5
		2·40	13·0
		2·44	26·9
120·45	66·90	2·40	13·6
		2·44	25·4
		2·40	13·7
		2·44	26·7
120·65	67·15	2·40	15·8
		2·44	27·5
		2·40	21·3
		2·40	20·5
120·47	66·87	2·421	20·64

Counterpoise balances Q and wire in water (G=120·47, L=66·8)+2·4163 grains in air.

In right-hand pan.

		gr.	Scale.
	A+B+D+G+K+M+S+(16)	+2·08	20·30
		+2·08	18·30
D=18·5, C=18·6, F=765·4, E=20·4.		+2·08	19·02
		+2·08	18·52
		+2·04	2·95
		+2·12	33·57
		2·08	18·78

C.poise balances A+B+D+G+K+M+S+(16)+2·0806 grs. in air (D=18·5, C=18·6, F=765·4, E=20·4).

Q and wire in water.		In right-hand pan.	
G.	L.	gr.	Scale.
121·05	67·80	2·38	22·9
		2·34	9·0
		2·38	25·6
		2·34	9·7
		2·38	26·7
121·25	68·05	2·34	13·9
		2·38	26·9
		2·34	15·5
		2·38	26·8
		2·34	17·3
		2·38	26·1
121·40	68·40	2·34	18·3
121·23	68·08	2·360	19·9

Counterpoise balances Q and wire in water (G=121·23, L=68·08)+2·3574 grains in air.

Q and wire in water (t=17·71)≏A+B+D+G+K+M+S+(16)−0·3062 gr. in air (t=18·59, b=763·33).

July 30.

Q and wire in water.		In right-hand pan.	
G.	L.	gr.	Scale.
120·10	66·25	2·48	27·5
		2·44	20·1
		2·40	6·9
		2·40	5·0
		2·44	24·3
		2·40	8·0
120·25	66·50	2·43	21·9
		2·43	20·8
		2·43	21·2
120·40	66·70	2·42	19·5
120·25	66·48	2·427	17·52

Counterpoise balances Q and wire in water (G=120·25, L=66·48)+2·4312 grains in air.

	In right-hand pan.	
	gr.	Scale.
A+B+D+G+K+M+S+(16)	+2·08	18·25
	+2·08	18·45
D=18·6, C=18·6, F=764·2, E=22·3.	+2·08	18·97
	2·08	18·56

C:poise bal. A+B+D+G+K+M+S+(16)+2·0813 grs. in air (D=18·6, C=18·6, F=764·2, E=22·3).

Q and wire in water.		In right-hand pan.	
G.	L	gr.	Scale.
120·6	67·2	2·40	22·5
		2·37	12·5
		2·40	23·1
		2·38	18·1
120·85	67·5	2·38	18·2
		2·38	18·9
		2·38	21·0
121·0	67·8	2·38	21·7
		2·36	16·7
120·82	67·5	2·3811	19·19

Counterpoise balances Q and wire in water (G=120·82, L=67·5)+2·3806 grains in air.

Q and wire in water (t=17·65)≏A+B+D+G+K+M+S+(16)−0·3246 grain in air (t=18·64, b=761·90).

Apparent weight of Q in water, weighed with platinum weights.

Water. t.	gr.	Air. t.	b.
17·3	Q≏4362·5201	18·41	759·79
17·44	Q≏4362·5942	18·54	759·79
18·14	Q≏4362·8345	18·51	766·91
17·46	Q≏4362·5878	18·44	763·99
17·71	Q≏4362·6886	18·59	763·33
17·65	Q≏4362·6702	18·64	761·90

Comparison of Q *with* PS.

During the comparison of Q with PS, the zero of the scale of the barometer was 305 millimètres above the centre of gravity of the weights, consequently 0·03 mm. must be added to the reading of the barometer. F denotes the reading of the barometer, E that of its attached thermometer. The thermometers C, D were suspended in the balance-case. PS will be designated by the letter I. The weight of 0·4 grain used with Q on the 3rd and 10th of August, was the 0·4 grain of ROBINSON's $5\frac{1}{2}$-inch balance; on the 5th it was the 0·4 grain of BARROW's balance; on the 6th it was the sum of the 0·3 grain and 0·1 grain of BARROW's balance; on the 7th it was the 0·4 grain of ROBINSON's $10\frac{1}{2}$-inch balance; on the 11th it was the sum of the 0·3 grain and 0·1 grain of ROBINSON's $5\frac{1}{2}$-inch balance. Z=0·4 gr.+0·3 gr.+0·1 gr. of ROBINSON's $5\frac{1}{2}$-inch balance+0·4 gr.+0·3 gr.+0·1 gr. of BARROW's balance+0·4 gr. of ROBINSON's $10\frac{1}{2}$-inch balance; (2) the 2-grain weight of BARROW's balance. The readings increase on placing a small weight in the left-hand pan.

100 parts=0·399 grain.

Z, (2).	(2), Z.
3·21	3·07
3·22	3·01
3·16	2·90
9·59	8·98

6Z =6(2)+0·00243 grain.

Z=(2)+0·000406 gr., (2)=1·999597 gr., Z=2·000003 grains, $\frac{1}{5}$Z=0·4000006 gr. Hence the mean value of the 0·4 grain used may be taken as exact.

August 3, 1846. 100 parts=0·31355 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, Q+0·4 gr.+X	21·16	Q+0·4 gr.+X, I+Y	18·57
				....	21·64	....	17·95
20·4	20·3			....	22·15	....	17·30
		760·0	21·5	....	22·15	....	16·57
20·6	20·5			....	22·42	....	17·51
20·65	20·6			I+X, Q+0·4 gr.+Y	23·12	Q+0·4 gr.+Y, I+X	16·41
20·65	20·6			I+0·02 gr.+X, Q+0·4 gr.+Y	17·09	Q+0·4 gr.+Y, I+0·02 gr.+X	22·51
		760·5	21·5	I+0·01 gr.+X, Q+0·4 gr.+Y	19·55	Q+0·4 gr.+Y, I+0·01 gr.+X	19·41
				....	20·19	....	19·49
				....	19·37	....	19·24
20·57	20·5	760·25	21·5	10I+0·05 gr., 10(Q+0·4 gr.)	208·84	10(Q+0·4 gr.), 10I+0·05 gr.	184·96

20(Q+0·4 gr.)≏20I+0·1 gr.+23·88 pts.

100 parts=0·31355 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, Q+0·4 gr.+Y	15·55	Q+0·4 gr.+Y, I+X	25·50
19·85	19·8	764·72	20·9	I+X, Q+0·42 gr.+Y	22·24	Q+0·42 gr.+Y, I+X	19·27
				I+X, Q+0·41 gr.+Y	18·59	Q+0·41 gr.+Y, I+X	22·29
				....	19·56	....	22·20
20	19·9			....	19·20	....	21·21
				I+Y, Q+0·41 gr.+X	19·94	Q+0·41 gr.+X, I+Y	20·75
20·05	19·95			....	18·79	....	20·47
		764·7	20·8	....	18·32	....	19·52
				....	18·96	....	20·29
20·15	20·05			....	18·65	....	19·50
20·01	19·92	764·71	20·85	10I, 10(Q+0·4 gr.)+0·1 gr.	189·80	10(Q+0·4 gr.)+0·1 gr., 10I	211·00

20(Q+0·4 gr.)≏20I−0·2 gr.−21·2 pts.

40(Q+0·4 gr.)≏40I−0·1 gr.+2·68 pts. in air (D=20·29, C=20·21, F=762·48, E=21·17).

Q+0·4 gr.≏I−0·00229 gr. in air (t=20·31, b=760·35).

August 5. 100 parts=0·31808 grain.

D.	C.	F.	E.		Scale.		Scale.
19·7	19·6			I+Y, Q+0·4 gr.+X	17·52	Q+0·4 gr.+X, I+Y	23·05
				....	18·07	....	22·82
				....	17·55	....	22·06
				....	18·31	....	22·44
20·0	19·87	762·2	20·4	....	19·44	....	22·47
				....	19·00	....	22·01
				....	19·47	....	22·06
				....	19·31	....	21·19
				....	19·35	....	20·11
20·2	20·1			....	19·81	....	19·65
19·97	19·86	762·2	20·4	10(I+Y), 10(Q+0·4gr.+X)	187·83	10(Q+0·4gr.+X), 10(I+Y)	217·86

20(Q+0·4 gr.+X)≏20(I+Y)−30·03 pts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, Q+0·4 gr.+Y	22·32	Q+0·4 gr.+Y, I+X	19·42
20·5	20·43			....	21·19	....	18·55
				....	22·05	....	18·31
				....	21·17	....	18·21
20·6	20·5	761·8	21	....	22·36	....	18·46
				....	22·11	....	19·97
				....	20·15	....	18·10
				....	20·01	....	17·60
20·65	20·6			....	20·04	....	17·71
				....	19·59	....	17·57
20·58	20·51	761·8	21	10(I+X), 10(Q+0·4 gr.+Y)	210·99	10(Q+0·4 gr.+Y), 10(I+X)	183·90

20(Q+0·4 gr.+Y)≏20(I+X)+27·09 pts.

40(Q+0·4 gr.)≏40I−2·94 pts. in air (D=20·27, C=20·18, F=762·0, E=20·7).

Q+0·4 gr.≏I−0·00023 grain in air (t=20·29, b=759·93).

August 6. 100 parts=0·31808 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, Q+0·4 gr.+Y	21·24	Q+0·4 gr.+Y, I+X	16·51
				....	21·17	....	16·71
20·7	20·6			....	22·05	....	18·66
				....	22·05	....	15·84
				I+0·01 gr.+X, Q+0·4 gr.+Y	18·57	Q+0·4 gr.+Y, I+0·01 gr.+X	20·01
		760·9	21·95	....	18·21	....	17·92
				....	18·92	....	17·54
21	20·93			....	18·55	....	17·57
				....	18·65	....	17·34
				....	19·59	....	17·00
20·85	20·76	760·9	21·95	10(I+X)+0·06 gr., 10(Q+0·4 gr.+Y)	199·00	10(Q+0·4 gr.+Y), 10(I+X)+0·06 gr.	175·10

20(Q+0·4 gr.+Y)≏20(I+X)+0·12 gr.+23·9 pts.

D.	C.	F.	E.		Scale.		Scale.
				I+0·02 gr.+Y, Q+0·4 gr.+X	16·69	Q+0·4 gr.+X, I+0·02 gr.+Y	22·27
				....	16·37	....	20·55
21·1	21·06			....	16·32	....	21·22
				....	16·79	....	20·72
				I+0·01 gr.+Y, Q+0·4 gr.+X	20·10	Q+0·4 gr.+X, I+0·01 gr.+Y	18·49
		760·9	22	....	19·99	....	18·19
				....	20·52	....	18·34
21·2	21·15			....	19·75	....	17·44
				....	20·61	....	17·25
				....	19·56	....	18·77
21·15	21·1	760·9	22	10(I+Y)+0·14 gr., 10(Q+0·4 gr.+X)	186·70	10(Q+0·4 gr.+X), 10(I+Y)+0·14 gr.	193·24

20(Q+0·4 gr.+X) ≏ 20(I+Y)+0·28 gr.−6·54 pts.

40(Q+0·4 gr.) ≏ 40I+0·4 gr.+17·36 pts. in air (D=21, C=20·93, F=760·9, E=21·97).

Q+0·4 gr. ≏ I+0·01138 grain in air (t=21·03, b=758·68).

August 7. 100 parts=0·31299 grain.

D.	C.	F.	E.		Scale.		Scale.
20·2	20·1			I+Y, Q+0·4 gr.+X	25·40	Q+0·4 gr.+X, I+Y	16·72
20·35	20·3			I+0·01 gr.+Y, Q+0·4 gr.+X	22·02	Q+0·4 gr.+X, I+0·01 gr.+Y	18·61
		757·7	20·95	....	23·20	....	18·06
20·55	20·45			I+0·02 gr.+Y, Q+0·4 gr.+X	20·20	Q+0·4 gr.+X, I+0·02 gr.+Y	22·12
20·6	20·5			....	20·05	....	20·45
				....	20·49	....	21·22
20·7	20·6			....	20·17	....	19·90
20·85	20·7	757·6	21·2	....	20·17	....	20·89
20·95	20·9			....	20·91	....	19·72
				....	21·25	....	19·36
20·6	20·51	757·65	21·07	10(I+Y)+0·16 gr., 10(Q+0·4 gr.+X)	213·86	10(Q+0·4 gr.+X), 10(I+Y)+0·16 gr.	197·05

20(Q+0·4 gr.+X) ≏ 20(I+Y)+0·32 gr.+16·81 pts.

D.	C.	F.	E.		Scale.		Scale.
20·92	20·85			I+0·02 gr.+X, Q+0·4 gr.+Y	20·82	Q+0·4 gr.+Y, I+0·02 gr.+X	19·54
		757·3	21·4	....	21·42	....	18·45
20·93	20·85			....	21·31	....	19·76
				....	21·76	....	18·67
20·97	20·9	757·2	21·5	....	22·11	....	19·42
				....	22·56	....	19·30
20·97	20·9			....	21·37	....	19·51
		756·83	21·5	....	21·67	....	20·60
20·95	20·9			....	23·00	....	19·46
				....	21·65	....	18·44
20·95	20·88	757·11	21·47	10(I+X)+0·2 gr., 10(Q+0·4+Y)	217·67	10(Q+0·4 gr.+Y), 10(I+X)+0·2 gr.	193·15

20(Q+0·4 gr.+Y) ≏ 20(I+X)+0·4 gr.+24·52 pts.

40(Q+0·4 gr.) ≏ 40I+0·72 gr.+41·33 pts. in air (D=20·77, C=20·7, F=757·38, E=21·27).

Q+0·4 gr. ≏ I+0·02123 grain in air (t=20·8, b=755·26).

August 11. 100 parts=0·31299 grain.

D.	C.	F.	E.		Scale.		Scale.
19·4	19·3			I+Y, Q+0·42 gr.+X	20·55	Q+0·42 gr.+X, I+Y	22·32
				....	20·06	....	20·12
19·6	19·5	765·52	20·6	....	19·97	....	19·37
				....	19·87	....	18·82
19·9	19·77			....	19·52	....	18·60
				....	19·05	....	16·22
19·98	19·85			....	20·61	....	16·50
		765·42	20·8	....	19·90	....	16·09
20	19·87			....	19·39	....	17·22
				....	20·19	....	16·50
19·78	19·66	765·47	20·7	10(I+Y), 10(Q+0·42 gr.+X)	199·11	10(Q+0·42 gr.+X), 10(I+Y)	181·76

20(Q+0·4 gr.+X) ≏ 20(I+Y)−0·4 gr.+17·35 pts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, Q+0·42 gr.+Y	20·51	Q+0·42 gr.+Y, I+X	19·01
20·1	20	765·5	20·9	....	20·90	....	17·91
				....	20·49	....	17·27
20·15	20·03			....	20·81	....	17·40
		765·58	20·95	....	20·87	....	17·01
20·2	20·1			....	20·47	....	16·45
				....	20·31	....	16·95
20·25	20·1	765·43	20·9	....	20·95	....	16·31
				....	20·79	....	16·20
20·25	20·13			....	20·62	....	17·05
20·2	20·07	765·5	20·91	10(I+X), 10(Q+0·42 gr.+Y)	206·72	10(Q+0·42 gr.+Y), 10(I+Y)	171·56

20(Q+0·4 gr.+Y)≏20(I+Y)−0·4 gr.+35·16 pts.

40(Q+0·4 gr.)≏40I−0·8 gr.+52·51 pts. in air (D=19·99, C=19·86, F=765·48, E=20·8).

Q+0·4 gr.≏I−0·01589 grain in air (t=19·99, b=762·92).

gr.	t.	b.
Q≏I−0·40229	20·31	760·35
Q≏I−0·40023	20·29	759·93
Q≏I−0·38862	21·03	758·68
Q≏I−0·37877	20·80	755·26
Q≏I−0·41589	19·99	762·92

According to STEINHEIL, the density of quartz at 0° is 2·650962 of the maximum density of water, and its cubic expansion for 1° C. is 0·00003255.

The weighings of Q in water give

	Water at 18° displaced by Q.	Logarithms.
	2639·831	3·4215760
	2639·809	3·4215725
	2639·838	3·4215774
	2639·825	3·4215751
	2639·819	3·4215742
	2639·814	3·4215733
Mean	2639·823	3·4215748
Maximum density of water : Δ water at 18		0·0005867
vQ at 18	2643·391	3·4221615
Q	7002·368	3·8452449
ΔQ at 18 : maximum density of water	2·649009	0·4230834
ΔQ at 18 : Δ water at 18	2·652590	0·4236701

The density of STEINHEIL's kilogramme of quartz at 18° in terms of the density of water at 18° is 2·652908.

The logarithm of the volume of Q at $t°$, in volumes of a grain of water at its maximum density, will be $3{\cdot}4221615+(t-18)\times 0{\cdot}00001413$. The comparisons of Q with I in air, reduced with this value of the volume of Q, give for the absolute weight of Q in a vacuum,

	gr.	No. of comparisons.
	Q=I+2·36801	40
	Q=I+2·36871	40
	Q=I+2·36817	40
	Q=I+2·36782	40
	Q=I+2·36715	40
Mean	Q=I+2·36797	200

Secondary Standards.

The comparison of different brass weights with platinum weights, shows that, however carefully preserved, they are liable to gain from 0·01 grain to 0·02 grain or more in the course of a few years. Brass, therefore, unless very well protected by gilding, is quite unfit to be used in the construction of weights having that degree of accuracy which is required in secondary standards. Electro-gilding was tried in the first instance. It failed, however, to afford a sufficient protection to the metal underneath, and the weight of the pound when the gilding was completed, was so very uncertain as to render its adjustment an extremely troublesome process. It was afterwards discovered that these evils were due to the want of skill of the person employed as a gilder by Mr. Barrow, and not to any inherent defect in electro-gilding itself. Mr. Barrow then resorted to amalgam gilding. In order to adjust a weight, its under surface, which was slightly concave, was more thickly coated with gold than the rest of its surface. If too heavy, a little of the gold was removed by rubbing it with charcoal; if too light, more amalgam was added, and the mercury driven off by heat. Directions had been given that the weights should be made of the alloy used by Mr. Baily for the standard yard bars, consisting of thirty-two parts of copper, five of tin, and two of zinc. The densities, however, of the greater part of them indicate that these proportions have not been strictly observed. The lbs. numbered 31 up to 36, protected by electro-gilding, were constructed by Messrs. Ladd and Streathfield. The weighings were reduced with the expansions given in Table III.

The observations for finding the densities of the secondary standards, and for comparing them with the platinum standard, were made in a room in the basement of my own house in Cambridge, the brick floor of which afforded a perfectly firm foundation for a strong table on which the balance was mounted.

The weights employed in some of the weighings are of bronze, for which $\log \Delta = 0·92250$.

Apparent weights of the secondary standards in water.

No. of lb.	Water. t.	Platinum. gr.	Bronze. gr.	Air. t.	Air. b.
1	15·11	6163·2354		17·03	755·10
2	18·41	6161·5534		19·86	760·32
3	17·75	6157·7809		19·94	753·04
4	15·35	6163·6422		15·62	767·02
5	14·69	6132·0586		16·45	755·40
6	14·31	6155·7369		16·34	758·92
7	14·24	6138·4869		15·17	760·01
8	17·48	6143·1594		19·10	760·86
9	16·31	6047·7747		17·01	763·14
10	14·71	6155·3347		16·88	758·12
11	17·80	6163·6244		18·49	750·29
12	15·28	6159·0015		16·09	767·20
13	10·79	6169·9472		12·20	734·36
14	14·42	6161·9845		15·54	755·94
15	10·23	6162·9240		11·01	751·19
16	17·43	6133·5979		17·93	754·30
17*a*	11·15	38·3615	6099·9740	12·42	758·02

17*b*	10·75	6182·2705		11·44	738·69
18	17·20	6157·6295		19·82	757·27
19	15·39	6161·0862		17·01	767·23
21	15·59	6122·6308		15·94	771·23
22	17·59	6146·8491		19·45	756·99
23	16·08	6141·7878		16·34	763·64
24	17·46	6140·9677		19·42	758·65
25	17·91	6136·6810		20·04	758·03
26	12·21	6141·5668		14·10	754·92
27	16·27	6142·9019		16·52	763·47
28	17·28	6139·2058		18·23	761·17
29	11·35	45·4802	6099·9740	13·35	758·40
30	16·18	6141·9389		17·06	763·07
31	14·53	78·7565	6099·9776	15·05	741·30
32	14·53	72·5862	6099·9776	15·11	741·50
33	14·63	75·3351	6099·9776	15·11	741·88
34	15·03	78·8455	6099·9776	15·01	752·51
35	15·03	74·5158	6099·9776	15·41	752·88
36	14·93	77·0155	6099·9776	15·31	753·07

Densities and volumes of the secondary standards at 0° C., the density being expressed in terms of the maximum density of water, and the volume in volumes of a grain of water at its maximum density.

No.	ΔG.	*v*G.	log *v*G.
1	8·36134	837·186	2·922822
2	8·34161	839·162	2·923846
3	8·30462	842·904	2·925778
4	8·36500	836·820	2·922632
5	8·06122	868·356	2·938698
6	8·28779	844·614	2·926658
7	8·12163	861·898	2·935456
8	8·16317	857·512	2·933240
9	7·37614	952·897	2·979046
10	8·28375	845·024	2·926869
11	8·36302	837·013	2·922732
12	8·31919	841·426	2·925016
13	8·43179	830·187	2·919176
14	8·34955	838·365	2·923433
15	8·36107	837·211	2·922835
16	8·07354	867·028	2·938033
17*a*	8·11718	862·365	2·935691
17*b*	8·55888	817·858	2·912678
18	8·30369	842·999	2·925827
19	8·33969	839·358	2·923947
21	7·97370	877·890	2·943440
22	8·19859	853·804	2·931358
23	8·15141	858·748	2·933866
24	8·14286	859·645	2·934319
25	8·10164	864·022	2·936525
26	8·15218	858·665	2·933824
27	8·16186	857·650	2·933310
28	8·12604	861·428	2·935219
29	8·18446	855·279	2·932108
30	8·15292	858·588	2·933785
31	8·51444	822·129	2·914940
32	8·47042	828·302	2·918189
33	8·47902	825·563	2·916750
34	8·51472	822·103	2·914926
35	8·47019	826·424	2·917203
36	8·49601	823·912	2·915881

Secondary standards weighed in air.

The small weights R were made by winding fine silvered brass wire, to which a weight was attached nearly as heavy as the wire could sustain, round a piece of steel pinion-wire. The pinion-wire, with the brass wire wound round it, and still stretched by the weight, was then clamped between two pieces of wood, and the rings separated by placing the back of a sharp penknife in one of the grooves of the pinion-wire, and pushing it on in the groove till all the coils of the fine wire were cut through. The weight of eighty-eight of the rings was found to be 0·99595 grain. Hence $R=0{\cdot}011313$ grain. The weights S of 0·01136 grain each, used in two of the observations, were made in the same manner. $I \mathrel{\hat{=}} T+Q+D-0{\cdot}00077$ grain in air ($t=19{\cdot}63$, $b=764{\cdot}43$), $\log v(I-Q)=2{\cdot}51960$, $\log v(T+D)=2{\cdot}51942$. Hence $I=T+Q+D-0{\cdot}00060$ gr. $T+D=I-0{\cdot}64449$ gr. Let W be taken to denote the commercial lb. Then, in air, at 65°·66 F., bar. 29·75 inches ($t=18{\cdot}7$, $b=755{\cdot}64$), at Somerset House, or in air for which $\log \Delta=7{\cdot}07832-10$, $W \mathrel{\hat{=}} I-Q+0{\cdot}01102$ gr. Also, since $T+Q+D \mathrel{\hat{=}} I+0{\cdot}00077$ gr., $W \mathrel{\hat{=}} T+D+0{\cdot}01025$ grain. $I-Q$ displaces 0·39641 grain, and $T+D$ displaces 0·39624 grain of air, for which $\log \Delta=7{\cdot}07832-10$. M denotes the mean of the readings of the thermometers D, C.

G= lb. No. 1.

100 parts + 0·27297 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+Y, G+X	28·10	G+X, T+D+R+Y	18·87
				....	24·96	....	19·45
13·15	13·2			....	26·60	....	17·14
				....	27·77	....	18·81
				....	28·34	....	20·32
		752·4	13·85	T+D+2R+Y, G+X	21·12	G+X, T+D+2R+Y	22·47
				....	19·90	....	19·34
13·3	13·35			....	18·57	....	19·31
				....	19·47	....	17·77
				....	21·77	....	19·55
					236·60		193·03

$20(G+X) \mathrel{\hat{=}} 20(T+D+Y)+30R+43{\cdot}57$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+2R+X, G+Y	17·52	G+Y, T+D+2R+X	18·95
				....	19·25	....	20·47
13·5	13·5			....	20·00	....	22·09
				....	19·67	....	23·02
				....	20·06	....	22·66
		751·75	14	....	20·82	....	20·39
				....	21·55	....	22·15
13·55	13·55			....	20·50	....	22·75
				....	22·27	....	23·37
				....	23·22	....	22·55
					204·86		218·40

$20(G+Y) \mathrel{\hat{=}} 20(T+D+X)+40R-13{\cdot}54$ parts.

$40G \mathrel{\hat{=}} 40(T+D)+70R+30{\cdot}03$ parts, $M=13{\cdot}39$, $F=752{\cdot}07$, $E=13{\cdot}92$.

$G \mathrel{\hat{=}} T+D+0{\cdot}02183$ grain in air ($t=13{\cdot}42$, $b=750{\cdot}83$).

$G=I-0{\cdot}00732$ grain.

$G \mathrel{\hat{=}} W+0{\cdot}01956$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G= lb. No. 2.

100 parts $=0{\cdot}27039$ grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+R+Y	19·86	G+R+Y, T+D+X	19·05
				....	22·37	....	18·31
12·9	12·95			....	21·82	....	18·95
				....	19·64	....	17·31
				....	17·72	....	13·54
		748·5	13·4	....	18·00	....	13·36
				T+D+X, G+Y	13·06	G+Y, T+D+X	19·32
				....	13·27	....	18·47
13·1	13·13			....	13·96	....	19·71
				....	14·97	....	19·10
				....	15·60	....	18·87
					190·27		195·99

$$22(G+Y)+12R \bumpeq 22(T+D+X)-5{\cdot}72 \text{ parts.}$$

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	18·00	G+X, T+D+Y	23·42
				....	20·61	....	22·85
13·45	13·48			....	18·47	....	21·16
				....	18·55	....	22·22
				....	18·84	....	22·49
		748·9	13·65	....	19·70	....	22·22
				....	17·44	....	20·54
				....	17·22	....	19·47
13·4	13·45			....	18·24	....	22·22
				....	18·17	....	21·35
				....	16·81	....	22·67
					202·05		240·61

$22(G+X)\bumpeq 22(T+D+Y)-38{\cdot}56$ parts.

$44G\bumpeq 44(T+D)-12R-44{\cdot}28$ parts, $M=13{\cdot}23$, $F=748{\cdot}7$, $E=13{\cdot}52$.

$G\bumpeq T+D-0{\cdot}00587$ grain in air ($t=13{\cdot}26$, $b=746{\cdot}51$).

$G=I-0{\cdot}03582$ grain.

$G\bumpeq W-0{\cdot}01132$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G= lb. No. 3.

100 parts$=0{\cdot}27397$ grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+Y, G+X	28·30	G+X, T+D+R+Y	23·15
				....	28·00	....	19·65
12·55	12·55			....	29·51	....	21·54
				....	27·62	....	20·90
				....	28·07	....	18·70
		755·8	13·3	T+D+2R+Y, G+X	23·04	G+X, T+D+2R+Y	24·64
				....	24·89	....	24·05
12·85	12·85			....	21·67	....	20·79
				....	22·80	....	20·29
				....	22·16	....	20·30
					256·06		214·01

$$20(G+X)\bumpeq 20(T+D+Y)+30R+42{\cdot}05 \text{ parts.}$$

D.	C.	F.	E.		Scale.		Scale.
				T+D+2R+X, G+Y	19·74	G+Y, T+D+2R+X	19·60
				....	22·09	....	22·70
13	13·05			....	23·47	....	22·87
				....	20·30	....	21·32
				....	23·27	....	22·47
		756·3	13·6	....	22·25	....	21·30
				....	23·35	....	20·89
13·4	13·45			....	20·16	....	22·34
				....	20·62	....	21·24
				....	19·62	....	18·97
					214·87		213·70

$20(G+Y) \bumpeq 20(T+D+X)+40R+1{\cdot}17$ part.

$40G \bumpeq 40(T+D)+70R+43{\cdot}22$ parts, $M=12{\cdot}96$, $F=756{\cdot}05$, $E=13{\cdot}45$.

$G \bumpeq T+D+0{\cdot}02276$ grain in air ($t=12{\cdot}99$, $b=754{\cdot}86$).

$G=I+0{\cdot}00510$ grain.

$G \bumpeq W+0{\cdot}02512$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 4.

100 parts=0·27450 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+X, G+Y	30·12	G+Y, T+D+R+X	23·31
				....	22·02	....	17·34
12·75	12·78			....	25·09	....	16·42
				....	23·71	....	13·98
				....	21·66	....	15·15
		764·6	13·5	....	22·77	....	14·45
				T+D+2R+X, G+Y	15·36	G+Y, T+D+2R+X	14·84
				....	15·14	....	14·35
13·25	13·25			....	17·54	....	15·31
				....	15·86	....	16·81
				....	16·65	....	13·57
					225·92		175·53

$22(G+Y) \bumpeq 22(T+D+X)+32R+50{\cdot}32$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+2R+Y, G+X	17·09	G+X, T+D+2R+Y	16·57
				....	16·90	....	16·45
13·2	13·25			....	17·29	....	14·69
				....	15·44	....	15·50
				....	14·87	....	14·94
		762·8	13·8	....	16·12	....	13·61
				....	17·60	....	15·07
				....	14·97	....	14·47
13·4	13·4			....	15·95	....	14·94
				....	15·10	....	12·30
				....	14·45	....	14·72
					175·78		163·26

$22(G+X) \bumpeq 22(T+D+Y)+44R+12{\cdot}52$ parts.

$44G \bumpeq 44(T+D)+76R+62{\cdot}84$ parts, $M=13{\cdot}16$, $F=763{\cdot}7$, $E=13{\cdot}65$.

$G \bumpeq T+D+0{\cdot}02346$ grain in air ($t=13{\cdot}19$, $b=762{\cdot}47$).

$G=I+0{\cdot}00425$ grain.

$G \bumpeq W+0{\cdot}03157$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 5.

100 parts = 0·27485 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+R+X	16·60	G+R+X, T+D+Y	16·00
				....	19·16	....	15·17
13	13·03			....	18·36	....	12·96
				....	15·47	....	10·62
				....	16·62	....	10·46
		767·55	13·8	T+D+Y, G+X	18·25	G+X, T+D+Y	17·74
				....	16·22	....	19·70
13·45	13·45			....	18·75	....	21·37
				....	18·15	....	21·35
				....	19·32	....	21·15
					176·90		166·52

$$20(G+X) \bumpeq 20(T+D+Y) - 10R + 10{\cdot}38 \text{ parts.}$$

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	15·50	G+Y, T+D+X	20·17
				....	17·69	....	19·17
13·7	13·75			....	16·21	....	17·95
				....	15·30	....	18·06
				....	16·90	....	17·57
		767·5	14·1	....	16·35	....	19·62
				....	14·67	....	19·17
13·87	13·87			....	16·95	....	19·42
				....	16·60	....	19·15
				....	14·95	....	18·87
					161·12		189·15

$20(G+Y) \bumpeq 20(T+D+X) - 28{\cdot}03$ parts.

$40G \bumpeq 40(T+D) - 10R - 17{\cdot}65$ parts, $M = 13{\cdot}51$, $F = 767{\cdot}52$, $E = 13{\cdot}95$.

$G \bumpeq T + D - 0{\cdot}00404$ grain in air ($t = 13{\cdot}54$, $b = 766{\cdot}24$).

$G = I + 0{\cdot}01783$ grain.

$G \bumpeq W + 0{\cdot}00734$ grain in air ($\log \Delta = 7{\cdot}07832 - 10$).

G = lb. No. 6.

100 parts = 0·28770 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	17·49	G+Y, T+D+X	23·77
				....	20·07	....	23·05
14·35	14·35			....	18·75	....	20·84
				....	18·49	....	19·84
				....	18·56	....	22·10
		764·85	14·9	....	18·61	....	22·77
				....	19·65	....	23·31
14·6	14·6			T+D+X, G+R+Y	23·04	G+R+Y, T+D+X	18·54
				....	21·77	....	18·62
				....	21·16	....	17·37
					197·59		210·21

$$20(G+Y) \bumpeq 20(T+D+X) - 6R - 12{\cdot}62 \text{ parts.}$$

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	17·65	G+X, T+D+Y	21·09
				....	18·20	....	21·02
15	15			....	19·61	....	21·05
				....	17·54	....	20·52
				....	17·60	....	19·56
		764·6	15·3	....	18·65	....	19·72
				....	18·62	....	20·19
15·1	15·3			....	15·72	....	19·12
				....	16·90	....	18·64
				....	16·50	....	19·12
					176·99		200·03

$20(G+X) \bumpeq 20(T+D+Y) - 23{\cdot}04$ parts.

$40G \bumpeq 40(T+D) - 6R - 35{\cdot}66$ parts, $M=14{\cdot}77$, $F=764{\cdot}72$, $E=15{\cdot}1$.

$G \bumpeq T+D-0{\cdot}00426$ grain in air ($t=14{\cdot}79$, $b=763{\cdot}30$).

$G=I-0{\cdot}01714$ grain.

$G \bumpeq W+0{\cdot}00083$ grain in air ($\log \Delta = 7{\cdot}07832-10$).

G=lb. No. 7.

100 parts=0·26008 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+Y, G+X	18·15	G+X, T+D+R+Y	20·82
				....	21·24	....	22·49
14·2	14·2			....	20·66	....	22·74
				....	19·31	....	21·29
				....	18·46	....	19·34
		766·4	15·2	....	20·11	....	21·54
				....	21·06	....	20·34
14·6	14·6			....	20·77	....	21·36
				....	19·32	....	18·69
				....	20·36	....	19·67
					199·44		208·28

$20(G+X) \bumpeq 20(T+D+Y)+20R-8{\cdot}84$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+X, G+Y	18·22	G+Y, T+D+R+X	19·10
				....	18·72	....	19·19
14·83	14·83			....	15·92	....	17·21
				....	17·34	....	17·72
				....	17·62	....	19·79
		765·9	15·2	....	18·66	....	20·21
				....	16·44	....	20·20
15	15			....	18·30	....	19·95
				....	16·19	....	19·72
				....	16·39	....	20·10
					173·80		193·19

$20(G+Y) \bumpeq 20(T+D+X)+20R-19{\cdot}39$ parts.

$40G \bumpeq 40(T+D)+40R-28{\cdot}23$ parts, $M=14{\cdot}66$, $F=766{\cdot}15$, $E=15{\cdot}2$.

$G \bumpeq T+D+0{\cdot}00948$ grain in air ($t=14{\cdot}68$, $b=764{\cdot}72$).

$G=I+0{\cdot}01933$ grain.

$G \bumpeq W+0{\cdot}01658$ grain in air ($\log \Delta = 7{\cdot}07832-10$).

G=lb. No. 8.

100 parts=0·28329 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+X, G+Y	30·32	G+Y, T+D+R+X	24·32
				....	20·42	....	12·95
14·8	14·8			....	25·95	....	19·30
				....	24·67	....	17·81
				....	23·59	....	16·55
		752·4	15·5	T+D+2R+X, G+Y	20·24	G+Y, T+D+2R+X	21·62
				....	21·17	....	20·57
15·25	15·25			....	19·71	....	20·37
				....	19·76	....	20·56
				....	20·34	....	19·32
					226·17		193·37

$20(G+Y) \bumpeq 20(T+D+X)+30R+32{\cdot}8$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+2R+Y, G+X	18 22	G+X, T+D+2R+Y	15·90
				....	17·50	....	16·85
15·95	15·95			....	18·31	....	17·54
				....	16·31	....	15·22
				....	16·27	....	13·00
		753·0	16	....	14·76	....	14·10
				....	14·07	....	12·54
16·35	16·35			....	14·56	....	12·82
				....	16·40	....	12·46
				....	13·77	....	12·21
					160·17		142·64

$20(G+X) \bumpeq 20(T+D+Y)+40R+17{\cdot}53$ parts.

$40G \bumpeq 40(T+D)+70R+50{\cdot}33$ parts, $M=15{\cdot}59$, $F=752{\cdot}7$, $E=15{\cdot}75$.

$G \bumpeq T+D+0{\cdot}02336$ grain in air ($t=15{\cdot}61$, $b=751{\cdot}24$).

$G=I+0{\cdot}01428$ grain.

$G \bumpeq W+0{\cdot}01679$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 9.

100 parts=0·40919 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	24·34	G+Q+X, I+Y	21·67
				....	23·52	....	20·59
9·9	9·9	740·6	11	....	23·17	....	19·27
				....	22·55	....	18·57
				....	21·87	....	17·36
10·27	10·3	740·0	11·1	....	21·47	....	16·66
				....	20·94	....	16·31
				....	20·57	....	15·49
10·53	10·57	739·95	11·4	....	17·94	....	12·92
				....	17·34	....	12·49
					213·71		171·33

$20(G+Q+X) \bumpeq 20(I+Y)+42{\cdot}38$ parts

D.	C.	F.	E.		Scale.		Scale.
				I+R+X, G+Q+Y	18·77	G+Q+Y, I+R+X	19·70
10·87	10·83	741·6	11·6	....	18·87	....	19·67
				....	17·32	....	19·02
				....	17·72	....	18·55
11·1	11·1			....	17·30	....	19·17
		742·0	11·6	....	16·82	....	18·09
				....	16·46	....	18·39
				....	16·55	....	18·24
11·15	11·2	742·9	11·7	....	16·05	....	17·36
				....	15·89	....	17·74
					171·75		185·93

20(G+Q+Y)≏20(I+X)+20R−14·18 parts.

40(G+Q)≏40I+20R+28·2 parts, M=10·64, F=741·17, E=11·42.

G+Q≏I+0·00854 grain in air (t=10·65, b=740·25).

G=I+0·11611 grain.

G≏W+0·00426 grain in air (log Δ=7·07832−10).

G=lb. No. 10.

May 28, 1854. 100 parts=0·28329 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+2R+X	21·55	G+2R+X, T+D+Y	19·56
14·6	14·6			....	23·96	....	21·32
				....	24·06	....	21·27
				....	22·36	....	19·46
				....	22·62	....	20·05
		756·2	15	T+D+Y, G+R+X	18·17	G+R+X, T+D+Y	24·45
				....	18·79	....	24·76
				....	17·95	....	24·06
				....	17·15	....	23·40
14·9	14·87			....	17·92	....	23·05
					204·53		221·38

20(G+X)≏20(T+D+Y)−30R−16·85 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+2R+Y	19·82	G+2R+Y, T+D+X	17·85
15·05	15·05			....	20·87	....	18·61
				....	21·75	....	20·41
				....	22·16	....	20·15
				....	23·66	....	20·34
		756·3	15·4	....	22·85	....	20·20
				....	21·69	....	19·75
				....	19·79	....	17·37
15·2	15·2			....	20·71	....	17·49
				....	21·24	....	18·70
					214·54		190·87

20(G+Y)≏20(T+D+X)−40R+23·67 parts.

40G≏40(T+D)−70R+6·82 parts, M=14·94, F=756·25, E=15·2.

G≏T+D−0·01931 grain in air (t=14·96, b=754·84).

G=I−0·03910 grain.

G≏W−0·02162 grain in air (log Δ=7·07832−10).

G=lb. No. 11.

January 6, 1853. 100 parts=0·41459 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+2R+Y	25·89	G+Q+2R+Y, I+X	24·55
10	10	749·9	10·8	....	25·85	....	23·57
				....	25·57	....	22·34
				....	24·59	....	21·65
				....	24·52	....	21·04
10·27	10·27	749·6	10·9	....	22·55	....	20·85
				....	23·19	....	19·86
				....	22·55	....	19·75
10·45	10·45	749·03	10·9	....	22·27	....	19·62
				....	22·51	....	19·32
					239·49		212·55

$$20(G+Q+Y) \bumpeq 20(I+X)-40R+26{\cdot}94 \text{ parts.}$$

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+R+X	18·57	G+Q+R+X, I+Y	20·96
10·53	10·6	748·8	11	....	19·01	....	21·40
				....	18·15	....	20·30
				....	17·57	....	20·86
				....	17·21	....	19·17
10·8	10·8	748·8	11·1	....	17·25	....	19·42
				....	16·37	....	18·12
				....	16·26	....	18·75
10·85	10·85	748·75	11·5	....	16·04	....	17·74
				....	15·35	....	17·32
					171·78		194·04

$20(G+Q+X) \bumpeq 20(I+Y)-20R-22{\cdot}26$ parts.

$40(G+Q) \bumpeq 40I-60R+4{\cdot}68$ parts, $M=10{\cdot}49$, $F=749{\cdot}14$, $E=11{\cdot}03$.

$G+Q \bumpeq I-0{\cdot}01648$ grain in air ($t=10{\cdot}49$, $b=748{\cdot}25$).

$G=I-0{\cdot}04208$ grain.

$G \bumpeq W-0{\cdot}01499$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 12.

December 28, 1852. 100 parts=0·39337 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	17·57	G+Q+X, I+Y	19·24
				....	16·66	....	17·70
10·4	10·4	752·4	11·4	....	17·92	....	19·00
				....	17·77	....	18·54
				....	20·52	....	21·50
10·8	10·8	752·5	11·5	....	19·99	....	20·17
				....	19·57	....	19·70
				....	19·79	....	19·59
10·73	10·73	752·5	11·5	....	19·40	....	18·62
				....	18·72	....	18·90
					187·91		192·96

$$20(G+Q+X) \bumpeq 20(I+Y)-5{\cdot}05 \text{ parts.}$$

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+Y	16·96	G+Q+Y, I+X	18·99
				....	17·02	....	18·89
11·3	11·3	752·9	11·65	....	18·06	....	19·40
				....	18·61	....	20·12
				....	17·29	....	19·34
11·5	11·5	753·3	12	....	16·70	....	19·50
				....	19·31	....	18·71
				....	17·45	....	18·85
11·6	11·55	753·65	12	....	16·72	....	18·76
				....	16·77	....	17·91
					174·89		190·47

$20(G+Q+Y) \stackrel{\wedge}{=} 20(I+X)-15{\cdot}58$ parts.

$40(G+Q) \stackrel{\wedge}{=} 40I-20{\cdot}63$ parts, $M=11{\cdot}1$, $F=752{\cdot}88$, $E=11{\cdot}69$.

$G+Q \stackrel{\wedge}{=} I-0{\cdot}00203$ grain in air ($t=11{\cdot}11$, $b=751{\cdot}9$).

$G=I-0{\cdot}02060$ grain.

$G \stackrel{\wedge}{=} W+0{\cdot}00118$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 13.

December 29, 1852. 100 parts=0·38625 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+R+Y	23·04	G+Q+R+Y, I+X	20·54
				....	22·84	....	20·85
9·95	9·95	755·3	10·6	....	20·79	....	19·35
				....	20·90	....	18·17
				....	20·22	....	17·50
10·45	10·5	754·4	10·9	....	19·19	....	16·82
				....	19·45	....	15·65
				....	19·00	....	15·27
10·52	10·52	754·29	10·95	....	19·01	....	15·25
				....	19·82	....	14·71
					204·26		174·11

$20(G+Q+Y) \stackrel{\wedge}{=} 20(I+X)-20R+30{\cdot}15$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+R+X	19·26	G+Q+R+X, I+Y	15·19
				....	19·70	....	13·60
10·65	10·65	753·85	11·1	....	18·45	....	12·29
				....	18·30	....	12·64
				I+Y, G+Q+X	14·89	G+Q+X, I+Y	14·94
10·8	10·8	753·7	11·3	....	14·49	....	14·61
				I+R+Y, G+Q+X	11·41	G+Q+X, I+R+Y	18·45
				....	12·54	....	18·01
11	11	753·6	11·2	I+Y, G+Q+X	14·11	G+Q+X, I+Y	14·92
				....	15·19	....	14·41
					158·34		149·06

$20(G+Q+X) \stackrel{\wedge}{=} 20(I+Y)-4R+9{\cdot}28$ parts.

$40(G+Q) \stackrel{\wedge}{=} 40I-24R+39{\cdot}43$ parts, $M=10{\cdot}67$, $F=754{\cdot}00$, $E=11{\cdot}07$.

$G+Q \stackrel{\wedge}{=} I-0{\cdot}00298$ grain in air ($t=10{\cdot}68$, $b=753{\cdot}21$).

$G=I-0{\cdot}03331$ grain.

$G \stackrel{\wedge}{=} W+0{\cdot}00195$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 14.

January 5, 1853. 100 parts=0·42206 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	17·67	G+Q+X, I+Y	21·62
				....	17·20	....	21·20
10·43	10·47	753·85	11·3	....	15·97	....	19·99
				....	16·19	....	19·24
				....	14·34	....	18·17
10·75	10·8	754·1	11·6	....	14·39	....	17·70
				I+Y, G+Q+R+X	16·40	G+Q+R+X, I+Y	15·35
				....	16·40	....	14·50
11	11	754·0	11·8	....	16·10	....	14·31
				....	16·10	....	14·39
					160·76		176·47

$20(G+Q+X) \bumpeq 20(I+Y)-8R-15{\cdot}71$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+R+Y	21·02	G+Q+R+Y, I+X	19·56
				....	21·41	....	19·59
11·35	11·35	754·0	12	....	20·50	....	18·84
				....	20·71	....	19·49
				....	21·32	....	18·70
11·35	11·35	754·2	12	....	21·05	....	18·22
				....	20·59	....	17·52
				....	20·01	....	17·55
11·45	11·45	754·25	12	....	19·89	....	17·05
				....	19·90	....	16·74
					206·40		183·26

$20(G+Q+Y) \bumpeq 20(I+X)-20R+23{\cdot}14$ parts.

$40(G+Q) \bumpeq 40I-28R+7{\cdot}43$ parts, $M=11{\cdot}07$, $F=754{\cdot}06$, $E=11{\cdot}78$.

$G+Q \bumpeq I-0{\cdot}00713$ grain in air ($t=11{\cdot}08$, $b=753{\cdot}07$).

$G=I-0{\cdot}02844$ grain.

$G \bumpeq W-0{\cdot}00297$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 15.

January 30, 31, 1851. 100 parts=0·34349 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	18·80	G+Q+X, I+Y	18·90
				....	17·84	....	17·04
				....	17·26	....	16·46
				....	17·66	....	15·89
10·15	10·2	749·7	11	....	18·30	....	15·54
				....	18·22	....	15·00
				....	16·89	....	15·01
				....	17·49	....	14·70
				....	16·35	....	13·69
				....	16·07	....	13·07
					174·88		155·30

$20(G+Q+X) \bumpeq 20(I+Y)+19{\cdot}58$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+Y	22·60	G+Q+Y, I+X	16·50
				....	24·50	....	16·98
				....	23·96	....	17·96
				....	23·75	....	18·21
9·0	9·03	742·75	9·5	I+0·01 gr.+X, G+Q+Y	19·55	G+Q+Y, I+0·01 gr.+X	19·95
				....	19·79	....	20·50
				....	20·50	....	19·30
				....	20·82	....	19·74
				....	20·92	....	21·31
				....	21·75	....	20·45
					218·14		190·90

$20(G+Q+Y) \bumpeq 20(I+X)+0·12$ gr. $+27·24$ parts.

$40(G+Q) \bumpeq 40I+0·12$ gr. $+46·82$ parts, $M=9·59$, $F=746·22$, $E=10·25$.

$G+Q \bumpeq I+0·00702$ grain in air ($t=9·59$, $b=745·43$).

$G=I-0·01864$ grain.

$G \bumpeq W+0·00820$ grain in air ($\log \Delta=7·07832-10$).

G=lb. No. 15.

March 18, 1854. 100 parts=0·25957 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+2R+Y	20·12	G+2R+Y, T+D+X	15·65
				....	21·75	....	14·62
				....	22·95	....	15·50
				....	21·20	....	16·15
10·15	10·2			T+D+X, G+R+Y	17·20	G+R+Y, T+D+X	18·65
				....	17·42	....	20·20
				....	16·47	....	17·02
				....	18·50	....	17·95
				....	16·30	....	17·45
		764·0	10·9	....	15·55	....	18·17
					189·46		171·36

$20(G+Y) \bumpeq 20(T+D+X)-28R+18·10$ parts.

D.	C.	F.	E.		Scale.		Scale.
		764·0	10·9	T+D+Y, G+R+X	18·80	G+R+X, T+D+Y	16·29
				....	18·39	....	14·27
				....	19·54	....	16·00
				....	21·42	....	16·25
10·48	10·53			T+D+Y, G+X	14·57	G+X, T+D+Y	20·87
				....	18·82	....	19·56
				....	16·06	....	20·95
				....	14·95	....	20·12
				....	16·15	....	19·50
				....	15·72	....	20·51
					170·42		184·32

$20(G+X) \bumpeq 20(T+D+Y)-8R-13·9$ parts.

$40G \bumpeq 40(T+D)-36R+4·2$ parts, $M=10·34$, $F=764·0$, $E=10·9$.

$G \bumpeq T+D-0·00991$ grain in air ($t=10·34$, $b=763·1$).

$G=I-0·02180$ grain.

$G \bumpeq W+0·00504$ grain in air ($\log \Delta=7·07832-10$).

Means of both series of comparisons:—

$G=I-0·02022$ grain.

$G \bumpeq W+0·00662$ grain in air ($\log \Delta=7·07832-10$).

G=lb. No. 16.

December 31, 1852.

100 parts=0·38541 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+5R+Y	24·06	G+Q+5R+Y, I+X	24·01
				...	22·67	...	21·71
10·4	10·4	766·2	11·1	...	22·04	...	21·29
				...	21·89	...	19·82
				...	21·70	...	18·44
10·75	10·75	765·87	11·5	...	17·86	...	16·67
				...	23·24	...	20·17
				...	21·85	...	19·97
11·07	11·07	765·7	11·7	...	21·62	...	18·71
				...	22·19	...	18·37
					219·12		199·16

$20(G+Q+Y) \bumpeq 20(I+X)-100R+19{\cdot}96$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+4R+X	19·42	G+Q+4R+X, I+Y	22·02
				...	17·55	...	22·55
11·35	11·3	765·6	11·8	...	17·96	...	19·89
				...	17·77	...	20·22
				...	18·82	...	19·19
11·4	11·4	765·5	11·8	...	18·84	...	21·27
				...	17·95	...	21·50
				...	17·07	...	20·57
11·5	11·5	765·35	11·9	...	17·15	...	19·65
				...	17·59	...	20·25
					180·12		207·11

$20(G+Q+X) \bumpeq 20(I+Y)-80R-26{\cdot}99$ parts.

$40(G+Q) \bumpeq 40I-180R-7{\cdot}03$ parts, $M=11{\cdot}07$, $F=765{\cdot}7$, $E=11{\cdot}61$.

$G+Q \bumpeq I-0{\cdot}05159$ grain in air ($t=11{\cdot}08$, $b=764{\cdot}71$).

$G=I-0{\cdot}02747$ grain.

$G \bumpeq W-0{\cdot}03636$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 17*a*.

April 8, 1854.

100 parts=0·30837 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+2R+X	14·39	G+2R+X, T+D+Y	24·65
				...	15·44	...	25·15
				...	16·15	...	24·29
				...	16·17	...	23·70
				...	15·65	...	24·47
14·2	14·2			T+D+Y, G+3R+X	18·64	G+3R+X, T+D+Y	19·80
				...	18·69	...	20·37
				...	18·47	...	20·04
				...	18·65	...	18·67
		767·1	14·7	...	17·37	...	16·20
					168·59		217·34

$20(G+X)+50R \bumpeq 20(T+D+Y)-47{\cdot}75$ parts.

D.	C.	F.	E.		Scale.		Scale.
		767·1	14·7	T+D+X, G+3R+Y	14·32	G+3R+Y, T+D+X	19·57
				....	15·20	....	20·25
				....	15·27	....	19·20
				....	15·15	....	20·15
				....	16·45	....	20·47
14·5	14·55			....	15·41	....	19·50
				....	15·42	....	18·80
				....	14·40	....	17·31
				....	13·90	....	19·25
				....	15·00	....	18·32
					150·52		192·82

$20(G+Y)+60R \bumpeq 20(T+D+X)-42{\cdot}3$ parts.

$40G+110R \bumpeq 40(T+D)-90{\cdot}05$ parts, $M=14{\cdot}36$, $F=767{\cdot}1$, $E=14{\cdot}7$.

$G \bumpeq T+D-0{\cdot}03811$ grain in air ($t=14{\cdot}38$, $b=765{\cdot}72$).

$G=I-0{\cdot}02614$ grain.

$G \bumpeq W-0{\cdot}02944$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 17*b*.

100 parts = 0·29133 grain.

	Scale.		Scale.
T+D+X, G+Y	16·97	G+Y, T+D+X	19·52
....	19·65	....	20·65
....	18·15	....	20·07
T+D+Y, G+X	20·82	G+X, T+D+Y	14·12
T+D+R+Y, G+X	17·10	G+X, T+D+R+Y	21·55
T+D+Y, G+X	21·15	G+X, T+D+R+Y	21·01
	113·84		116·92

$12G \bumpeq 12(T+D)+3R-3{\cdot}08$ parts, $M=11{\cdot}91$, $F=755{\cdot}1$, $E=12{\cdot}6$.

$G \bumpeq T+D+0{\cdot}00208$ grain in air ($t=11{\cdot}93$, $b=754{\cdot}01$).

$G=I-0{\cdot}04475$ grain.

$G \bumpeq W+0{\cdot}00529$ grain in air ($\log \Delta=7{\cdot}07832-10$).

100 parts = 0·27861 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	19·97	G+Y, T+D+X	13·44
				....	19·15	....	12·12
				....	18·30	....	13·85
				....	20·05	....	14·44
13·15	13·2			....	19·52	....	14·16
		749·1	13·65	T+D+R+X, G+Y	16·41	G+Y, T+D+R+X	16·87
				....	13·89	....	14·95
				....	16·11	....	15·26
				....	17·76	....	15·59
				....	14·71	....	17·81
					172·87		148·49

$20(G+Y) \bumpeq 20(T+D+X)+10R+24{\cdot}38$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+R+Y, G+X	22·82	G+X, T+D+R+Y	23·71
				....	23·50	....	23·90
				....	23·74	....	25·10
				....	25·00	....	24·09
13·45	13·45			....	23·41	....	23·62
		749·3	13·8	....	21·66	....	21·80
				....	24·37	....	21·55
				....	22·85	....	23·91
				....	25·50	....	22·32
				....	26·22	....	24·00
					239·07		234·00

$20(G+X) \bumpeq 20(T+D+Y)+20R+5{\cdot}07$ parts.

$40G \bumpeq 40(T+D)+30R+29{\cdot}45$ parts, $M=13{\cdot}31$, $F=749{\cdot}2$, $E=13{\cdot}72$.

$G \bumpeq T+D+0{\cdot}01054$ grain in air ($t=13{\cdot}34$, $b=747{\cdot}99$).

$G=I-0{\cdot}04413$ grain.

$G \bumpeq W+0{\cdot}00591$ grain in air ($\log \Delta=7{\cdot}07832-10$).

Means ......$G=I-0{\cdot}04427$ grain.

$G \bumpeq W+0{\cdot}00577$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 18.

100 parts = 0·41249 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+S+Y, G+Q+X	16·34	G+Q+X, I+S+Y	17·80
11·2	11·2	752·75	12	....	16·25	....	18·25
				....	16·59	....	18·02
11·37	11·37	752·8	11·9	....	16·35	....	17·54
				....	15·57	....	17·82
				....	14·77	....	15·14
11·5	11·5	752·8	11·95	....	15·06	....	15·84
				....	14·47	....	16·00
11·55	11·55	752·83	12	....	14·66	....	14·90
				....	14·47	....	15·82
					154·53		167·13

$20(G+Q+X) \bumpeq 20(I+Y)+20S+12{\cdot}6$ parts.

D.	C.	F.	E.		Scale.		Scale,
				I+S+X, G+Q+Y	20·34	G+Q+Y, I+S+X	18·12
11·65	11·65	753·6	12·1	....	21·20	....	16·65
				I+2S+X, G+Q+Y	17·55	G+Q+Y, I+2S+X	18·92
11·75	11·75	753·7	12·2	....	17·74	....	19·89
				....	17·07	....	18·21
				....	16·86	....	19·29
11·9	12	753·7	12·5	....	15·69	....	17·97
				....	14·52	....	17·40
12	12	753·7	12·5	....	15·90	....	17·84
				....	16·41	....	17·62
					173·28		181·91

$20(G+Q+Y) \bumpeq 20(I+X)+36S-8{\cdot}63$ parts.

$40(G+Q) \bumpeq 40I+56S+3{\cdot}97$ parts, $M=11{\cdot}61$, $F=753{\cdot}23$, $E=12{\cdot}13$.

$G+Q \bumpeq I+0{\cdot}01631$ grain in air ($t=11{\cdot}63$, $b=752{\cdot}2$).

$G=I-0{\cdot}00129$ grain.

$G \bumpeq W+0{\cdot}01862$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 19

100 parts=0·40766 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+S+Y, G+Q+X	19·66	G+Q+X, I+S+Y	18·52
				...	18·54	...	17·50
11·55	11·6	747·6	12·6	...	17·76	...	17·34
				...	18·75	...	17·57
				...	19·04	...	17·80
11·93	11·93	747·4	12·6	...	17·50	...	16·52
				...	16·96	...	15·60
				...	16·92	...	16·44
12·07	12·15	747·5	12·9	...	15·17	...	15·55
				...	16·16	...	14·74
					176·46		167·58

$20(G+Q+X) \bumpeq 20(I+Y)+20S+8{\cdot}88$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+2S+X, G+Q+Y	18·80	G+Q+Y, I+2S+X	17·69
12·8	12·8	736·9	13·35	...	18·72	...	17·54
				...	17·44	...	18·07
				...	17·50	...	16·12
				...	17·02	...	16·92
12·95	12·95	737·1		...	16·62	...	17·44
				...	16·01	...	17·27
				...	17·54	...	16·87
13	13·05	737·2	13·1	...	15·81	...	16·45
				...	17·87	...	16·27
					173·33		170·64

$20(G+Q+Y) \bumpeq 20(I+X)+40S+2{\cdot}69$ parts.

$40(G+Q) \bumpeq 40I+60S+11{\cdot}57$ parts, $M=12{\cdot}4$, $F=742{\cdot}28$, $E=12{\cdot}87$.

$G+Q \bumpeq I+0{\cdot}01822$ grain in air ($t=12{\cdot}42$, $b=741{\cdot}22$).

$G=I-0{\cdot}01473$ grain.

$G \bumpeq W+0{\cdot}00954$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 21.

100 parts=0·38653 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	22·66	G+Q+X, I+Y	18·64
				...	22·77	...	18·89
11·25	11·25	750·9	11·8	I+R+Y, G+Q+X	19·71	G+Q+X, I+R+Y	20·71
				...	19·69	...	21·15
				...	19·15	...	20·87
11·4	11·4	750·8	12	...	19·17	...	21·00
				...	18·80	...	20·56
				...	19·35	...	20·20
11·5	11·5	750·45	12	...	19·14	...	20·85
				...	19·12	...	20·04
					199·53		202·91

$20(G+Q+X) \bumpeq 20(I+Y)+16R-3{\cdot}38$ parts.

D.	C.	F.	E.	I+X, G+Q+Y	Scale.	G+Q+Y, I+X	Scale.
					18·21		17·90
				....	19·07	....	17·22
10·5	10·5	751·63	11·4	....	18·87	....	16·16
				....	18·09	....	15·85
				....	18·52	....	15·96
10·9	10·9	751·33	11·6	....	16·75	....	14·95
				....	17·06	....	14·10
				....	16·82	....	13·62
11·05	11·05	751·20	11·7	....	16·27	....	13·50
				....	16·36	....	13·25
					176·02		152·51

20(G+Q+Y)≏20(I+X)+23·51 parts.

40(G+Q)≏40I+16R+20·13 parts, M=11·1, F=751·05, E=11·76.

G+Q≏I+0·00647 grain in air (t=11·2, b=760·07).

G=I+0·03971 grain.

G≏W+0·01779 grain in air (log Δ=7·07832−10).

G=lb. No. 22.

100 parts=0·38541 grain.

D.	C.	F.	E.	I+Y, G+Q+R+X	Scale.	G+Q+R+X, I+Y	Scale.
					23·66		24·26
10·87	10·87	756·65	11·8	....	20·81	....	22·32
				....	21·47	....	22·30
				....	20·87	....	21·62
11·35	11·35	756·85	11·8	....	19·40	....	20·37
				....	18·45	....	20·29
				....	19·39	....	19·45
11·45	11·47	757·20	11·9	....	19·45	....	19·62
				....	18·84	....	18·35
				....	20·21	....	18·92
					202·55		207·50

20(G+Q+X)+20R≏20(I+Y)−4·95 parts.

D.	C.	F.	E.	I+X, G+Q+R+Y	Scale.	G+Q+R+Y, I+X	Scale.
					17·39		19·59
				....	18·72	....	19·37
11·55	11·57	758·63	12·1	....	17·41	....	17·49
				....	16·92	....	17·19
				....	16·65	....	16·36
11·7	11·7	758·65	12·2	....	17·96	....	16·99
				....	17·74	....	17·82
				....	18·02	....	18·12
11·85	11·8	758·73	12·3	....	17·96	....	18·12
				....	17·52	....	17·96
					176·29		179·01

20(G+Q+Y)+20R≏20(I+X)−2·72 parts.

40(G+Q)+40R≏40I−7·67 parts, M=11·46, F=757·77, E=12·06.

G+Q≏I−0·01205 grain in air (t=11·47, b=756·74).

G=I−0·01214 grain.

G≏W−0·00519 grain in air (log Δ=7·07832−10).

G=lb. No. 23.

100 parts=0·38429 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	19·82	G+Q+X, I+Y	16·15
				....	18·96	....	15·99
10·53	10·6	759·15	11·2	....	18·55	....	15·10
				....	18·50	....	13·62
				....	15·56	....	12·00
11	11	759·39	11·6	....	14·80	....	11·57
				....	14·15	....	10·59
				....	14·67	....	10·63
11·2	11·2	759·7	11·6	....	20·57	....	16·59
				....	20·82	....	16·36
					173·40		138·62

$20(G+Q+X) \bumpeq 20(I+Y)+34{\cdot}78$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+R+X, G+Q+Y	18·02	G+Q+Y, I+R+X	19·60
				....	17·87	....	20·19
11·25	11·25	759·9	11·4	....	17·65	....	20·15
				....	17·94	....	20·02
				....	17·72	....	20·25
11·4	11·4	760·3	11·5	....	17·21	....	19·49
				....	17·79	....	19·65
				....	17·27	....	19·55
11·5	11·5	760·65	11·6	....	17·01	....	18·97
				....	17·01	....	18·71
					175·49		196·58

$20(G+Q+Y) \bumpeq 20(I+R+X)-21{\cdot}09$ parts.

$40(G+Q) \bumpeq 40I+20R+13{\cdot}69$ parts, $M=11{\cdot}15$, $F=759{\cdot}85$, $E=11{\cdot}48$.

$G+Q \bumpeq I+0{\cdot}00697$ grain in air ($t=11{\cdot}16$, $b=758{\cdot}88$).

$G=I+0{\cdot}01557$ grain.

$G \bumpeq W+0{\cdot}01659$ grain in air ($\log\Delta=7{\cdot}07832-10$).

G=lb. No. 24.

100 parts=0·40700 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+4R+Y	22·24	G+Q+4R+Y, I+X	22·84
9·6	9·65	747·9	10·6	....	22·27	....	22·74
				....	22·02	....	21·02
				....	20·24	....	21·97
				....	20·47	....	18·56
10	10	747·9	10·6	....	20·47	....	20·32
				....	19·75	....	19·41
				....	20·50	....	19·07
10·15	10·15	747·8	10·8	....	19·12	....	18·20
				....	19·25	....	18·17
					206·33		202·30

$20(G+Q+Y)+80R \bumpeq 20(I+X)+4{\cdot}03$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+4R+X	20·41	G+Q+4R+X, I+Y	16·90
				....	20·36	....	16·67
10·4	10·4	747·8	11	....	19·90	....	16·42
				....	20·02	....	16·79
				I+Y, G+Q+3R+X	15·47	G+Q+3R+X, I+Y	16·36
10·55	10·6	747·7	11·1	....	14·56	....	16·55
				....	14·71	....	16·41
				....	14·72	....	16·05
10·65	10·7	747·7	11·2	....	14·24	....	15·69
				....	15·09	....	15·85
					169·48		163·69

$20(G+Q+X)+68R \bumpeq 20(I+Y)+5{\cdot}79$ parts.

$40(G+Q)+148R \bumpeq 40I+9{\cdot}82$ parts, $M=10{\cdot}23$, $F=747{\cdot}8$, $E=11$.

$G+Q \bumpeq I-0{\cdot}04086$ grain in air ($t=10{\cdot}23$, $b=746{\cdot}91$).

$G=I-0{\cdot}03932$ grain.

$G \bumpeq W-0{\cdot}03937$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 25.

100 parts$=0{\cdot}39825$ grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+2R+Y	17·65	G+Q+2R+Y, I+X	16·42
				....	17·00	....	15·80
10·52	10·55	768·7	11·5	....	15·40	....	14·55
				....	15·61	....	13·94
				....	17·21	....	16·42
10·85	10·85	769·4	11·65	....	16·11	....	15·86
				....	18·01	....	18·65
				....	17·84	....	18·59
11·1	11·1	769·7	11·9	....	16·86	....	18·07
				....	18·62	....	18·20
					170·31		166·50

$20(G+Q+2R) \bumpeq 20(I+X)+3{\cdot}81$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+2R+X	18·35	G+Q+2R+X, I+Y	18·09
				....	18·91	....	17·57
11·3	11·3	770·2	11·9	....	16·17	....	18·04
				....	18·06	....	18·01
				....	17·82	....	17·70
11·45	11·45	770·3	12	....	18·26	....	17·37
				....	17·22	....	16·91
				....	17·70	....	17·00
11·6	11·6	770·5	12·1	....	15·77	....	14·80
				....	14·51		15·64
					172·77		171·13

$20(G+Q+2R+X) \bumpeq 20(I+Y)+1{\cdot}64$ part.

$40G+Q+2R \bumpeq 40I+5{\cdot}45$ parts, $M=11{\cdot}13$, $F=769{\cdot}8$, $E=11{\cdot}84$.

$G+Q \bumpeq I-0{\cdot}02218$ grain in air ($t=11{\cdot}14$, $b=768{\cdot}77$).

$G=I+0{\cdot}00163$ grain.

$G \bumpeq W-0{\cdot}00368$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 25.

100 parts=0·39510 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+R+X	24·76	G+Q+R+X, I+Y	24·95
				....	24·41	....	24·65
9·35	9·35	751·3	10·2	....	24·32	....	23·77
				....	24·12	....	23·04
				....	23·01	....	21·67
9·55	9·6	750·9	10·5	....	22·61	....	20·61
				....	22·32	....	19·95
				....	21·91	....	19·86
9·9	9·9	749·8	10·6	....	21·30	....	18·89
				....	21·44	....	18·59
					230·20		215·98

20(G+Q+X)+20R≏20(I+Y)+14·22 parts.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+Y	16·95	G+Q+Y, I+X	18·52
				....	16·72	....	18·64
10·2	10·2	748·5	10·6	....	15·85	....	18·66
				....	15·22	....	18·11
				....	15·51	....	17·77
10·35	10·35	748·0	10·65	....	16·04	....	18·56
				....	15·64	....	17·96
				....	16·24	....	17·95
10·4	10·4	747·55	10·65	....	17·52	....	18·37
				....	16·64	....	18·21
					162·33		182·75

20(G+Q+Y)≏20(I+X)−20·42 parts.

40(G+Q)+20R≏40I−6·2 parts, M=9·96, F=749·34, E=10·53.

G+Q≏I−0·00627 grain in air (t=9·96, b=748·51).

G=I+0·00265 grain.

G≏W−0·00266 grain in air (log Δ=7·07832−10).

G=lb. No. 25.

100 parts=0·26918 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+3R+Y	17·50	G+3R+Y, T+D+X	13·80
				T+D+X, G+R+Y	7·12	G+R+Y, T+D+X	19·19
				T+D+X, G+2R+Y	18·80	G+2R+Y, T+D+X	22·75
				....	19·09	....	24·15
8·55	8·6			....	22·35	....	25·40
				....	22·24	....	23·15
				....	22·10	....	22·65
				....	19·49	....	22·72
				T+D+X, G+3R+Y	23·37	G+3R+Y, T+D+X	17·81
		762·8	9·3	....	23·25	....	18·24
					194·31		208·84

20(G+Y)+44R≏20(T+D+X)−14·53 parts.

D.	C.	F.	E.		Scale.		Scale.
		762·8	9·3	T+D+Y, G+2R+X	19·27	G+2R+X, T+D+Y	20·91
				....	20·49	....	19·02
				....	21·62	....	20·00
				....	20·50	....	18·87
				....	20·65	....	18·45
8·9	8·93			....	20·70	....	19·07
				....	20·99	....	18·81
				....	20·49	....	17·77
				....	22·22	....	19·32
				....	22·02	....	18·80
					208·95		191·02

20(G+X)+40R≏20(T+D+Y)+17·93 parts.

40G+84R≏40(T+D)+3·4 parts, M=8·74, F=762·8, E=9·3.

G≏T+D−0·02353 grain in air (t=8·73, b=762·1).

G=I+0·00113 grain.

G≏W−0·00418 grain in air (log Δ=7·07832−10).

Means of the three series of comparisons:—

G=I+0·00180 grain.

G≏W−0·00351 grain in air (log Δ=7·07832−10).

G=lb. No. 26.

100 parts=0·28116 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	21·20	G+Y, T+D+X	19·67
14·3	14·35			....	20·32	....	18·80
				....	20·97	....	19·42
				....	22·27	....	19·46
				....	21·47	....	19·67
		753·9	14·8	....	18·75	....	17·71
				....	19·15	....	17·26
				....	20·20	....	18·81
				....	18·62	....	18·07
14·55	14·55			....	18·25	....	18·27
				....	19·91	....	17·40
					221·12		204·54

22(G+Y)≏22(T+D+X)+16·58 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	17·16	G+X, T+D+Y	16·56
·14·9	14·87			....	19·72	....	15·00
				....	19·97	....	15·16
				....	19·60	....	14·45
				....	18·54	....	17·47
		754·4	15·2	....	19·29	....	18·56
				....	20·31	....	16·69
				....	19·67	....	16·29
				....	18·86	....	13·37
15·3	15·3			....	16·87	....	13·92
				....	19·02	....	13·77
					209·01		171·24

22(G+X)≏22(T+D+Y)+37·77 parts.

44G≏44(T+D)+53·35 parts, M=14·76, F=754·15, E=15.

G≏T+D+0·00347 grain in air (t=14·78, b=752·77).

G=I−0·00112 grain.

G≏W+0·00002 grain in air (log Δ=7·07832−10).

G = lb. No. 27.

100 parts = 0·39809 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+2R+Y, G+Q+X	18·09	G+Q+X, I+2R+Y	17·52
				....	18·32	....	17·06
11·8	11·8	742·6	12	....	16·30	....	16·75
				....	16·15	....	16·41
				....	16·76	....	17·07
11·95	11·95	743·6	12·1	....	17·12	....	15·97
				....	16·36	....	15·24
				....	15·91	....	14·84
12	12	744·4	12·2	....	14·85	....	15·40
				....	14·65	....	15·34
					164·51		161·60

$20(G+Q+X) \bumpeq 20(I+2R+Y)+2{\cdot}91$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+2R+X, G+Q+Y	18·46	G+Q+Y, I+2R+X	17·12
				....	17·15	....	16·05
11·05	11·1	739·83	12	....	14·47	....	12·70
				....	13·45	....	13·91
				....	19·89	....	18·46
11·47	11·5	740·7	12	....	18·46	....	19·22
				....	17·14	....	17·47
				....	18·30	....	18·02
11·6	11·63	740·97	12	....	17·11	....	17·25
				....	18·31	....	15·51
					172·74		167·71

$20(G+Q+Y) \bumpeq 20(I+2R+X)+5{\cdot}03$ parts.

$40(G+Q) \bumpeq 40(I+2R)+7{\cdot}94$ parts, $M=11{\cdot}65$, $F=742{\cdot}01$, $E=12{\cdot}05$.

$G+Q \bumpeq I+0{\cdot}02342$ grain in air ($t=11{\cdot}67$, $b=741{\cdot}02$).

$G=I+0{\cdot}01405$ grain.

$G \bumpeq W+0{\cdot}01640$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 28.

100 parts = 0·30166 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+2R+Y	20·67	G+2R+Y, T+D+X	20·67
				....	20·21	....	22·60
				....	19·37	....	21·49
				....	20·55	....	19·46
				....	20·22	....	18·54
14·4	14·4			....	17·62	....	19·52
				....	18·75	....	18·80
				....	19·57	....	21·97
				....	17·46	....	21·19
		772·7	14·1	....	20·50	....	20·72
					194·92		204·96

$20(G+Y)+40R \bumpeq 20(T+D+X)-10{\cdot}04$ parts.

D.	C.	F.	E.		Scale.		Scale.
		772·7	14·1	T+D+Y, G+2R+X	21·59	G+2R+X, T+D+Y	18·40
				....	21·65	....	17·57
				....	21·47	....	16·25
				....	21·46	....	18·16
				....	20·44	....	16·97
13·95	13·95			T+D+Y, G+R+X	17·70	G+R+X, T+D+Y	21·74
				....	17·30	....	19·05
				....	16·71	....	19·22
				....	15·39	....	19·66
				....	15·81	....	19·36
					189·52		186·38

$20(G+X)+30R \bumpeq 20(T+D+Y)+3{\cdot}14$ parts.

$40G+70R \bumpeq 40(T+D)-6{\cdot}9$ parts, $M=14{\cdot}17$, $F=772{\cdot}7$, $E=14{\cdot}1$.

$G \bumpeq T+D-0{\cdot}02032$ grain in air ($t=14{\cdot}19$, $b=771{\cdot}39$).

$G=I-0{\cdot}00416$ grain.

$G \bumpeq W-0{\cdot}00635$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 29.

100 parts = 0·29084 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+R+Y	20·79	G+R+Y, T+D+X	22·32
				....	22·06	....	24·92
				....	21·35	....	25·32
				....	20·75	....	23·36
				....	19·75	....	21·85
13·3	13·3	768·8	13·85	....	22·22	....	24·47
				....	21·07	....	20·56
				....	21·25	....	23·87
				....	20·67	....	22·71
				....	21·07	....	23·31
				....	20·35	....	23·19
					231·33		255·88

$22(G+Y)+22R \bumpeq 22(T+D+X)-24{\cdot}55$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+R+X	22·19	G+R+X, T+D+Y	20·37
				....	23·92	....	19·62
				....	24·87	....	19·45
				....	25·59	....	18·32
				....	23·96	....	16·42
13·65	13·65	767·8	14	T+D+Y, G+X	19·92	G+X, T+D+Y	20·35
				....	20·27	....	20·97
				....	19·24	....	20·59
				....	19·24	....	23·22
				....	20·24	....	20·88
				....	19·95	....	20·20
					238·39		126·21

$22(G+X)+10R \bumpeq 22(T+D+Y)+18{\cdot}0$ parts.

$44G+32R \bumpeq 44(T+D)-6{\cdot}55$ parts, $M=13{\cdot}47$, $F=768{\cdot}3$, $E=13{\cdot}92$.

$G \bumpeq T+D-0{\cdot}00867$ grain in air ($t=13{\cdot}5$, $b=767{\cdot}04$).

$G=I-0{\cdot}00222$ grain.

$G \bumpeq W+0{\cdot}00296$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 30.

100 parts = 0·41249 grain.

D.	C.	F.	E.		Scale.		Scale.
				I+X, G+Q+Y	17·00	G+Q+Y, I+X	20·96
11·65	11·65	754·7	12·8	…………………	18·65	…………………	20·11
				…………………	17·57	…………………	20·35
11·93	11·93	755·0	12·9	…………………	16·87	…………………	19·82
				…………………	16·20	…………………	19·86
				…………………	16·10	…………………	18·70
12·10	12·10	755·0	12·9	…………………	15·36	…………………	18·09
				…………………	14·97	…………………	17·04
12·25	12·3	754·9	13	…………………	14·79	…………………	17·76
				…………………	14·37	…………………	17·84
					161·88		190·53

$20(G+Q+Y) \bumpeq 20(I+X)-28{\cdot}65$ parts.

D.	C.	F.	E.		Scale.		Scale.
				I+Y, G+Q+X	16·10	G+Q+X, I+Y	16·75
12·55	12·55	754·5	13·05	…………………	15·79	…………………	16·97
				…………………	14·90	…………………	16·50
12·55	12·6	754·25	13·1	…………………	14·77	…………………	16·42
				…………………	15·06	…………………	15·24
				…………………	15·05	…………………	15·71
12·65	12·7	754·1	13·1	…………………	15·32	…………………	15·51
				…………………	14·74	…………………	14·45
12·75	12·8	754·1	13·0	…………………	14·62	…………………	14·67
				…………………	14·87	…………………	14·99
					151·22		157·21

$20(G+Q+X) \bumpeq 20(I+Y)-5{\cdot}99$ parts.
$40(G+Q) \bumpeq 40I-34{\cdot}64$ parts, $M=12{\cdot}31$, $F=754{\cdot}57$, $E=12{\cdot}98$.
$G+Q \bumpeq I-0{\cdot}00357$ grain in air ($t=12{\cdot}33$, $b=753{\cdot}42$).
$G=I-0{\cdot}00269$ grain.
$G \bumpeq W-0{\cdot}00140$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 30.

100 parts = 0·24737 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+2R+X	20·32	G+2R+X, T+D+Y	17·90
				………………………	21·12	………………………	16·85
				………………………	19·79	………………………	14·86
				………………………	19·31	………………………	15·82
				T+D+Y, G+R+X	15·86	G+R+X, T+D+Y	21·00
7·55	7·6			………………………	15·37	………………………	20·59
				………………………	14·62	………………………	21·20
				………………………	16·02	………………………	20·15
				………………………	16·67	………………………	21·00
		762·2	8·3	………………………	16·10	………………………	19·57
					175·18		188·94

$20(G+X)+28R \bumpeq 20(T+D+Y)-13{\cdot}76$ parts.

D.	C.	F.	E.		Scale.		Scale.
		762·2	8·3	T+D+X, G+2R+Y	14·95	G+2R+Y, T+D+X	15·34
				....	14·74	....	15·55
				....	15·31	....	15·06
				....	16·15	....	16·41
7·9	7·93			....	16·26	....	18·32
				....	16·02	....	17·14
				....	17·05	....	16·45
				....	16·75	....	17·97
				....	15·60	....	16·45
				....	15·30	....	16·65
					158·12		165·34

20(G+Y)+40R ≏ 20(T+D+X)−7·22 parts.

40G+68R ≏ 40(T+D)−20·98 parts, M=7·74, F=762·2, E=8·3.

G ≏ T+D−0·02053 grain in air (t=7·72, b=761·62).

G=I−0·00072 grain.

G ≏ W+0·00052 grain in air (log Δ=7·07832−10).

A mean of the two series of comparisons gives—

G=I−0·00170 grain.

G ≏ W−0·00044 grain in air (log Δ=7·07832−10).

G=lb. No. 31.

100 parts =0·28531 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	18·60	G+Y, T+D+X	19·81
7·6	7·6			....	18·57	....	19·26
				....	19·04	....	19·11
		739·7	8·2	....	18·40	....	19·06
				....	17·55	....	18·46
7·9	7·85			....	17·44	....	18·52
				....	17·04	....	17·22
				....	17·86	....	17·39
8	8			....	17·67	....	16·47
		739·6	8·4	....	16·70	....	16·45
				....	16·60	....	16·92
				....	17·01	....	16·11
					212·48		214·78

24(G+Y) ≏ 24(T+D+X)−2·3 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	15·80	G+X, T+D+Y	15·45
8·13	8·1			....	16·16	....	15·16
				....	16·85	....	14·75
		739·65	8·6	....	14·84	....	14·41
8·2	8·2			....	16·70	....	14·90
				....	15·46	....	14·02
8·3	8·25			....	16·79	....	14·67
				....	15·02	....	14·02
		739·65	8·6	....	16·41	....	13·45
				....	15·21	....	14·70
8·35	8·3			....	15·57	....	13·59
				....	15·75	....	13·89
					190·56		173·01

24(G+X) ≏ 24(T+D+Y)+17·55 parts.

48G ≏ 48(T+D)+15·25 parts, M=8·01, F=739·65, E=8·45.

G ≏ T+D+0·00091 grain in air (t=8·0, b=739·08).

G=I−0·04410 grain.

G ≏ W+0·00088 grain in air (log Δ=7·07832−10).

G = lb. No. 32.

100 parts = 0·31852 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	17·60	G+X, T+D+Y	13·85
				T+D+R+Y, G+X	18·42	G+X, T+D+R+Y	20·95
15·9	15·8			....	18·60	....	21·72
				....	18·61	....	21·22
				T+D+Y, G+X	21·56	G+X, T+D+Y	18·04
				....	21·86	....	18·65
16·03	15·95			....	22·50	....	18·94
		757·8	16	T+D+R+Y, G+X	18·97	G+X, T+D+R+Y	20·12
				....	18·07	....	21·66
				....	17·35	....	21·96
16	15·9			....	17·92	....	20·42
				T+D+Y, G+X	22·29	G+X, T+D+Y	17·97
				....	20·90	....	17·05
					254·65		252·55

$26(G+X) \bumpeq 26(T+D+Y)+14R+2·1$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	24·36	G+Y, T+D+X	19·81
				....	24·32	....	21·04
15·85	15·8			....	23·51	....	21·35
				....	24·89	....	19·42
				....	23·80	....	21·05
16	15·95			....	23·69	....	19·55
		758·9	16·4	T+D+R+X, G+Y	19·82	G+Y, T+D+R+X	22·54
				....	20·35	....	21·74
16·15	16·05			....	19·42	....	22·47
				....	20·62	....	22·59
				....	20·11	....	22·49
				....	20·04	....	23·10
				....	19·01	....	23·31
					283·94		280·46

$26(G+Y) \bumpeq 26(T+D+X)+14R+3·48$ parts.

$52G \bumpeq 52(T+D)+28R+5·58$ parts, M = 15·94, F = 758·35, E = 16·2.

$G \bumpeq T+D+0·00643$ grain in air (t = 15·96, b = 756·81).

G = I − 0·03431.

G = lb. No. 32.

100 parts = 0·31974 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	24·51	G+Y, T+D+X	26·27
				....	26·07	....	26·64
15·3	15·2			....	26·04	....	26·74
				....	26·12	....	26·10
15·4	15·4			....	26·41	....	26·61
				....	26·19	....	26·66
15·55	15·5			....	25·81	....	25·47
		764·1	16	....	24·31	....	24·99
15·7	15·65			....	25·00	....	25·22
				....	25·10	....	25·52
15·8	15·75			....	24·12	....	23·97
				....	24·04	....	24·22
				....	25·05	....	24·70
					328·77		333·11

$26(G+Y) \bumpeq 26(T+D+X)-4·34$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	25·15	G+X, T+D+Y	27·26
15·25	15·2			....	25·72	....	27·59
				....	26·32	....	27·80
15·5	15·4			....	24·44	....	26·40
				....	25·21	....	26·85
15·6	15·5	768·8	16	....	24·84	....	26·37
				....	24·55	....	25·45
15·75	15·7			....	25·40	....	26·56
				....	24·79	....	26·46
15·95	15·85			....	24·50	....	25·12
				....	24·76	....	25·26
16·03	15·98			....	24·05	....	24·75
				....	24·35	....	24·47
					325·08		340·34

$26(G+X) \bumpeq 26(T+D+Y)-15{\cdot}26$ parts.

$52G \bumpeq 52(T+D)-19{\cdot}60$ parts, $M=15{\cdot}58$, $F=766{\cdot}45$, $E=16$.

$G \bumpeq T+D-0{\cdot}00121$ grain in air ($t=15{\cdot}6$, $b=764{\cdot}92$). $G=I-0{\cdot}03465$ grain.

Mean ...$G=I-0{\cdot}03448$ grain. $G \bumpeq W+0{\cdot}00304$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G=lb. No. 33.

100 parts=0·31374 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	17·00	G+X, T+D+Y	20·30
13·05	13·0			....	15·95	....	19·35
				....	16·09	....	20·39
				....	16·70	....	19·27
13·3	13·25			....	16·77	....	19·97
		761·1	14·1	....	16·45	....	19·89
				....	16·84	....	20·50
13·5	13·5			T+D+Y, G+R+X	20·40	G+R+X, T+D+Y	17·05
				....	19·79	....	16·65
		761·0	14·4	....	20·80	....	16·02
				....	20·52	....	16·27
13·7	13·65			....	19·39	....	14·15
				....	19·91	....	15·54
13·75	13·7			T+D+Y, G+X	16·95	G+X, T+D+Y	19·02
					253·56		254·37

$28(G+X)+12R \bumpeq 28(T+D+Y)-0{\cdot}81$ part.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	19·19	G+Y, T+D+X	23·41
12·85	12·8			....	19·85	....	24·49
				T+D+X, G+R+Y	23·47	G+R+Y, T+D+X	19·09
13·0	13·0			....	22·44	....	18·21
		759·7	14	....	22·20	....	18·79
13·25	13·2			....	22·37	....	18·51
				....	21·57	....	18·52
13·4	13·35			....	21·25	....	16·90
				T+D+X, G+Y	18·01	G+Y, T+D+X	20·46
13·5	13·45			....	16·72	....	19·65
		758·5	14·4	....	17·77	....	19·62
				....	18·57	....	20·75
				....	17·20	....	18·34
				....	17·16	....	18·29
					277·77		275·03

$28(G+Y)+12R \bumpeq 28(T+D+Y)+2{\cdot}74$ parts.

$56G+24R \bumpeq 56(T+D)+1{\cdot}93$ part, $M=13{\cdot}31$, $F=760{\cdot}07$, $E=14{\cdot}22$.

$G \bumpeq T+D-0{\cdot}00474$ grain in air ($t=13{\cdot}34$, $b=758{\cdot}77$).

$G=I-0{\cdot}04144$ grain. $G \bumpeq W-0{\cdot}00063$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 34.

100 parts = 0·28057 grain.

D.	C.	F.	E.		Scale.		Scale.
6·85	6·8			T+D+X, G+R+Y	20·10	G+R+Y, T+D+X	28·05
				T+D+X, G+2R+Y	25·47	G+2R+Y, T+D+X	23·94
7·05	7·0			....	24·92	....	23·44
				....	24·96	....	23·62
7·25	7·25			....	22·87	....	20·95
		760·0	7·8	....	22·60	....	21·32
7·5	7·45			....	20·92	....	19·29
				....	20·72	....	19·47
7·1	7·1			....	20·15	....	19·86
				....	21·42	....	20·17
7·4	7·35			....	21·25	....	19·52
		759·8	8	....	20·64	....	20·05
				....	20·99	....	19·55
				....	21·50	....	18·44
					308·51		297·67

$28(G+Y)+52R \bumpeq 28(T+D+X)+10{\cdot}84$ parts.

D.	C.	F.	E.		Scale.		Scale.
6·35	6·3			T+D+Y, G+2R+X	28·31	G+2R+X, T+D+Y	26·89
				....	28·52	....	26·29
6·55	6·5			....	20·42	....	16·22
				....	20·65	....	17·15
				....	18·99	....	17·17
6·65	6·6			....	18·47	....	15·84
6·65	6·6			T+D+Y, G+R+X	13·35	G+R+X, T+D+Y	19·14
		757·5	7·1	....	14·25	....	18·75
				....	13·16	....	17·32
6·85	6·8			....	15·27	....	18·69
				....	13·50	....	17·81
6·5	6·5			....	18·02	....	22·20
				....	17·92	....	23·11
6·7	6·65			....	19·35	....	23·32
					260·18		279·90

$28(G+X)+40R \bumpeq 28(T+D+Y)-19{\cdot}72$ parts.

$56G+92R \bumpeq 56(T+D)-8{\cdot}88$ parts, $M=6{\cdot}87$, $F=758{\cdot}7$, $E=7{\cdot}45$.

$G \bumpeq T+D-0{\cdot}01904$ grain in air ($t=6{\cdot}84$, $b=758{\cdot}23$).

$G=I-0{\cdot}04584$ grain.

$G \bumpeq W-0{\cdot}00089$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 35.

100 parts = 0·30126 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+Y	21·55	G+Y, T+D+X	27·80
14·8	14·7			T+D+X, G+R+Y	25·27	G+R+Y, T+D+X	25·90
				....	25·50	....	24·81
15	15			....	24·47	....	25·29
				....	23·45	....	23·77
15	14·95			....	24·11	....	22·99
		771·9	15·6	....	24·57	....	23·46
				....	23·92	....	24·04
15·05	15·05			....	24·82	....	23·22
				....	23·89	....	24·51
				....	23·81	....	24·10
14·95	14·9			....	24·40	....	23·66
				....	24·42	....	23·56
					315·18		317·11

$26(G+Y)+24R \bumpeq 26(T+D+X)-1{\cdot}93$ part.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+X	21·72	G+X, T+D+Y	27·60
14·7	14·6			T+D+Y, G+R+X	25·81	G+R+X, T+D+Y	24·32
				....	25·66	....	25·32
				....	24·35	....	24·16
14·9	14·85			....	26·31	....	25·01
				....	24·80	....	24·60
		772	15·6	....	24·45	....	24·26
15·05	15			....	25·42	....	24·16
				....	24·02	....	23·64
				....	24·29	....	23·12
15·13	15·1			....	24·59	....	23·52
				....	25·40	....	24·25
				....	24·90	....	23·56
					321·72		317·52

$26(G+X)+24R \bumpeq 26(T+D+Y)+4{\cdot}2$ parts.

$52G+48R \bumpeq 52(T+D)+2{\cdot}27$ parts, $M=14{\cdot}92$, $F=771{\cdot}95$, $E=15{\cdot}6$.

$G \bumpeq T+D-0{\cdot}01031$ grain in air ($t=14{\cdot}95$, $b=770{\cdot}45$).

$G=I-0{\cdot}04019$ grain.

$G \bumpeq W-0{\cdot}00041$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 36.

100 parts = 0·307298 grain.

D.	C.	F.	E.		Scale.		Scale.
14·8	14·7			T+D+Y, G+R+X	24·50	G+R+X, T+D+Y	26·19
				....	25·12	....	25·70
				....	25·31	....	26·95
				....	25·17	....	26·07
15·15	15·1			....	23·89	....	26·46
				....	24·86	....	25·65
		772·4	16·2	....	25·05	....	26·60
15·35	15·25			....	25·61	....	24·61
				....	26·57	....	26·56
		772·4	16·4	....	24·52	....	24·94
15·35	15·25			....	25·25	....	26·05
				....	25·77	....	26·11
				....	25·75	....	24·92
					327·37		336·81

$26(G+X)+26R \bumpeq 26(T+D+Y)-9{\cdot}44$ parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+R+Y	18·24	G+R+Y, T+D+X	22·09
14·55	14·5			....	17·56	....	20·46
				....	18·91	....	20·46
		776·3	15·5	....	16·92	....	18·44
				....	16·11	....	19·29
14·7	14·6			....	15·76	....	18·11
				T+D+X, G+2R+Y	19·71	G+2R+Y, T+D+X	15·59
				....	19·66	....	15·31
14·9	14·8			....	20·56	....	14·60
		776·1	15·5	T+D+X, G+R+Y	16·15	G+R+Y, T+D+X	18·60
				....	16·99	....	19·12
15	14·9			....	16·67	....	19·34
				....	17·05	....	18·40
					230·29		239·81

$26(G+Y)+32R \bumpeq 26(T+D+X)-9{\cdot}52$ parts.

$52G+58R \bumpeq 52(T+D)-18{\cdot}96$ parts, $M=14{\cdot}93$, $F=774{\cdot}3$, $E=15{\cdot}9$.

$G \bumpeq T+D-0{\cdot}01374$ grain in air ($t=14{\cdot}95$, $b=772{\cdot}76$).

$G=I-0{\cdot}04488$ grain. $G \bumpeq W-0{\cdot}00209$ grain in air ($\log \Delta=7{\cdot}07832-10$).

G = lb. No. 36.

100 parts = 0·28057 grain.

D.	C.	F.	E.		Scale.		Scale.
				T+D+Y, G+R+X	23·57	G+R+X, T+D+Y	28·29
				....	19·09	....	23·11
7·4	7·35			....	18·51	....	22·09
		755·6	8	....	19·22	....	21·70
				....	17·69	....	22·11
7·55	7·5			....	17·90	....	21·05
					115·98		138·35

12(G+X+R) ≏ 12(T+D+X) − 22·37 parts.

D.	C.	F.	E.		Scale.		Scale.
				T+D+X, G+R+Y	16·91	G+R+Y, T+D+X	20·04
7·65	7·6			....	16·97	....	20·06
				....	17·64	....	20·52
		755·6	8·2	....	17·76	....	20·50
				....	16·61	....	20·45
7·8	7·75			....	16·54	....	20·10
					102·43		121·67

12(G+Y+R) ≏ 12(T+D+X) − 19·24 parts.

24(G+R) ≏ 24(T+D) − 41·61 parts, M = 7·5, F = 755·6, E = 8·1.

G ≏ T+D − 0·01618 grain in air (t = 7·48, b = 755·05).

G = I − 0·04480 grain.

G ≏ W − 0·00206 grain in air (log Δ = 7·07832 − 10).

Mean...G = I − 0·04484 grain. G ≏ W − 0·00207 grain in air (log Δ = 7·07832 − 10).

The 10-*lb. Weight.*

A weight of 10 lbs., or the weight of a gallon of water, protected by electro-gilding, was constructed by Mr. Oertling. Let D denote the sum of the secondary standards, Nos. 9, 11, 12, 13, 14, 16, 17*a*, 17*b*, 29, 30; E the sum of Nos. 18, 19, 21, 22, 23, 24, 25, 27, 28, together with a weight Pd, for which vPd = 822·792, Pd ≏ W + 0·00169 grain in air (log Δ = 7·07832 − 10); F the sum of Nos. 1, 2, 3, 4, 5, 6, 7, 8, 10, 26; T the weight of 10 lbs. The balance employed, made by Mr. Oertling and lent by him for the purpose of weighing T, has pans capable of being interchanged without taking out the weights contained in them. The weights of the pans will be denoted by X and Y. Each end of the beam carries a pointer, behind which is fixed an ivory scale with about 50 divisions to an inch. Every fifth division is marked by a longer line. The interval between two such consecutive lines is one part of the scale. The reading increases on placing a small weight in the left-hand pan. The index and scale were viewed through a telescope placed at a distance of about 12 feet from the scale. The absolute weight of water (t = 14·87) displaced by T was 8378·8 grains. Hence vT = 8379·4. vD = 8561·006, vE = 8538·336, vF = 8492·141. In air for which log Δ = 7·07832 − 10, D ≏ 10W − 0·06794 grain, E ≏ 10W + 0·02612 grain, F ≏ 10W + 0·08487 grain.

1 part=0·1355 grain.

	Scale.		Scale.
T+Y, D+0·06 gr.+X	10·27	D+0·06 gr.+X, T+Y	9·65
....	9·75	....	9·54
....	9·84	....	9·14
....	9·45	....	9·28
....	9·65	....	9·12
T+X, D+0·06 gr.+Y	8·95	D+0·06 gr.+Y, T+X	9·56
....	9·21	....	9·60
....	8·60	....	9·74
....	8·71	....	9·35
....	8·78	....	9·43
T+Y, D+X	10·51	D+X, T+Y	7·37
....	9·58	....	5·62
....	9·72	....	8·16
....	11·11	....	6·12
....	10·82	....	7·75
....	10·79	....	7·95
T+X, D+Y	10·49	D+Y, T+X	8·47
....	10·71	....	8·01
T+X, D+0·1 gr.+Y	10·10	D+0·1 gr.+Y, T+X	8·87
....	9·71	....	8·60
T+X, D+0·2 gr.+Y	9·06	D+0·2 gr.+Y, T+X	9·04
....	9·54	....	8·72
T+Y, D+0·1 gr.+X	10·91	D+0·01 gr.+X, T+Y	8·70
....	11·01	....	8·06
T+Y, D+0·2 gr.+X	10·17	D+0·2 gr.+X, T+Y	9·22
....	10·51	....	9·36
....	10·10	....	8·91
....	9·97	....	9·06
T+X, D+0·2 gr.+Y	10·65	D+0·2 gr.+Y, T+X	9·95
....	10·94	....	10·17
....	10·46	....	9·65
....	10·67	....	10·05
....	10·89	....	10·10
....	10·15	....	10·24
	341·78		302·56

$68T \bumpeq 68D + 6{\cdot}8$ grain $+ 39{\cdot}22$ parts.

$T \bumpeq D + 0{\cdot}1781$ grain in air ($t=16{\cdot}9$, $b=759{\cdot}4$).

$T \bumpeq 10W + 0{\cdot}1080$ grain in air ($\log \Delta = 7{\cdot}07832 - 10$).

	Scale.		Scale.
T+Y, E+X	11·45	E+X, T+Y	9·55
....	11·21	....	9·65
....	11·27	....	9·42
....	10·76	....	9·70
....	10·91	....	9·46
T+X, E+Y	10·96	E+Y, T+X	9·47
....	10·44	....	10·62
....	10·85	....	9·90
....	11·96	....	10·09
....	12·92	....	10·00
	112·73		97·86

$20T \bumpeq 20E + 14{\cdot}87$ parts.

$T \bumpeq E + 0{\cdot}1007$ grain in air ($t=17{\cdot}2$, $b=755{\cdot}0$).

$T \bumpeq 10W + 0{\cdot}1262$ grain in air ($\log \Delta = 7{\cdot}07832 - 10$).

	Scale.		Scale.
T+X, F+Y	10·41	F+Y, T+X	10·36
....	10·52	....	11·76
....	11·30	....	11·84
....	10·89	....	10·41
....	10·86	....	11·67
T+Y, F+X	10·96	F+X, T+Y	10·15
....	10·00	....	9·91
....	10·30	....	9·62
....	10·94	....	9·35
....	10·40	....	9·45
	106·58		104·52

$20\mathrm{T} \stackrel{\frown}{=} 20\mathrm{F} + 2{\cdot}06$ parts.

$\mathrm{T} \stackrel{\frown}{=} \mathrm{F} + 0{\cdot}0140$ grain in air ($t=17{\cdot}6$, $b=754{\cdot}9$).

$\mathrm{T} \stackrel{\frown}{=} 10\mathrm{W} + 0{\cdot}0986$ grain in air ($\log \Delta = 7{\cdot}07832-10$).

Mean......$\mathrm{T} \stackrel{\frown}{=} 10\mathrm{W} + 0{\cdot}111$ grain in air ($\log \Delta = 7{\cdot}07832-10$).

Hence the 10-lb. weight appears to be 0·111 grain heavier than ten commercial pounds of the density of the lost standard, weighed in air, thermometer 65°·67 F., barometer 29·78 inches, the mercury being reduced to the freezing-point ($t=18{\cdot}7$, $b=755{\cdot}64$) at Somerset House.

Exchequer Kilogramme.

A kilogramme of gun-metal protected by electro-gilding, was constructed by Mr. Oertling for the use of the Exchequer. The absolute weight of the water ($t=17{\cdot}29$) displaced by it was 1852·212 grains. Hence, denoting this kilogramme by $\mathfrak{K}$, $v\mathfrak{K}=1852{\cdot}82$, $\log v\mathfrak{K}=3{\cdot}267834$, $\Delta\mathfrak{K}=8{\cdot}32910$. Let $\mathfrak{A}$ denote the Kilogramme des Archives, $\mathfrak{C}$ the English kilogramme of platinum. $\mathfrak{L}$ the kilogramme type laiton, and let $\mathfrak{O}$ denote $\mathfrak{K}$ together with a bit of platinum wire, the weight of which was 1·40002 grain, by a mean of six comparisons with the following weights belonging to Barrow's balance:—1 gr.+0·4 gr., and 1 gr.+0·3 gr.+0·1 gr.

100 parts=0·68159 grain.

$\mathfrak{O}$+X, $\mathfrak{C}$+Y.		$\mathfrak{C}$+X, $\mathfrak{O}$+Y.	$\mathfrak{O}$+Y, $\mathfrak{C}$+X.		$\mathfrak{C}$+X, $\mathfrak{O}$+Y.
13·76		14·07	15·72		18·26
19·24		16·94	17·17		18·42
18·70	C= 14·7, D=14·65	18·05	15·12	C= 15·3, D=15·3	18·17
18·05		17·50	16·69		18·24
18·61		19·25	17·96		17·72
17·94		19·29	15·27		18·15
17·64	F=762·6, E=15·5	19·19	17·00	F=762·6, E=15·6	17·99
18·10		18·05	16·07		16·31
18·01		17·70	14·51		15·47
17·17	C= 15·1, D=15·1	18·54	14·45	C= 15·55, D=15·57	18·21
16·50		18·45	15·87		16·19
17·99		18·36	15·89		17·00
211·71		215·39	191·72		210·13

$48\mathfrak{O} \stackrel{\frown}{=} 48\mathfrak{C} + 22{\cdot}09$ parts.

$\mathfrak{K} + 1{\cdot}39687$grain $\stackrel{\frown}{=} \mathfrak{C}$ in air ($t=15{\cdot}17$, $b=761{\cdot}13$).

$\mathfrak{K}$ displaces 2·26684 grs. of air ; $\mathfrak{C}-1{\cdot}39687$ gr. of platinum displaces 0·89281 gr. of air. Therefore $\mathfrak{K}=\mathfrak{C}-0{\cdot}02284$ gr. But $\mathfrak{C}=\mathfrak{A}-0{\cdot}02412$ gr. Hence

$$\mathfrak{K}=\mathfrak{A}-0{\cdot}04696 \text{ grain.}$$

In air for which $\log \Delta = 7{\cdot}07832 - 10$, $\mathfrak{K}$ displaces 2·2211 grs., and $\mathfrak{A} - 1{\cdot}4$ gr. of platinum displaces 0·89981 gr. Therefore $\mathfrak{K} + 1{\cdot}3683 \text{ gr.} \bumpeq \mathfrak{A}$. But $\mathfrak{A} \bumpeq \mathfrak{L} + 1{\cdot}3673$ gr. Hence, when weighed in air of the above-mentioned density, $\mathfrak{K}$ appears to be 0·0010 gr. lighter than $\mathfrak{L}$.

Troy Ounce Weights.

A series of troy ounce weights from 500 ounces down to 0·001 ounce, of gun-metal protected by electro-gilding, was constructed by Messrs. LADD and STREATHFIELD for the use of the Exchequer. Each weight will be designated by the number expressing its approximate value in troy ounces, with a letter to distinguish it from other weights of nearly the same value. For the weights marked s, t, z $\log \Delta = 0{\cdot}92768$; for those marked m, n $\log \Delta = 0{\cdot}92501$. T denotes the platinum troy pound; U the lost standard troy pound, or 12 troy ounces. The comparisons of different combinations of these weights with T, and with one another, gave the following results:—

No. of Comp.		gr.	t.	b.
28	$4n+4t+2n+2t$	$\bumpeq T - 0{\cdot}02166$	18·6	754·6
24	$10n+\frac{1}{2}(2n+2t)$	$\bumpeq T + 0{\cdot}00078$	19·14	759·47
20	$10t+1s+1x$	$\bumpeq T + 0{\cdot}01370$	17·13	751·52
24	$5s+4t+1s+2t$	$\bumpeq T + 0{\cdot}00228$	17·78	760·1
48	$10s+1s+1x$	$\bumpeq T + 0{\cdot}00774$	17·9	755·68
32	$10m+2m$	$\bumpeq T + 0{\cdot}00060$	17·66	763·62
32	$4s+4m+2s+2m$	$\bumpeq T - 0{\cdot}00511$	16·5	763·44

In air for which $\log \Delta = 7{\cdot}07832 - 10$,

	gr.
$4n+4t+2n+2t$	$\bumpeq T - 0{\cdot}02133$
$10n+\frac{1}{2}(2n+2t)$	$\bumpeq T + 0{\cdot}00253$
$10t+1s+1x$	$\bumpeq T + 0{\cdot}01387$
$5s+4t+1s+2t$	$\bumpeq T + 0{\cdot}00688$
$10s+1s+1x$	$\bumpeq T + 0{\cdot}00925$
$10m+2m$	$\bumpeq T + 0{\cdot}00781$
$4s+4m+2s+2m$	$\bumpeq T + 0{\cdot}00639$
T	$\bumpeq U - 0{\cdot}00745$

Since these weights, the densities of which do not differ much from that of the lost standard U, are intended for use as commercial weights without applying any correction for the weight of the air displaced, $=$ may be substituted for $\bumpeq$ in expressing their values in terms of U in air for which $\log \Delta = 7{\cdot}07832 - 10$.

	oz. gr.	
$4n+4t+2n+2t$	$= 12 - 0{\cdot}02878$	1
$10n+\frac{1}{2}(2n+2t)$	$= 12 - 0{\cdot}00492$	2
$10t+1s+1x$	$= 12 + 0{\cdot}00642$	3
$5s+4t+1s+2t$	$= 12 - 0{\cdot}00057$	4
$10s+1s+1x$	$= 12 + 0{\cdot}00180$	5
$10m+2m$	$= 12 + 0{\cdot}00036$	6
$4s+4m+2s+2m$	$= 12 - 0{\cdot}00106$	7

No. of Comp.			gr.	
10	$4t$	$=2t+2n$	$+0{\cdot}01685$	8
10	$4n$	$=2t+2n$	$-0{\cdot}00090$	9
26	$2t$	$=2n$	$+0{\cdot}01121$	10
24	$10n$	$=5s+5m$	$+0{\cdot}00188$	11
16	$5s$	$=5m$	$+0{\cdot}00041$	12
10	$1s$	$=1m$	$+0{\cdot}00046$	13

50	$1x$	$=1s$	$+0{\cdot}00074$	14
12	$4t+1s$	$=5s$	$+0{\cdot}00224$	15
12	$10t$	$=5s+4t+1s$	$+0{\cdot}00270$	16
12	$4t$	$=2t+1s+1x$	$+0{\cdot}00217$	17
28	$1s+1x$	$=2t$	$+0{\cdot}00322$	18
8	$3s$	$=2t+1x$	$+0{\cdot}00125$	19
16	$3m$	$=2m+1s$	$+0{\cdot}00030$	20
10	$1s$	$=1z$	$+0{\cdot}00017$	21
20	$4m$	$=2s+2m$	$+0{\cdot}00059$	22
20	$4s$	$=2s+2m$	$+0{\cdot}00059$	23
36	$2m$	$=2s$	$+0{\cdot}00032$	24
20	$1s+1x$	$=2(0{\cdot}5s+0{\cdot}5m)$	$+0{\cdot}00040$	25
20	$0{\cdot}5s$	$=0{\cdot}5m$	$+0{\cdot}00001$	26
6	$0{\cdot}4m+0{\cdot}4s$	$=2(0{\cdot}2n+0{\cdot}2s)$	$+0{\cdot}00013$	27
10	$1s$	$=0{\cdot}4s+0{\cdot}4m+0{\cdot}2s$	$+0{\cdot}00016$	28
16	$0{\cdot}2s$	$=0{\cdot}2n$	$+0{\cdot}00049$	29
10	$0{\cdot}4s$	$=0{\cdot}4m$	$+0{\cdot}00029$	30
10	$0{\cdot}2s$	$=0{\cdot}1s+0{\cdot}1m$	$+0{\cdot}00024$	32
10	$0{\cdot}1m$	$=0{\cdot}1s$	$+0{\cdot}00070$	33
6	$0{\cdot}3s+0{\cdot}3m$	$=2\times 0{\cdot}2s+0{\cdot}1s+0{\cdot}1m$	$+0{\cdot}00045$	34
6	$0{\cdot}1s+0{\cdot}1m$	$=2(0{\cdot}05s+0{\cdot}05m)$	$+0{\cdot}00019$	35
10	$0{\cdot}05m$	$=0{\cdot}05s$	$+0{\cdot}00005$	36
6	$0{\cdot}05s$	$=0{\cdot}025s+0{\cdot}025m$	$-0{\cdot}00013$	37
6	$0{\cdot}025m$	$=0{\cdot}025s$	$+0{\cdot}00018$	38
4	$0{\cdot}1s$	$=0{\cdot}04s+0{\cdot}04m+\frac{1}{2}(0{\cdot}02s+0{\cdot}02m)$	$+0{\cdot}00020$	39
	$0{\cdot}02s$	$=0{\cdot}02m$	$+0{\cdot}00022$	40
	$0{\cdot}04s$	$=0{\cdot}02s+0{\cdot}02m$	$+0{\cdot}00017$	41
	$0{\cdot}04m$	$=0{\cdot}04s$	$+0{\cdot}00010$	42
4	$0{\cdot}02s$	$=0{\cdot}01s+0{\cdot}01m$	$-0{\cdot}00051$	43
4	$0{\cdot}01s$	$=0{\cdot}01m$	$+0{\cdot}00038$	44
8	$0{\cdot}03s$	$=0{\cdot}02s+0{\cdot}01s$	$+0{\cdot}00054$	45
20	$0{\cdot}01s$	$=0{\cdot}004s+0{\cdot}004m+\frac{1}{2}(0{\cdot}002s+0{\cdot}002m)$	$+0{\cdot}00030$	46
10	$0{\cdot}004s$	$=0{\cdot}002s+0{\cdot}002m$	$-0{\cdot}00041$	47
10	$0{\cdot}004m$	$=0{\cdot}002s+0{\cdot}002m$	$-0{\cdot}00032$	48
6	$0{\cdot}002s$	$=0{\cdot}001s+0{\cdot}001n$	$-0{\cdot}00004$	49
6	$0{\cdot}001m$	$=0{\cdot}001s$	$+0{\cdot}00003$	50
4	$0{\cdot}003s$	$=0{\cdot}002s+\frac{1}{2}(0{\cdot}001s+0{\cdot}001n)$	$+0{\cdot}00007$	51
8	$0{\cdot}005s$	$=0{\cdot}004s+\frac{1}{2}(0{\cdot}001s+0{\cdot}001n)$	$-0{\cdot}00068$	52

		oz.	gr.	
(3), (14), (15), (16), (17), (18)	$2t$	$=2$	$-0{\cdot}00175$	
(4), (14), (15), (17), (18)	$2t$	$=2$	$-0{\cdot}00193$	
(1), (8), (9), (10)	$2t$	$=2$	$-0{\cdot}00185$	
Mean	$2t$	$=2$	$-0{\cdot}00184$	53
(18), (53)	$1s+1x$	$=2$	$+0{\cdot}00138$	54
(5), (54)	$10s$	$=10$	$+0{\cdot}00042$	55
(14), (54)	$1s$	$=1$	$+0{\cdot}00032$	56
(7), (22), (23), (24)	$2m$	$=2$	$-0{\cdot}00022$	57
	$2s$	$=2$	$-0{\cdot}00053$	58
	$4m$	$=4$	$-0{\cdot}00016$	59
	$4s$	$=4$	$-0{\cdot}00016$	60
(6), (60)	$10m$	$=10$	$+0{\cdot}00058$	61
(10), (53)	$2n$	$=2$	$-0{\cdot}01305$	62
(53), (62), (2)	$10n$	$=10$	$+0{\cdot}00253$	63
(14), (53)	$1x$	$=1$	$+0{\cdot}00106$	64
(3), (54)	$10t$	$=10$	$+0{\cdot}00504$	65
(63), (11), (12)	$5s$	$=5$	$+0{\cdot}00053$	
(65), (15), (16)	$5s$	$=5$	$+0{\cdot}00005$	
(4), (15), (53)	$5s$	$=5$	$-0{\cdot}00048$	
Mean	$5s$	$=5$	$+0{\cdot}00003$	66
(12), (66)	$5m$	$=5$	$-0{\cdot}00038$	67
(20), (56), (57)	$3m$	$=3$	$+0{\cdot}00040$	68
(19), (53), (64)	$3s$	$=3$	$+0{\cdot}00047$	69
(21), (56)	$1z$	$=1$	$+0{\cdot}00015$	70
(25), (26), (54)	$0{\cdot}5s$	$=0{\cdot}5$	$+0{\cdot}00013$	71

(56), (27), (28), (29), (30)	0·4s	=	0·4	+0·00013	72
	0·2s	=	0·2	+0·00018	73
(32), (33), (73)	0·1m	=	0·1	+0·00032	74
	0·1s	=	0·1	−0·00038	75
(45), (73), (75)	0·3s	=	0·3	+0·00037	76
(74), (75), (35), (36)	0·05s	=	0·05	−0·00008	77
(77), (37), (38)	0·025s	=	0·025	−0·00007	78
(39), (75), (40), (41)	0·02s	=	0·02	−0·00066	79
	0·04s	=	0·04	+0·00015	80
(79), (43), (44)	0·01s	=	0·01	+0·00012	81
(45), (79), (81)	0·03s	=	0·03	+0·00000	82
(81), (46), (47), (48)	0·004s	=	0·004	−0·00019	83
	0·002s	=	0·002	+0·00018	84
(84), (49), (50)	0·001s	=	0·001	+0·00012	85
	0·001n	=	0·001	+0·00009	86
(51), (84), (85), (86)	0·003s	=	0·003	+0·00036	
(52), (83), (85), (86)	0·005s	=	0·005	−0·00076	

By a mean of 12 comparisons $20s=10s+10m-0{\cdot}00386$ gr. $=20$ ounces$-0{\cdot}00286$ gr.
By a mean of 12 comparisons $30s=20s+10m+0{\cdot}00361$ gr. $=30$ ounces$+0{\cdot}00133$ gr.

Let W denote a commercial lb. of the same density as the lost standard troy pound. In air for which log $\Delta=7{\cdot}07832-10$, the lb. Pd appears to be equal to W$+0{\cdot}00169$ gr. Let (1000), [1000] denote two brass weights of nearly 1000 grains each; (2000), (4000) two brass weights of nearly 2000 and 4000 grains each respectively. Then.

No. of Comp.			gr.
12	$(4000)+(2000)+\frac{1}{2}\{(1000)+[1000]\}$	$=$Pd	−0·00582
12	(4000)	$=(2000)+(1000)+[1000]$	−0·00550
10	(2000)	$=(1000)+[1000]$	+0·00154
4	(1000)	$=[1000]$	+0·01360

Hence

		gr.
[1000]	=	999·99296
(1000)	=	1000·00656
(2000)	=	2000·00105
(4000)	=	3999·99506

The bronze weight (200) $=199{\cdot}9971$ grains (in air).

By a mean of 28 comparisons, $40s=$ sum of secondary standard lbs. Nos. 9 and 11 $+(4000)+(1000)+(200)-0{\cdot}0171$ grain $=40$ ounces$-0{\cdot}0298$ grain.

By a mean of 40 comparisons, $50s=$ sum of secondary standard lbs. Nos. 9, 11, 12 $+(2000)+(1000)+0{\cdot}0073$ grain $=50$ ounces$+0{\cdot}0054$ grain.

By a mean of 60 comparisons, $100s=$ sum of secondary standard lbs. Nos. 9, 11, 12, 13, 14, 16$+(4000)+(2000)+0{\cdot}0701$ grain $=100$ ounces$+0{\cdot}0193$ grain.

By a mean of 68 comparisons, $200s=$ sum of secondary standard lbs. Nos. 9, 11, 12, 13, 14, 16, 17a, 17b, 29, 30, 19, 21, 28$+(4000)+(1000)+0{\cdot}023$ grain $=200$ ounces $-0{\cdot}022$ grain.

By a mean of 26 comparisons, $300s=200s+100s+0{\cdot}0304$ gr. $=300$ oz.$+0{\cdot}027$ gr.

By a mean of 20 comparisons, $400s=300s+100s-0{\cdot}046$ gr. $=400$ oz.$+0{\cdot}000$ gr.

By a mean of 20 comparisons, $500s=300s+200s-0{\cdot}059$ gr. $=500$ oz.$-0{\cdot}054$ gr.

The troy ounce weights marked s are deposited in the Exchequer.

The weights (12800), (6400), (3200), (1600), (800), (400), (200), (100), and a weight of 100 grains unmarked, which accompany BARROW's balance, are of bronze, for which $\log\Delta=0\cdot92260$. Let them be denoted by W, V, U, T, S, R, Q, P, O respectively. It was found that

No. of Comp.		gr.
4	P = O	−0·0001
4	Q = P+O	−0·0037
4	R = Q+P+O	+0·0108
4	S = R+Q+P+O	−0·0009
4	T = S+R+Q+P+O	−0·0155
4	U = T+S+R+Q+P+O	−0·0207
4	V = U+T+S+R+Q+P+O	−0·0030
4	W = V+U+T+S+R+Q+P+O	−0·0516

Hence

	gr.
P =	O−0·0001
Q =	2O−0·0038
R =	4O+0·0069
S =	8O+0·0021
T =	16O−0·0104
U =	32O−0·0260
V =	64O−0·0343
W =	128O−0·1172

By a mean of 4 comparisons, the bronze weights U+T+S+P+platinum weights (32)+(16)+(8)+(4)+0·0133 gr. ≏ platinum troy pound T in air ($t=10\cdot45, b=758\cdot28$). Hence U+T+S+P=5699·9724 grains.

U+T+S+P=57O−0·0344 grain. Therefore 57O=5700·0068 grains. Hence

	gr.		gr.
(100) =	100·0000	(1600) =	1599·9915
(200) =	199·9964	(3200) =	3199·9778
(400) =	400·0074	(6400) =	6399·9733
(800) =	800·0031	(12800) =	12799·8981

The values assigned to these weights in computing the densities of some of the old troy pounds and secondary standard lbs., were obtained upon the supposition that the absolute weight of the platinum troy pound T was about 0·002 grain less than it afterwards appeared to be on reducing the observations with the weight of air as determined by REGNAULT. The error, which was not discovered till after going to press, is much too small to affect the last figure of the calculated densities.

Probable Errors of the Platinum Weights.

Probable errors of the comparisons of T, and of the auxiliary weights.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison.	Probable error of n comparisons.
			gr.	gr.
815	T, Sp	168	0·000982	0·000076
818	T, RS	78	0·000727	0·000083
824	T, A+B+C+D+F	40	0·000540	0·000086
825	A, F+G	22	0·000287	0·000061
825	B, F+G	22	0·000560	0·000123
826	C, F+G	22	0·000521	0·000114
826	D, F+G	22	0·000669	0·000146
826	F, G+H	31	0·000458	0·000084
821	G, H+K	10	0·000170	0·000057
821	G, H+L	10	0·000154	0·000051

822	G, H+M	10	0·000159	0·000053
822	G, H+N	10	0·000111	0·000037
822	H, K+L+M+N+R	17	0·000298	0·000073
822	H, K+L+M+N+S	17	0·000241	0·000060
822	K, R+S	12	0·000134	0·000040
822	L, R+S	12	0·000178	0·000054
823	M, R+S	12	0·000139	0·000042
823	N, R+S	12	0·000122	0·000037
828	R, Y+Z	6	0·000072	0·000032
828	S, Y+Z	6	0·000063	0·000028
828	Y, W+V+Q	20	0·000097	0·000022
828	Z, W+V+Q	20	0·000046	0·000034
829	W, V+10Q	6	0·000053	0·000024
829	V, 10Q	6	0·000119	0·000053

According to the 300 comparisons of U, the lost troy pound, with Sp, and the 140 comparisons with RS, made in 1829, RS=Sp+0·00515 grain. According to the comparisons of T with Sp and RS in 1845, RS=Sp+0·00534 gr., a result which differs from the former by only 0·00019 gr. Giving to the comparisons of U and T with Sp twice the weight of their comparisons with RS, because the number of comparisons of U and T with Sp is about twice as great as the number of comparisons with RS, the probable error of T, in terms of Sp and RS, will be 0·000058 gr. The probable error of $\frac{1}{4}$(A+B+C+D) is 0·000026 gr., and that of Q is 0·000004 gr. Hence the probable error of T+Q+$\frac{1}{4}$(A+B+C+D) is 0·000064 gr.

Probable errors of the comparisons of I with T and the auxiliary weights.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison.	Probable error of n comparisons.
			gr.	gr.
858	I, T+Q+A	40	0·000569	0·000091
859	I, T+Q+B	40	0·000504	0·000081
859	I, T+Q+C	40	0·000648	0·000104
860	I, T+Q+D	50	0·000618	0·000083

Hence the probable error of the comparison of I with T+Q+$\frac{1}{4}$(A+B+C+D) is 0·000038 gr. But the probable error of T+Q+$\frac{1}{4}$(A+B+C+D) is 0·000064 gr. Therefore the probable error of I is 0·000074 grain. If we substitute for A, B, C, D their values in terms of F+G, the resulting values of I will be affected by the errors of the comparisons of A, B, C, D with F+G, combined with the errors of the comparisons of I with T+Q+A, T+Q+B, T+Q+C, T+Q+D. The differences of these values of I from the mean are −0·00097 grain, +0·00008 grain, +0·00011 grain, +0·00078 grain respectively.

Probable errors of the comparisons of I with K, L, M, N, the platinum lbs. Nos. 1, 2, 3, 4, and with Sp+V, Professor SCHUMACHER's lb.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison.	Probable error of n comparisons.
			gr.	gr.
862	I, K	100	0·000514	0·000052
864	I, L	108	0·000659	0·000064
866	I, M	102	0·000485	0·000048
868	I, N	102	0·000555	0·000055
872	I, Sp+V	100	0·000495	0·000050

Probable errors of the weights (64), (32), (16), (8), (4), (2), J.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison.	Probable error of n comparisons.
			gr.	gr.
829	$(64)+16, \frac{1}{3}(K+L+M+N+R+S)$	10	0·000150	0·000050
830	$(64), (32)+(16)+(8)+(4)+(2)+2J$	5	0·000144	0·000072
830	$(32), (16)+(8)+(4)+(2)+2J$	5	0·000095	0·000047
830	$(16), (8)+(4)+(2)+2J$	5	0·000082	0·000041
830	$(8), (4)+(2)+2J$	5	0·000036	0·000018
830	$(4), (2)+2J$	5	0·000031	0·000016
830	$(2), 2J$	5	0·000141	0·000070

The probable error of K+L+M+N+R+S is 0·000019 grain, and the probable error of J is 0·000015 grain.

Probable errors of the comparisons of $\mathfrak{A}$, the Kilogramme des Archives, with K+L+B, and with the platinum kilogramme $\mathfrak{C}$, and of the comparisons of $\mathfrak{C}$ with I+K+A, I+L+B, I+M+Γ, I+N+Δ.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison.	Probable error of n comparisons.
			gr.	gr.
891	$\mathfrak{A}$, K+L+B	60	0·001769	0·000230
882	$\mathfrak{A}$, $\mathfrak{C}$	200	0·001594	0·000113
886	$\mathfrak{C}$, I+K+A	34	0·001296	0·000226
886	$\mathfrak{C}$, I+L+B	40	0·000790	0·000127
887	$\mathfrak{C}$, I+M+Γ	40	0·001333	0·000213
888	$\mathfrak{C}$, I+N+Δ	42	0·000987	0·000154

Probable errors of the auxiliary weights used in obtaining the value of $\mathfrak{C}$ in terms of I.

Page.	Weights compared.	No. of comparisons (n).	Probable error of one comparison.	Probable error of n comparisons.
			gr.	gr.
889	I, A+B+Γ+Δ	24	0·000469	0·000097
889	A, Z+Θ	10	0·000373	0·000124
889	B, Z+Θ	10	0·000510	0·000170
890	Γ, Z+Θ	10	0·000372	0·000124
890	Δ, Z+Θ	10	0·000467	0·000146

Θ = sum of weights L, M of nearly 80 grains each, J, the mean of two grain weights, and Q, the mean of the ten weights of 0·64509 grain, the probable errors of which are 0·000083 grain, 0·000066 grain, 0·000015 grain, 0·000004 grain respectively. Therefore the probable error of Θ is 0·000107 grain. Hence the probable error of $\mathfrak{C}$ in terms of I is 0·000193 grain. But the probable error of the comparison of $\mathfrak{C}$ with $\mathfrak{A}$ is 0·000113 grain. Therefore the probable error of the value of $\mathfrak{A}$ in grains of which I contains 7000·00000, is 0·000224 grain.

The four different values of $\mathfrak{C}$ obtained by substituting for K, L, M, N their values in terms of I, and for A, B, Γ, Δ their values in terms of Z+Θ, are affected by the errors of the comparisons of $\mathfrak{C}$ with I+K+A, I+L+B, I+M+Γ, I+N+Δ, combined with the errors of the comparisons of A, B, Γ, Δ with Z+Θ. Their differences from the mean are +0·00176 grain, +0·00026 grain, −0·00105 grain, −0·00098 grain respectively.

The probable error of the value of $\mathfrak{A}$ in grains of which I contains 7000·00000, obtained by comparing it with K+L+B, is 0·000334 grain.

The differences between two or more series of comparisons of the same weights, though small, are larger than the probable error of each series would lead us to expect. Of the errors which affect the results of weighing, some partake too much of the nature of constant errors to be fairly estimated by the method of least squares. Of this kind is the error due to small differences of temperature of the weights. Whenever it was practicable, the weights to be compared were left in the balance-case during the night previous to the day on which they were compared. This precaution, however, was in some measure defeated, when a single weight was compared with the sum of several others; for the latter would be in advance of the former in following the changes of temperature during the time occupied by the comparisons. The effect of temperature on the apparent weight of any object appears to be due to currents of air ascending or descending, according as the weight is hotter or colder than the air in the balance-case. A brass kilogramme that had been left for several hours in the balance-case where the temperature was 5°·2 C., appeared to be about 5 milligrammes lighter after it had been heated up to 16°·4 C. The hygroscopic matter contained in some of the auxiliary weights, from which it was difficult to free them entirely by digestion in boiling water, may also have introduced a small error in one direction. In order to diminish, as much as possible, any inaccuracy resulting from this cause, all the more important weighings into which these weights entered, were made within the narrowest practicable limits of time. Many observations that could not be brought within such limits of time as were considered satisfactory, were rejected, the chance of a larger irregular error belonging to a small number of comparisons being considered less injurious to accuracy than the error in one direction to be apprehended in a larger number, extending over a considerable interval of time. That part of the weighing which depended upon the performance of the balances was most satisfactory. When the large balance, constructed by Mr. Barrow, was loaded with a pound in each pan, the probable error of a single comparison, by Gauss's method, was 0·00056 grain, or less than one-12 millionth of the weight in either pan; with a kilogramme in each pan, the probable error of a single comparison, by Borda's method, was 0·00162 grain, or less than one-9 millionth part of the weight in either pan; by Gauss's method it was 0·00112 grain, or one-14 millionth of the weight in either pan.

Legalization of the new Standards.

Legal authority has been given to the new Standard lb. and its four copies in platinum by an Act of Parliament, entitled "An Act for legalizing and preserving the restored Standards of Weights and Measures." The most important of those provisions of the Act which relate exclusively to the Standards of Weight, are contained in the following extracts:—

"Whereas by an Act of the Fifth Year of the Reign of King George the Fourth, Chapter Seventy-four, ... it was enacted ... that from and after the First Day of May

1825, the Standard Brass Weight of One Pound Troy Weight made in the Year 1758, then in the Custody of the Clerk of the House of Commons, should be and the same was thereby declared to be the original and genuine Standard Measure of Weight, and that such Brass Weight should be the 'Imperial Standard Troy Pound,' and should be and the same was declared to be the Unit or only Standard Measure of Weight from which all other Weights should be derived, computed, and ascertained, and that $\frac{1}{12}$ of the said Troy Pound should be an Ounce, and that $\frac{1}{20}$ of such Ounce should be a Pennyweight, and that $\frac{1}{24}$ of such Pennyweight should be a Grain, so that 5760 such Grains should be a Troy Pound, and that 7000 such Grains should be and they were thereby declared to be a Pound Avoirdupois: And whereas by the said Act Provision was made for restoring the said Imperial Troy Pound, in case of Loss, Destruction, Defacement, or other Injury, by Reference to the Weight of a Cubic Inch of Water: And whereas the said Standard Pound Troy (was) destroyed in the Fire at the Houses of Parliament: And whereas by the Researches of Scientific Men Doubts were thrown on the Accuracy of the Methods provided by the said Act for the Restoration of the said Standard: And whereas there exist Weights which had been accurately compared with the said Standard Pound Troy, which afforded sufficient Means for restoring such original Standard: And it having been deemed expedient that the Standard for Reference as a Measure of Weight should be a Pound Avoirdupois, there has been constructed a Pound Weight Avoirdupois equivalent to the Pound Avoirdupois of 7000 such Grains as are mentioned in the said recited Act, and Four accurate Copies of the said Pound Avoirdupois so constructed: And whereas the Standard Pound Avoirdupois so constructed as aforesaid, and the Copies thereof, are of Platinum, the Form being that of a Cylinder nearly 1·35 Inch in Height and 1·15 Inch in Diameter, with a Groove or Channel round it whose Middle is about 0·34 Inch below the Top of the Cylinder, for insertion of the Points of the Ivory Fork by which it is to be lifted; the Edges are carefully rounded off: And whereas the said Standard of Weight marked P.S. 1844, 1 lb. has been deposited in the Office of the Exchequer at *Westminster*, and One of the said Copies of the Standard of Weight marked No. 1. P.C. 1844, 1 lb. has been deposited at the Royal Mint; and One other of the said Copies of the Standard of Weight marked No. 2. P.C. 1844, 1 lb. has been delivered to the Royal Society of *London*; and One other of the said Copies of the Standard of Weight marked No. 3. P.C. 1844, 1 lb. has been deposited in the Royal Observatory of *Greenwich*; and the other of the said Copies of the Standard of Weight marked No. 4. P.C. 1844, 1 lb. has been immured in the Cill of the Recess on the East Side of the lower Waiting Hall in the New Palace at *Westminster*: And whereas it is expedient to legalize the Standards so constructed and to provide for the Preservation thereof: Be it therefore enacted...as follows:—

I. So much of the said Act of the Fifth Year of King George the Fourth as relates to the Restoration of the Standard Troy Pound, in case of Loss, Destruction, Defacement, or other Injury, shall be repealed.

III. The said Weight of Platinum marked P.S. 1844, 1 lb., deposited in the Office of the Exchequer as aforesaid, shall be the legal and genuine Standard Measure of Weight, and shall be and be denominated the Imperial Standard Pound Avoirdupois, and shall be deemed to be the only Standard Measure of Weight from which all other Weights and other Measures having Reference to Weight shall be derived, computed, and ascertained, and One equal Seven Thousandth Part of such Pound Avoirdupois shall be a Grain, and Five Thousand seven hundred and sixty such Grains shall be and be deemed to be a Pound Troy.

VII. If at any Time hereafter the said Imperial Standard Pound Avoirdupois be lost, or in any Manner destroyed, defaced, or otherwise injured, the Commissioners of Her Majesty's Treasury may cause the same to be restored by Reference to or Adoption of any of the Copies so deposited as aforesaid, or such of them as may remain available for that Purpose."

Densities, Errors, and Distribution of the Standards of Weight.

The first column of the following Table contains the mark or designation of the weight; the second, its density at the temperature of melting snow (with one exception) in terms of the maximum density of water; the third, its error, in grains, when compared in a vacuum with PS, the new Imperial Standard lb.; the fourth, its error, in grains, when compared with W, the Commercial Standard lb., in air of the temperature 65°·55 FAHR., under the pressure of 29·750 inches of mercury at the temperature of melting snow ($t=18°\cdot7$ C., $b=\overset{mm}{755\cdot64}$), in Somerset House, or in air for which $\log\Delta=7\cdot07832-10$. The last column contains the name of the place of deposit, or of the country to which the weight has been sent.

Designation.	Density.	In a vacuum.	In air for which $\log\Delta=7\cdot07832-10$.	Distribution.
		gr.	gr.	
P.S.	21·1572	0·00000	0·63407 too heavy.	Exchequer.
No. 1. P.C.	21·1671	0·00051 too heavy.	0·63477 too heavy.	Royal Mint.
No. 2. P.C.	21·1640	0·00089 too light.	0·63331 too heavy.	Royal Society.
No. 3. P.C.	21·1615	0·00178 too light.	0·63237 too heavy.	Royal Observatory, Greenwich.
No. 4. P.C.	21·1516	0·00314 too light.	0·63090 too heavy.	New Palace, Westminster.
Sp.+V.	21·1321	0·00023 too light.	0·63336 too heavy.	Observatory, Altona.
Troy Pd. T.	21·1661	0·52934 too light.	0·00745 too light.	Royal Observatory, Greenwich.
Gilt lb. No. 1.	8·36134	0·00732 too light.	0·01956 too heavy.	India.
...... No. 2.	8·34161	0·03582 too light.	0·01132 too light.	Russia.
...... No. 3.	8·30462	0·00510 too heavy.	0·02512 too heavy.	Prussia.
...... No. 4.	8·36500	0·00425 too heavy.	0·03157 too heavy.	Bavaria.
...... No. 5.	8·06122	0·01783 too heavy.	0·00734 too heavy.	U.S. America.
...... No. 6.	8·28779	0·01714 too light.	0·00083 too heavy.	Edinburgh.
...... No. 7.	8·12163	0·01933 too heavy.	0·01658 too heavy.	Austria.
...... No. 8.	8·16317	0·01428 too heavy.	0·01679 too heavy.	Dublin.
...... No. 9.	7·37614	0·11611 too heavy.	0·00426 too heavy.	Canada.
...... No. 10.	8·28375	0·03910 too light.	0·02162 too light.	Cape of Good Hope.
...... No. 11.	8·36302	0·04208 too light.	0·01499 too light.	Sydney.
...... No. 12.	8·31919	0·02060 too light.	0·00118 too heavy.	Portugal.
...... No. 13.	8·43179	0·03331 too light.	0·00195 too heavy.	Spain.
...... No. 14.	8·34955	0·02844 too light.	0·00301 too light.	Holland.

Gilt lb. No. 15.	8·36107	0·02022 too light.	0·00667 too heavy.	France.
...... No. 16.	8·07354	0·02747 too light.	0·03640 too heavy.	Belgium.
...... No. 17*a*.	8·11718	0·02614 too light.	0·02948 too light.	Hanover.
...... No. 17*b*.	8·55888	0·04428 too light.	0·00542 too heavy.	Saxony.
...... No. 18.	8·30369	0·00129 too light.	0·01857 too heavy.	Sweden.
...... No. 19.	8·33969	0·01473 too light.	0·00950 too heavy.	Denmark.
...... No. 21.	7·97370	0·03971 too heavy.	0·01777 too heavy.	Switzerland.
...... No. 22.	8·19859	0·01214 too light.	0·00523 too light.	Sardinian States.
...... No. 23.	8·15141	0·01557 too heavy.	0·01655 too heavy.	Papal States.
...... No. 24.	8·14286	0·03932 too light.	0·03941 too light.	Naples.
...... No. 25.	8·10164	0·00180 too heavy.	0·00354 too light.	Turkey.
...... No. 26.	8·15218	0·00112 too light.	0·00001 too light.	Brazil.
...... No. 27.	8·16186	0·01405 too heavy.	0·01635 too heavy.	Buenos Ayres.
...... No. 28.	8·12604	0·00416 too light.	0·00638 too light.	Chili.
...... No. 29.	8·18446	0·00222 too light.	0·00293 too heavy.	Melbourne.
...... No. 30.	8·15292	0·00170 too light.	0·00050 too light.	Hobarton.
...... No. 31.	8·51444	0·04410 too light.	0·00080 too heavy.	Royal Observatory, Greenwich. Still disposable.
...... No. 32.	8·47042	0·03448 too light.	0·00304 too heavy.	
...... No. 33.	8·47902	0·04144 too light.	0·00063 too light.	
...... No. 34.	8·51472	0·04584 too light.	0·00089 too light.	
...... No. 35.	8·47019	0·04019 too light.	0·00041 too light.	Exchequer.
...... No. 36.	8·49601	0·04484 too light.	0·00207 too light.	R. Obs., Gr. Still disposable.
Quartz lb. (at 18° C.)	2·649009	2·36797 too heavy.	0·40147 too light.	Royal Observatory, Greenwich.
Platinum Kilogr. 𝕮.	21·13791	0·02412 too light.	....	Royal Observatory, Greenwich.
Gilt Kilogramme 𝕶.	8·32910	0·04696 too light.	0·0010 too light.	Exchequer.
10-lb. weight.	8·35382	....	0·111 too heavy.	Exchequer.
Auxiliary weights.	....	....	....	Royal Observatory, Greenwich.

Troy ounce weights deposited in the Office of the Exchequer.

Troy ounces.	gr.		Troy ounces.	gr.	
500	0·054	too light.	0·5	0·00013	too heavy.
400	0·000		0·4	0·00013	too heavy.
300	0·027	too heavy.	0·3	0·00037	too heavy.
200	0·022	too light.	0·2	0·00018	too heavy.
100	0·0193	too heavy.	0·1	0·00038	too light.
50	0·0054	too heavy.	0·05	0·00008	too light.
40	0·0298	too light.	0·04	0·00015	too heavy.
30	0·00138	too heavy.	0·03	0·00000	
20	0·00286	too light.	0·025	0·00007	too light.
10	0·00042	too heavy.	0·02	0·00066	too light.
5	0·00003	too heavy.	0·01	0·00012	too heavy.
4	0·00016	too light.	0·005	0·00076	too light.
3	0·00047	too heavy.	0·004	0·00019	too light.
2	0·00053	too light.	0·003	0·00036	too heavy.
1	0·00032	too heavy.	0·002	0·00018	too heavy.
oz.	0·00015	too heavy.	0·001	0·00012	too heavy.